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# Application of Pentagonal Fuzzy Number in Neural Network

**Research Article***

**A.Vigin Raj[1] and S.Karthik[1]**

1 Department of Mathematics, St. Joseph's College of Arts & Science, Manjakuppam, Cuddalore (Tamil Nadu), India.

**Abstract:** Neural Network is and used to be a principal component of mathematics education. Many models have been developed in the literature for the description of the neural network. In this paper, we use fuzzy number choose the best machine for a job by Feed-Forward Neural Network.

**Keywords:** Fuzzy Number, Pentagonal Fuzzy Number, Feed-Forward Neural Network.



## 1. Introduction

Fuzzy sets were introduced by zadeh in 1965 to represent and information possessing non-statistical certainties. However, the story of fuzzy logic started much earlier. The rotation of an infinite valued logic was "Fuzzy Set" where he described the mathematics of fuzzy set theory, and by extension fuzzy logic. Many operations were carried out using fuzzy numbers. The concept of fuzzy numbers is the generalization of concept of real numbers. Fuzzy neural networks are usually based on neural network architecture with fuzzification of inputs, outputs, weights, or rules that are applied using fuzzy system. Architecture of multilayer feed-forward algebraic neural network for fuzzy input vectors, fuzzy outputs and fuzzy weights is proposed. The input-output relations in this fuzzy neural network, using max/min operators, are defined by the extension principle of Zadeh. The paper is organized as follows: the basic definition and notation of fuzzy numbers are given. The pentagonal fuzzy numbers are defined and based on the function; the arithmetic operation such as addition, subtraction, scalar multiplication of pentagonal fuzzy numbers. Feed-forward (FFNN) model and procedure. In final, we derived conclusion and gives suggestion based on our study.

## 2. Basics Definition

**Definition 2.1** (Fuzzy set)**.** *A fuzzy set is any set that allows its members to have different degree of membership called membership function, in the interval [0,1]. A fuzzy set A is defined in the universal space X and denoted byan ordered set of pairs, the first element of which denotes the element and the second degree of membership i.e.,* $\tilde{A} = \{x, \mu_{\tilde{A}}(x); x \in X\}$.

**Definition 2.2** (Fuzzy number)**.** *A fuzzy number* $\tilde{A}$ *is a fuzzy set on the real line R, must satisfy the following conditions*

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1. $\mu_{\tilde{A}}(x_o)$ *is the piecewise continuous*

2. *there exist at least one* $x_o \in R$ *with* $\mu_{\tilde{A}}(x_o) = 1$

3. $\tilde{A}$ *must be normal and convex*

**Definition 2.3** (Triangular fuzzy number). *Triangular fuzzy number is defined as* $\widetilde{A} = \{a, b, c\}$ *where all a,b,c are real numbers and its membership function is given below*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & \text{for } a \le x \le b \\ \frac{(c-x)}{(c-b)}, & \text{for } b \le x \le c \\ 0, & \text{otherwise.} \end{cases}$$

**Definition 2.4** (Trapezoidal Fuzzy Number). *A fuzzy set* $\widetilde{A} = (a, b, c, d)$ *is said to trapezoidal fuzzy number if its membership function is given by where* $a \le b \le c \le d$.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{for } x < a \\ \frac{(x-a)}{(b-a)}, & \text{for } a \le x \le b \\ 1, & \text{for } b \le x \le c \\ \frac{(d-x)}{(d-c)}, & \text{for } c \le x \le d \\ 0, & \text{for } cx > d. \end{cases}$$

# 3. Pentagonal Fuzzy Number

**Definition 3.1** (Pentagonal Fuzzy Number). *A pentagonal fuzzy number of a fuzzy set* $\tilde{P}$ *is defined as* $\widetilde{P} = \{a, b, c, d, e\}$ *and its membership functions is given by*

$$\mu_{\tilde{P}}(X) = \begin{cases} 0, & \text{for } x < a \\ \frac{(x-a)}{(b-a)}, & \text{for } a \le x \le b \\ \frac{(x-b)}{(c-a)}, & \text{for } b \le x \le c \\ 1, & x = c \\ \frac{(d-x)}{(d-c)}, & \text{for } c \le x \le d \\ \frac{(e-x)}{(e-d)}, & \text{for } d \le x \le e \\ 0, & \text{for } x > e. \end{cases}$$

**Definition 3.2** (Condition on pentagonal fuzzy number). *A pentagonal fuzzy number* $\tilde{A}_p$ *should satisfy the following conditions:*

1. $\mu_{\tilde{A}p}(x)$ *is a continuous function in the interval [0, 1]*

2. $\mu_{\tilde{A}p}(x)$ *is strictly increasing and continuous function on [a, b] and [b, c]*

3. $\mu_{\tilde{A}}(p)$ *is strictly decreasing and continuous function on (c, d] and [d, e)*

### 3.1. Arithmetic Operations on Pentagonal Fuzzy Number (PFN)

**Definition 3.3** (Addition of Two Pentagonal Fuzzy Numbers). *If* $\tilde{A}_p = (a_1, b_1, c_1, d_1, e_1)$; $\tilde{O}_p = (a_2, b_2, c_2, d_2, e_2)$. *Then* $\tilde{A}_p + \tilde{O}_p = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2, e_1 + e_2)$.

