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Inter Relations Between Fuzzy Door Space and Some Fuzzy Topological Spaces

Research Article*

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Abstract: In this paper the concept of fuzzy Door space are studied. Several relations between fuzzy Door space and some fuzzy topological spaces are investigated with suitable examples in this paper.

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1. Introduction

The theory of fuzzy sets was initiated by L.A.Zadeh in his classical paper [14] in the year 1965 as an attempt to develop a mathematically precise framework in which to treat systems or phenomena which cannot themselves be characterized precisely. The potential of fuzzy notion was realized by the researchers and has successfully been applied for investigations in all the branches of Science and Technology. The paper of C.L.Chang [5] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. The concept of Door spaces have been studied in classical topology in McCartan in [6]. In this paper, the concept of Door spaces in fuzzy setting and investigate several characterizations of fuzzy Door spaces and the relations of fuzzy Door spaces and some fuzzy topological spaces are studied.

2. Preliminaries

Now we introduce some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang [5].

Definition 2.1 ([3]). *Let λ and μ be any two fuzzy sets in a fuzzy topological space (X, T). Then we define:*

- $\lambda \vee \mu : X \to [0,1]$ *as follows:* $(\lambda \vee \mu)(x) = max\{\lambda(x), \mu(x)\}$;
- $\lambda \wedge \mu : X \to [0,1]$ *as follows:* $(\lambda \wedge \mu)(x) = min\{\lambda(x), \mu(x)\}$;

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- $\mu = \lambda^c \Leftrightarrow \mu(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i / i \in I\}$ *of fuzzy sets in* (X, T)*, the union* $\psi = \vee_i \lambda_i$ *and intersection* $\delta = \wedge_i \lambda_i$ *are defined respectively as* $\psi(x) = sup_i\{\lambda_i(x), x \in X\}$ *and* $\delta(x) = inf_i\{\lambda_i(x), x \in X\}$.

Definition 2.2 ([3]). *Let* (X, T) *be a fuzzy topological space. For a fuzzy set* λ *of* X*, the interior and the closure of* λ *are defined respectively as* $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$ *and* $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

Definition 2.3 ([12]). *A fuzzy set* λ *in a fuzzy topological space* (X, T) *is called a fuzzy* G_δ*-set in* (X, T) *if* $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$*, where* $\lambda_i \in T$ *for* $i \in I$.

Definition 2.4 ([13]). *A fuzzy set* λ *in a fuzzy topological space* (X, T) *is called* **fuzzy dense** *if there exists no fuzzy closed set* μ *in* (X, T) *such that* $\lambda < \mu < 1$ *That is* $cl(\lambda) = 1$.

Definition 2.5 ([13]). *A fuzzy set* λ *in a fuzzy topological space* (X, T) *is called* **fuzzy nowhere dense** *if there exists no non-zero fuzzy open set* μ *in* (X, T) *such that* $\mu < cl(\lambda)$ *That is,* $int\ cl(\lambda) = 0$.

Definition 2.6 ([13]). *Let* (X, T) *be a fuzzy topological space. A fuzzy set* λ *in* (X, T) *is called* **fuzzy first category set** *if* $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$*, where* λ_i*'s are fuzzy nowhere dense sets in* (X, T)*. A fuzzy set which is not fuzzy first category set is called a* **fuzzy second category set** *in* (X, T).

Definition 2.7 ([13]). *A fuzzy topological space* (X, T) *is called* **fuzzy first category space** *if* $1 = \vee_{i=1}^{\infty}(\lambda_i)$ *where* λ_i*'s are fuzzy nowhere dense sets in* (X, T)*. A topological space which is not of fuzzy first category space, is said to be of* **fuzzy second category space**.

Definition 2.8 ([11]). *Let (X,T) be a fuzzy topological space. Then* (X, T) *is called a* **fuzzy Baire space** *if* $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$*, where*$(\lambda_i)$*'s are fuzzy nowhere dense sets in* (X, T).

