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Control of Epidemics Using SIS Model

Research Article*

T.Henson[1] and B.Suriyakala[1]

1 Department of Mathematics, St. Joseph's College of Arts & Science, Manjakuppam, Cuddalore (Tamil Nadu), India.

Abstract: A mathematical model is described for the system of epidemic by non-linear difference equations. In this paper, we have dealt with SIS epidemic Model in which study on the anticipation of epidemic by Population thresholds and control of epidemics.

Keywords: SIS model, deterministic chaos.

1. Introduction

The modeling of infectious disease is a tool which has been used to study the mechanisms by which disease spread, to predict the future course of an outbreak and to evaluate strategies to control an epidemic. The SIR model is a simple model, due to Kermack and Mckendrick, of an epidemic of an infectious disease in a population. Removing the equation representing the recovered population from the SIR model and adding those removed from the infected population into the susceptible population gives the SIS model. Some infectious, eg) those from the common cold and influenza, do not confer any long lasting immunity. Such infections do not give immunization upon recovery from infection, and individuals become susceptible again.

1.1. SIS Model

In a given population at time 't', S(t) be the susceptible set of people who are with less immunity and are likely to be infected but are not infected. I(t) be the set of people who are infected by disease. In this model, a person become infected depends on contact between susceptibles and infected (aSI) and recovery is at a constant rate, proportional to number of infected (bI) so that we get the model [1]

$$\frac{dS}{dt} = -aSI + bI \tag{1}$$

$$\frac{dI}{dt} = aSI - bI \tag{2}$$

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Let 'n' be the number of susceptible in the population and one infected person. We are assuming then

$$S\left(t\right)+I\left(t\right)=n+1$$
$$S\left(0\right)=S_0=n$$
$$I\left(0\right)=I_0=1$$

Therefore

$$S+I=N\Rightarrow S=N-I \tag{3}$$

Substitute (3) in (2), we get the equation

$$\begin{aligned}\frac{dI}{dt}&=a\left(N-I\right)I-bI\\&=aNI-aI^2-bI\\&=\left(aN-b\right)I-aI^2\end{aligned}$$

Therefore

$$\frac{dI}{dt}=AI-aI^2 \tag{4}$$

Where $A=aN-b$. Integrating on both sides we get

$$I=\frac{(aN-b)I_0e^{(aN-b)t}}{(aN-b)+aI_0(e^{(aN-b)t}-1)}$$

2. Prediction of Epidemic by Population Thresholds

We are considering a differential equation into a finite difference equation in SIS Model [2]

$$S\left(t+\Delta t\right)=S\left(t\right)-a\Delta tI\left(t\right)S\left(t\right)+b\Delta tI\left(t\right)(5)$$
$$I\left(t+\Delta t\right)=I\left(t\right)+a\Delta tI\left(t\right)S\left(t\right)-\ b\Delta tI\left(t\right) \tag{5}$$

Susceptibles ⟷ **Infectives**

In this model there will be no recovered person instead of that recovered person become susceptible therefore we have

$$R\left(t+\Delta t\right)=R\left(t\right)=0 \tag{6}$$

But the total population we have

$$\begin{aligned}N\left(t+\Delta t\right)&=S\left(t+\Delta t\right)+I\left(t+\Delta t\right)+R(t+\Delta t)\\&=S\left(t+\Delta t\right)+I\left(t+\Delta t\right)+0\\N(t+\Delta t)&=S\left(t+\Delta t\right)+I(t+\Delta t)\end{aligned}$$

At the initial conditions we have $\Delta t=0$. Therefore $N(t)=S(t)+I(t)=K(constant)$

$$S\left(t\right)+I\left(t\right)=K$$
$$S\left(t\right)=K-I\left(t\right) \tag{7}$$

Using (7) in (5) we get

$$I(t+\Delta t) = I(t) + a\Delta t I(t) S(t) - b\Delta t I(t)$$
$$= I(t) + a\Delta t I(t) [K - I(t)] - b\Delta t I(t)$$

The above equation is similar to the Pearl-Verhulst map-giving rise to Chaos [3]. After normalization of the given population we will get equation

$$i(t+1/b) = 4\delta i(t) [1 - i(t)] \tag{8}$$

This equation is called the Logistic equation. Since the value for $i(t+1/b)$ is lies between 0 and 1. Therefore the value for δ also lies between 0 and 1. Let us considering four different conditions

(i) When $\delta \leq 1/4$, then we are getting $I(t) = 0$.

(ii) When $1/4 \leq \delta \leq 3/4$, then we are getting

$$I(t) = K\left[1 - \frac{1}{4\delta}\right]$$

(iii) When $3/4 \leq \delta \leq 0.89$, then we are getting bifurcation with period doubling

(a) At the point $i(t) = 0.5$ and $\delta = 0.35,\ 0.45\ ,\ 0.6$ we are getting stable conditions.

(b) At the point $i(t) = 0.5$ and $\delta = 0.8$ we are getting periodic doubling. Since it has the values 0.7994688 and 05128841 are occurring twice as many iterations as before.

(iv) When $0.89 \leq \delta \leq 1$, then we are getting deterministic chaos.

3. Control of an Epidemic

The control of epidemic is done with respect to the vaccination. When we came to control of an epidemic, then we have to go for vaccination to make an infected person into immured by vaccination. Then SIS model become,

$$\frac{dS}{dt} = -aSI - v \tag{9}$$
$$\frac{dI}{dt} = aSI - bI \tag{10}$$
$$\frac{dR}{dt} = bI \tag{11}$$
$$\frac{dV}{dt} = v \tag{12}$$

By normalizing the above equation we get

$$\frac{d\overline{S}}{dt} = -\overline{a}\overline{SI} - \overline{v}$$
$$\frac{d\overline{I}}{dt} = \overline{a}\overline{SI} - b\overline{I}$$
$$\frac{d\overline{R}}{dt} = b\overline{I}$$
$$\frac{d\overline{V}}{dt} = \overline{v}$$

The control of epidemic stand up vaccination it involves the costs. To minimise the cost of vaccination we have assign the conditions. Let C(v) be the cost function. To control an epidemic we have two conditions

1. The sum of recovered person and infected person must be less than or equal to a particular number X for a period of time.

2. The maximum number of infected person must also less than or equal to a particular number Y for a period of time. This leads to an optimization problem is given by

$$I(t) + R(t) \leq X$$

$$\max_{[0,\ t]} I(t) \leq Y$$

Then cost function is minimum. Where initial conditions of S, I and X, Y are all non negatives values [5].

4. Conclusion

In this paper, the SIS model can interpretate any disease at a particular period of time in a fixed population period of time in a fixed population under some conditions. We have also studied the await of epidemic by population thresholds and control of epidemics.

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