Definition 2.9 ([9]). *A fuzzy topological space* (X, T) *is called a* **fuzzy D-Baire space** *if every fuzzy first category set in* (X, T) *is a fuzzy nowhere dense set in* (X, T).

Definition 2.10 ([10]). *A fuzzy topological space* (X, T) *is fuzzy Baire space. Then* (X, T) *is called a* **fuzzy D′-Baire space** *if every fuzzy set with empty interior is fuzzy nowhere dense in* (X, T).

Definition 2.11 ([1]). *A fuzzy topological space* (X, T) *is said to be a* **fuzzy Quasi-maximal space** *if for every fuzzy dense set* λ *in* (X, T) *with* $int(\lambda) \neq 0$ *(the null set),* $int(\lambda)$ *is also fuzzy dense in* (X, T).

Definition 2.12 ([2]). *A fuzzy topological space* (X, T) *is said to be a* **fuzzy Quasi-regular space** *for every non-empty fuzzy open set* λ *in* (X, T) *there is a non-empty open se* μ *in* (X, T) *with* $cl(\mu) \leq \lambda$.

Definition 2.13 ([8]). *A fuzzy topological space* (X, T) *is said to be a* **fuzzy P-space** *if every if countable intersection of fuzzy open sets in* (X, T) *is fuzzy open in* (X, T).

Definition 2.14 ([4]). *A fuzzy topological space* (X, T) *is said to be a* **fuzzy Hausdorf space** *if whenever* λ, μ *in* (X, T) *and* $\lambda \neq \mu$ *we can find the fuzzy open sets* γ *and* δ *such that* $\lambda \leq \gamma, \mu \leq \delta$ *and* $\gamma \wedge \delta = 0$.

Definition 2.15 ([12]). *A fuzzy topological space* (X, T) *is called a* **fuzzy nodec space** *if every non-zero fuzzy nowhere dense set* λ *is fuzzy closed in* (X, T)*. That is, if* λ *is a fuzzy nowhere dense set in* (X, T)*, then* $1 - \lambda \in T$.

3. Fuzzy Door Space

Motivated by the classical concept studied in [6] we shall now define:

Definition 3.1. *A fuzzy topological space (X, T) is said to be a* **fuzzy door space** *if every fuzzy sub set of X is either fuzzy open or fuzzy closed.*

Example 3.2. *Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and γ are defined on X as follows:*
$\lambda : X \to [0, 1]$ *defined as* $\lambda(a) = 0.8; \lambda(b) = 0.9; \lambda(c) = 0.2.$
$\mu : X \to [0, 1]$ *defined as* $\mu(a) = 0.7; \mu(b) = 0.8; \mu(c) = 0.1.$
$\gamma : X \to [0, 1]$ *defined as* $\gamma(a) = 0.7; \gamma(b) = 0.8; \gamma(c) = 0.1.$
Then $T = \{0, \lambda, \mu, \gamma, 1\}$ is a fuzzy topology on X. The fuzzy sets λ, μ, γ in (X, T) are fuzzy open sets and the fuzzy sets $1 - \lambda, 1 - \mu, 1 - \gamma$ in (X, T) are fuzzy closed sets, therefore the fuzzy sets in (X, T) are either fuzzy open or fuzzy closed. Hence (X, T) is a fuzzy Door space.

Example 3.3. *Let $X = \{a, b\}$. The fuzzy sets λ, μ, γ and δ are defined on X as follows:*
$\lambda : X \to [0, 1]$ *defined as* $\lambda(a) = 0.5; \lambda(b) = 0.$
$\mu : X \to [0, 1]$ *defined as* $\mu(a) = 0; \mu(b) = 0.4.$
$\gamma : X \to [0, 1]$ *defined as* $\gamma(a) = 1; \gamma(b) = 0.$
$\delta : X \to [0, 1]$ *defined as* $\delta(a) = 0; \delta(b) = 1.$
Then $T = \{0, \lambda, \mu, \gamma, \delta, (\lambda \vee \mu), (\lambda \vee \delta), (\mu \vee \gamma), 1\}$ is a fuzzy topology on X. Now consider the fuzzy sets in X as followes:
$\alpha : X \to [0, 1]$ *defined as* $\alpha(a) = 0.8; \alpha(b) = 0.9.$
$\beta : X \to [0, 1]$ *defined as* $\beta(a) = 0.7; \beta(b) = 0.8..$
$\delta : X \to [0, 1]$ *defined as* $\delta(a) = 0.7; \delta(b) = 0.8.$
The fuzzy sets $\lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \vee \delta), (\mu \vee \gamma)$ in (X, T) are fuzzy open sets and the fuzzy sets $1 - \lambda, 1 - \mu, 1 - \gamma, 1 - (\lambda \vee \mu), 1 - (\lambda \vee \delta), 1 - (\mu \vee \gamma)$ in (X, T) are fuzzy closed sets, but the fuzzy sets α, β, δ in (X, T) are not fuzzy open or not fuzzy closed. Hence (X, T) is not fuzzy Door space.

Proposition 3.4. *Let (X, T) be a fuzzy Door space, then (X, T) is a fuzzy Baire space. Consider the following example.*

Example 3.5. *Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and γ are defined on X as follows:*
$\lambda : X \to [0, 1]$ *defined as* $\lambda(a) = 0.8; \lambda(b) = 0.9; \lambda(c) = 0.6.$
$\mu : X \to [0, 1]$ *defined as* $\mu(a) = 0.7; \mu(b) = 0.8; \mu(c) = 0.6.$
$\gamma : X \to [0, 1]$ *defined as* $\gamma(a) = 0.6; \gamma(b) = 0.6; \gamma(c) = 0.6.$
Then $T = \{0, \lambda, \mu, \gamma, 1\}$ is a fuzzy topology on X. The fuzzy sets λ, μ, γ in (X, T) are fuzzy open sets and the fuzzy sets $1 - \lambda, 1 - \mu, 1 - \gamma$ in (X, T) are fuzzy closed sets, therefore the fuzzy sets in (X, T) are either fuzzy open or fuzzy closed, therefore (X, T) is a fuzzy Door space. Now $int\ cl(1 - \lambda) = 0, int\ cl(1 - \mu) = 0$ and $int\ cl(1 - \gamma) = 0$ therefore $1 - \lambda, 1 - \mu$ and $1 - \gamma$ are fuzzy nowhere dense set, $int[(1 - \lambda) \vee (1 - \mu) \vee (1 - \gamma)] = 1 - \gamma = 0$. Hence (X, T) is a fuzzy Door space, then (X, T) is fuzzy Baire space.

Converse need not be true. Consider the following example.

Example 3.6. *Let $X = \{a, b, c\}$. The fuzzy sets λ and μ are defined on X as follows:*
$\lambda : X \to [0, 1]$ *defined as* $\lambda(a) = 0.3; \lambda(b) = 0.6; \lambda(c) = 0.5$
$\mu : X \to [0, 1]$ *defined as* $\mu(a) = 0.5; \mu(b) = 0.4; \mu(c) = 0.7$
Then $T = \{0, \lambda, \mu, (\lambda \vee \mu), (\lambda \wedge \mu), 1\}$ is a fuzzy topology on X. Now consider the fuzzy sets in X as followes:

$\alpha : X \to [0,1]$ *defined as* $\alpha(a) = 0.4; \alpha(b) = 0.5; \alpha(c) = 0.2.$

$\beta : X \to [0,1]$ *defined as* $\beta(a) = 0.3; \beta(b) = 0.4; \beta(c) = 0.3.$

$\delta : X \to [0,1]$ *defined as* $\delta(a) = 0.5; \delta(b) = 0.6; \delta(c) = 0.1.$

The fuzzy sets $1 - \mu, 1 - \lambda \vee \mu$ *are fuzzy nowhere dense sets in* (X, T). *Now* $int[1 - \mu \vee (1 - (\lambda \vee \mu))] = 0$ *therefore* (X, T) *are fuzzy Baire space but not of fuzzy Door space. Since the fuzzy sets* α, β, δ *in* (X, T) *are not fuzzy open and not fuzzy closed. Hence* (X, T) *is not fuzzy Door space.*

Proposition 3.7. *Let* (X, T) *be a fuzzy Door space, then* (X, T) *is a fuzzy D-Baire space.*

Proof. By example 3.1, (X, T) be a fuzzy Door space. Now $1 - \lambda, 1 - \mu, 1 - \gamma$ are fuzzy nowhere dense sets in (X, T). Now $int\ cl[(1 - \lambda) \vee (1 - \mu) \vee (1 - \gamma)] = int\ cl(1 - \gamma) = 0$ therefore the fuzzy first category set is fuzzy nowhere dense. Hence (X, T) is fuzzy D-Baire space. □

Converse of the above proposition need not be true. Consider the following example.

Example 3.8. *Let* $X = \{a, b, c\}$. *The fuzzy sets* λ, μ *and* γ *are defined on* X *as follows:*

$\lambda : X \to [0,1]$ *defined as* $\lambda(a) = 0.5; \lambda(b) = 0.4; \lambda(c) = 0.6$

$\mu : X \to [0,1]$ *defined as* $\mu(a) = 0.6; \mu(b) = 0.6; \mu(c) = 0.6.$

$\gamma : X \to [0,1]$ *defined as* $\gamma(a) = 0.8; \gamma(b) = 0.6; \gamma(c) = 0.7.$

Then $T = \{0, \lambda, \mu, \gamma, 1\}$ *is a fuzzy topology on* X. *Now consider the fuzzy sets in* X *as followes:*

$\alpha : X \to [0,1]$ *defined as* $\alpha(a) = 0.5; \alpha(b) = 0.5.; \alpha(c) = 0.6.$

$\beta : X \to [0,1]$ *defined as* $\beta(a) = 0.5; \beta(b) = 0.5; \beta(c) = 0.4..$

$\delta : X \to [0,1]$ *defined as* $\delta(a) = 0.4; \delta(b) = 0.5; \delta(c) = 0.1.$

The fuzzy nowhere dense sets in (X, T) *are* $1 - \lambda, 1 - \mu, 1 - \gamma, \alpha, \beta, \delta$. *Now* $int\ cl[(1\lambda) \vee (1 - \mu) \vee (1 - \gamma) \vee \alpha \vee \beta \vee \delta] = int\ cl(1 - \gamma) = 0$ *therefore* (X, T) *is fuzzy D-Baire but not of fuzzy Door space. Since the fuzzy sets* α, β, δ *in* (X, T) *are not fuzzy open and not fuzzy closed. Hence* (X, T) *is not of fuzzy Door space.*

Proposition 3.9. *Let* (X, T) *be a fuzzy Door space, then* (X, T) *is a fuzzy* D'*-Baire space.*

Proof. By example 3.1, (X, T) be a fuzzy Door space. Now $1 - \lambda, 1 - \mu, 1 - \gamma$ are fuzzy nowhere dense sets in (X, T). Now the empty interior of (X, T) is $(1 - \lambda), (1 - \mu), (1 - \gamma)$ therefore $(1 - \lambda), (1 - \mu), (1 - \gamma)$ are fuzzy nowhere dense sets. Hence (X, T) is fuzzy D$'$-Baire space. □

Converse of the above proposition need not be true. Consider the following example.

Example 3.10. *Let* $X = \{a, b, c\}$. *The fuzzy sets* λ, μ, γ *and* δ *are defined on* X *as follows:*

$\lambda : X \to [0,1]$ *defined as* $\lambda(a) = 0.9; \lambda(b) = 0.8; \lambda(c) = 0.8$

$\mu : X \to [0,1]$ *defined as* $\mu(a) = 0.8; \mu(b) = 0.8; \mu(c) = 0.7.$

$\gamma : X \to [0,1]$ *defined as* $\gamma(a) = 0.8; \gamma(b) = 0.8; \gamma(c) = 0.6.$

Then $T = \{0, \lambda, \mu, \gamma, 1\}$ *is a fuzzy topology on* X. *Now consider the fuzzy sets in* X *as followes:*

$\alpha : X \to [0,1]$ *defined as* $\alpha(a) = 0.2; \alpha(b) = 0.2.; \alpha(c) = 0.2.$

$\beta : X \to [0,1]$ *defined as* $\beta(a) = 0.2; \beta(b) = 0.1; \beta(c) = 0.3..$

$\delta : X \to [0,1]$ *defined as* $\delta(a) = 0.1; \delta(b) = 0.1; \delta(c) = 0.4.$

Now the empty interior of (X, T) *is* $(1 - \lambda), (1 - \mu), (1 - \gamma), \beta, \delta$ *therefore* $(1 - \lambda), (1 - \mu), (1 - \gamma), \beta, \delta$ *are fuzzy nowhere dense sets. Hence* (X, T) *is fuzzy* D'*-Baire space but not of fuzzy Door space. Since the fuzzy sets* α, β, δ *in* (X, T) *are not fuzzy open and not fuzzy closed. Hence* (X, T) *is not of fuzzy Door space.*

Proposition 3.11. *Let (X,T) be a fuzzy Door space, then (X,T) is fuzzy Quasi-maximal space.*

Proof. By example 3.1, (X,T) be a fuzzy Door space. Now the fuzzy dense sets in (X,T) are λ, μ, γ. Now $int(\lambda) \neq 0, int(\mu) \neq 0, int(\gamma) \neq 0$ and $cl\ int(\lambda) = 1, cl\ int(\mu) = 1, cl\ int(\gamma) = 1$. Hence (X,T) is fuzzy Quasi-maximal space. □

Converse of the above proposition need not be true. Consider the following example.

Example 3.12. *Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and γ are defined on X as follows:*

$\lambda : X \to [0,1]$ *defined as* $\lambda(a) = 0.9; \lambda(b) = 0.7; \lambda(c) = 0.6$

$\mu : X \to [0,1]$ *defined as* $\mu(a) = 0.5; \mu(b) = 0.6; \mu(c) = 0.8.$

$\gamma : X \to [0,1]$ *defined as* $\gamma(a) = 0.7; \gamma(b) = 0.8; \gamma(c) = 0.9.$

Then $T = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \wedge \mu, \lambda \vee \gamma, \lambda \wedge \gamma, \mu \vee \lambda \wedge \gamma, 1\}$ *is a fuzzy topology on X. Now consider the fuzzy sets in X as followes:*

$\alpha : X \to [0,1]$ *defined as* $\alpha(a) = 0.8; \alpha(b) = 0.6.; \alpha(c) = 0.6.$

$\beta : X \to [0,1]$ *defined as* $\beta(a) = 0.7; \beta(b) = 0.6; \beta(c) = 0.7.$

$\delta : X \to [0,1]$ *defined as* $\delta(a) = 0.5; \delta(b) = 0.8; \delta(c) = 0.8.$

The fuzzy dense sets in (X,T) are $\lambda, \mu, \gamma, \lambda \vee \mu, \lambda \wedge \mu, \lambda \vee \gamma, \lambda \wedge \gamma, \mu \vee \lambda \wedge \gamma, \alpha, \beta, \delta$. *Now* $int(\lambda) \neq 0, int(\mu) \neq 0, int(\gamma) \neq 0, int(\lambda \vee \mu) \neq 0, int(\lambda \wedge \mu) \neq 0, int(\lambda \vee \gamma) \neq 0, int(\lambda \wedge \gamma) \neq 0, int(\mu \vee \lambda \wedge \gamma) \neq 0, int(\alpha) \neq 0, int(\beta) \neq 0, int(\delta) \neq 0$ *and* $cl\ int(\lambda) = 1, cl\ int(\mu) = 1, cl\ int(\gamma) = 1, cl\ int(\lambda \vee \mu) = 1, cl\ int(\lambda \wedge \mu) = 1, cl\ int(\lambda \vee \gamma) = 1, cl\ int(\lambda \wedge \gamma) = 1, cl\ int(\mu \vee \lambda \wedge \gamma) = 1, cl\ int(\alpha) = 1, cl\ int(\beta) = 1, cl\ int(\delta) = 1$. *Hence (X,T) is fuzzy Quasi-maximal space but not of fuzzy Door space. Since the fuzzy sets α, β, δ in (X,T) are not fuzzy open and not fuzzy closed. Hence (X,T) is not of fuzzy Door space.*

Proposition 3.13. *Let (X,T) be a fuzzy Door space, then (X,T) is a fuzzy Quasi-regular space.*

Proof. By example 3.1, (X,T) be a fuzzy Door space. Now $1-\lambda, 1-\mu, 1-\gamma$ are fuzzy nowhere dense sets in (X,T). Now $int\ cl[(1-\lambda) \vee (1-\mu) \vee (1-\gamma)] = int\ cl(1-\gamma) = 0$ therefore the fuzzy first category set is fuzzy nowhere dense. Hence (X,T) is fuzzy D-Baire space. □

Converse of the above proposition need not be true. Consider the following example.

Example 3.14. *Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and γ are defined on X as follows:*

$\lambda : X \to [0,1]$ *defined as* $\lambda(a) = 0.5; \lambda(b) = 0.5; \lambda(c) = 0.6.$

$\mu : X \to [0,1]$ *defined as* $\mu(a) = 0.6; \mu(b) = 0.6; \mu(c) = 0.5.$

$\gamma : X \to [0,1]$ *defined as* $\gamma(a) = 0.4; \gamma(b) = 0.4; \gamma(c) = 0.5.$

Then $T = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \wedge \mu, 1\}$ *is a fuzzy topology on X. The fuzzy sets* $\lambda, \mu, \gamma, \lambda \vee \mu, \lambda \wedge \mu$ *in (X,T) are fuzzy open set in (X,T) and the fuzzy sets* $1-\lambda, 1-\mu, 1-\gamma, 1-\lambda \vee \mu, 1-\lambda \wedge \mu$ *in (X,T) are fuzzy closed sets in (X,T). Hence (X,T) are fuzzy Door space. Now* $\lambda, \mu, \gamma, \lambda \vee \mu$ *and* $\lambda \wedge \mu$ *are fuzzy open set in (X,T) then* $cl(\lambda) \leq \lambda \vee \mu, cl(\mu) \leq \lambda \vee \mu, cl(\gamma) \leq \gamma, cl(\lambda \vee \mu) \leq \lambda \vee \mu, cl(\lambda \wedge \mu) \leq \lambda \wedge \mu$ *hence (X,T) is fuzzy Quasi-regular space.*

Proposition 3.15. *Let (X,T) be a fuzzy Door space, then (X,T) is a fuzzy Hausdorf space. Consider the following example.*

Example 3.16. *Let $X = \{a, b, c\}$. The fuzzy sets λ and μ are defined on X as follows:*

$\lambda : X \to [0,1]$ *defined as* $\lambda(a) = 1; \lambda(b) = 0; \lambda(c) = 0.$

$\mu : X \to [0,1]$ *defined as* $\mu(a) = 0; \mu(b) = 1; \mu(c) = 1.$

Then $T = \{0, \lambda, \mu, 1\}$ *is a fuzzy topology on X. The fuzzy sets λ, μ in (X,T) are fuzzy open set in (X,T) and the fuzzy sets*

$1-\lambda, 1-\mu$ *in* (X,T) *are fuzzy closed sets in* (X,T)*. Hence* (X,T) *are fuzzy Door space. Now if the fuuzy set* $1-\lambda, 1-\mu$ *in* (X,T) *and* $1-\lambda \neq 1-\mu$ *now the fuzzy open sets* λ *and* μ *such that* $1-\lambda \leq \mu$ *and* $1-\mu \leq \lambda$ *then* $\lambda \wedge \mu = 0$*. Hence* (X,T) *are fuzzy Hausdorf space.*

Converse of the above proposition need not be true. Consider the following example.

Example 3.17. *Let* $X = \{a,b,c\}$*. The fuzzy sets* λ *and* μ *are defined on* X *as follows:*
$\lambda : X \to [0,1]$ *defined as* $\lambda(a) = 1; \lambda(b) = 0; \lambda(c) = 0$
$\mu : X \to [0,1]$ *defined as* $\mu(a) = 0; \mu(b) = 1; \mu(c) = 1.$
Then $T = \{0, \lambda, \mu, 1\}$ *is a fuzzy topology on* X*. Consider the fuzzy sets* α, β, δ *in* X *as followes:*
$\alpha : X \to [0,1]$ *defined as* $\alpha(a) = 0.5; \alpha(b) = 0; \alpha(c) = 0$
$\beta : X \to [0,1]$ *defined as* $\beta(a) = 0; \beta(b) = 0.4; \beta(c) = 0.3.$
$\delta : X \to [0,1]$ *defined as* $\delta(a) = 0; \delta(b) = 0; \delta(c) = 0.1$
The fuzzy sets $1-\lambda \leq \mu, 1-\mu \leq \lambda$ *and* $1-\lambda \neq 1-\mu$ *then* $\lambda \wedge \mu = 0.$
$1-\lambda \leq \mu, \alpha \leq \lambda$ *and* $1-\lambda \neq \alpha$ *then* $\lambda \wedge \mu = 0.$
$\alpha \leq \lambda, \beta \leq \mu$ *and* $\alpha \neq \beta$ *then* $\lambda \wedge \mu = 0.$
$\alpha \leq \lambda, \delta \leq \mu$ *and* $\alpha \neq \delta$ *then* $\lambda \wedge \mu = 0.$
$1-\mu \leq \lambda, \beta \leq \mu$ *and* $1-\mu \neq \beta$ *then* $\lambda \wedge \mu = 0.$
$1-\mu \leq \lambda, \delta \leq \mu$ *and* $1-\mu \neq \delta$ *then* $\lambda \wedge \mu = 0$*. Therefore* (X,T) *is fuzzy Hausdorf space but not a fuzzy Door space. Since the fuzzy set* α, β, δ *in* (X,T) *is not a fuzzy open and not fuzzy closed in* (X,T)*.*

Proposition 3.18. *A fuzzy P-space* (X,T) *is not a fuzzy door space. Consider the following example.*

Example 3.19. *Let* $X = \{a,b,c\}$*. The fuzzy sets* λ, μ *and* γ *are defined on* X *as follows:*
$\lambda : X \to [0,1]$ *defined as* $\lambda(a) = 0.1; \lambda(b) = 0.3; \lambda(c) = 0.1.$
$\mu : X \to [0,1]$ *defined as* $\mu(a) = 0.2; \mu(b) = 0.2; \mu(c) = 0.3.$
$\gamma : X \to [0,1]$ *defined as* $\gamma(a) = 0.3; \mu(b) = 0.1; \mu(c) = 0.2.$
Then $T = \{0, \lambda, \mu, \gamma, \lambda \vee \mu, \lambda \wedge \mu, \lambda \vee \gamma, \lambda \wedge \gamma, \mu \vee \gamma, \mu \wedge \gamma, \lambda \vee \mu \vee \gamma, \lambda \vee \mu \wedge \gamma, \mu \wedge \lambda \vee \gamma, \gamma \vee \lambda \wedge \mu, 1\}$ *is a fuzzy topology on* X*. Consider the fuzzy sets* α, β, δ *in* X *as followes:*
$\alpha : X \to [0,1]$ *defined as* $\alpha(a) = 0.5; \alpha(b) = 0; \alpha(c) = 0$
$\beta : X \to [0,1]$ *defined as* $\beta(a) = 0; \beta(b) = 0.4; \beta(c) = 0.3.$
$\delta : X \to [0,1]$ *defined as* $\delta(a) = 0; \delta(b) = 0; \delta(c) = 0.1$
Now the fuzzy sets $\lambda \wedge \gamma = \lambda \wedge \gamma \wedge (\mu \vee \gamma) \wedge (\lambda \vee \mu)$ *and* $\mu \wedge (\lambda \vee \gamma) = \lambda \vee (\mu \wedge \gamma) \wedge [\gamma \vee (\lambda \wedge \mu)]$ *are fuzzy* G_δ *sets in* (X,T) *and* $\lambda \wedge \gamma, [\mu \wedge (\lambda \vee \gamma)]$ *are fuzzy open in* (X,T)*. Hence* (X,T) *is fuzzy P-space, but not of fuzzy Door space. Since the fuzzy sets* α, β *and* δ *are not fuzzy open and not fuzzy closed in* (X,T)*.*

Proposition 3.20. *Let* (X,T) *be a fuzzy Door space, then* (X,T) *is a fuzzy nodec space. Consider the following example.*

Example 3.21. *Let* $X = \{a,b,c\}$*. The fuzzy sets* λ *and* μ *are defined on* X *as follows:*
$\lambda : X \to [0,1]$ *defined as* $\lambda(a) = 0.3; \lambda(b) = 0.6; \lambda(c) = 0.5.$
$\mu : X \to [0,1]$ *defined as* $\mu(a) = 0.5; \mu(b) = 0.4; \mu(c) = 0.7.$
Then $T = \{0, \lambda, \mu, (\lambda \vee \mu), (\lambda \wedge \mu), 1\}$ *is a fuzzy topology on* X*. The fuzzy sets in* (X,T) *are fuzzy open or fuzzy closed, therefore* (X,T) *are fuzzy Door space. Now the fuzzy nowhere dense sets in* (X,T) *are* $1-\lambda, 1-\mu$ *is also fuzzy closed set in* (X,T)*. Hence* (X,T) *is a fuzzy nodec space.*

Converse need not be true. Consider the following example.

Example 3.22. *Let* $X = \{a, b, c\}$*. The fuzzy sets* λ *and* μ *are defined on* X *as follows:*

$\lambda : X \to [0, 1]$ *defined as* $\lambda(a) = 0.3; \lambda(b) = 0.6; \lambda(c) = 0.5$

$\mu : X \to [0, 1]$ *defined as* $\mu(a) = 0.5; \mu(b) = 0.4; \mu(c) = 0.7$

Then $T = \{0, \lambda, (\lambda \vee \mu), (\lambda \wedge \mu), 1\}$ *is a fuzzy topology on* X*. Now consider the fuzzy sets in* X *as followes:*

$\alpha : X \to [0, 1]$ *defined as* $\alpha(a) = 1; \alpha(b) = 0.5; \alpha(c) = 0.2.$

$\beta : X \to [0, 1]$ *defined as* $\beta(a) = 0.3; \beta(b) = 1; \beta(c) = 0.3.$

$\delta : X \to [0, 1]$ *defined as* $\delta(a) = 0.5; \delta(b) = 0.6; \delta(c) = 1.$

The fuzzy sets $1 - \mu, 1 - \lambda \vee \mu$ *are fuzzy nowhere dense sets and fuzzy closed in* (X, T)*, hence* (X, T) *is fuzzy nodec space, but not of fuzzy door space. Since the fuzzy sets* α, β, δ *are not fuzzy open or not fuzzy closed in* (X, T)*.*

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