**Definition 3.4** (Subtraction of Two Pentagonal Fuzzy Numbers). *If* $\tilde{A}_p = (a_1, b_1, c_1, d_1, e_1)$; $\tilde{O}_p = (a_2, b_2, c_2, d_2, e_2)$. *Then* $\tilde{A}_p - \tilde{O}_p = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2, e_1 - e_2)$.

**Definition 3.5** (Multiplication of Two Pentagonal Fuzzy Numbers). *If* $\tilde{A}_p = (a_1, b_1, c_1, d_1, e_1)$; $\tilde{O}_p = (a_2, b_2, c_2, d_2, e_2)$. *Then* $\tilde{A}_p * \tilde{O}_p = (a_1 * a_2, b_1 * b_2, c_1 * c_2, d_1 * d_2, e_1 * e_2)$.

**Definition 3.6** (Construction of Pentagonal Fuzzy Number). *The pentagonal fuzzy number is represented by the five parameters such as a, b, c, d and e, where a and b denote the smallest possible values, c the most promising value and d,e the largest possible value. Formula to generate fuzzy pentagonal number defined as follows*

$$\tilde{A} = (a-2, a-1, a, a+1, a+2), \quad \textit{for all} \;\; a = 3, 4, 5, 6, 7 \tag{1}$$

*Since fuzzy number scale is defined from 1 to 9.*

**Definition 3.7** (Membership Function for Pentagonal Fuzzy Number). *Membership function of* $\tilde{A}_p = (a, b, c, d, e)$ *is defined as*

$$\left(\frac{a}{10}, \frac{b}{10}, \frac{c}{10}, \frac{d}{10}, \frac{e}{10}\right), \quad \textit{if} \;\; 0 \leq a \leq b \leq c \leq d \leq e \leq 10 \tag{2}$$

*Where* $0 \leq \frac{a}{10} \leq \frac{b}{10} \leq \frac{c}{10} \leq \frac{d}{10} \leq \frac{e}{10} \leq 1$.

## 4. Neural Network

Artificial neural networks (ANN) are powerful tools that can be used to manage knowledge and solve problems. They are information processing systems that reproduce by computer the function of a very simplified biological neural network, composed of a certain number of interconnected neurons. Intelligent behavior springs from appropriate interactions between interconnected units. Feed-forward networks involve connections that always move towards the network output, i.e. there are no feedback loops. The nodes of a neural network are mostoften organized into layers where connections exist only between nodes in adjacent layers. A fully connected network contains links between all nodes in adjacent layers. A multi layer network contains at least one layer between the network input and output, i.e.inputs are not fed directly to output nodes. Figure 1 illustrates a fully connected two input, Single-output, feed-forward, multi layer network with a single hidden layer consisting of three nodes.

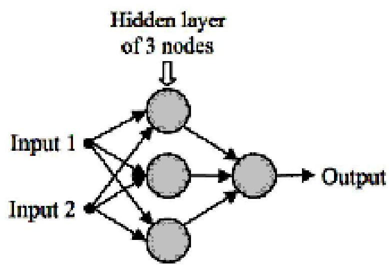


**Figure 1.**

### 4.1. Feed-Forward Fuzzy Neural Networks

There are three basically types of fuzzy neural networks depending on the type of fuzzification of inputs, outputs and weights (including biases): fuzzy weights and crisp inputs, crisp weights and fuzzy inputs and fuzzy weights and fuzzy inputs . In what follows we consider the most complete fuzzification of neural networks: fuzzy inputs, fuzzy weights and fuzzy outputs.

### 4.2. Activation Function

According to Faqs.org [2010] activations function are needed for hidden layer of the NN to introduce non- linearity. Without them NN would be same as plain perceptions. If linear function were used, NN would notbe as powerful as they are. Activation function can be linear, threshold or sigmoid function. Sigmoid activation function is usually used for hidden layer because it combines nearly linear behavior, curvilinear behavior and nearly constant behavior depending on the input value Larose. To explain activation function figure 2 will be used.

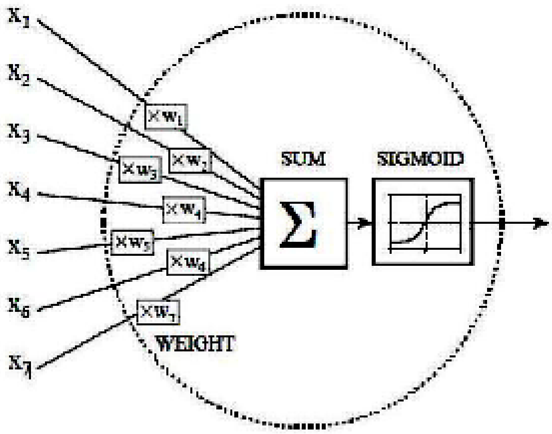


**Figure 2.**

SUM is collection of the output nodes from hidden layer that have been multiplied by connection weights,added to get single number and put through sigmoid function (activation function). Input to sigmoid is any value between negative infinity and positive infinity number while the output can only be a number between 0 and 1.

**Procedure**

Step 1: Gather the imprecise estimation needed for the problem which is in the form of pentagonal fuzzy number.

Step 2: Convert the element of pentagonal fuzzynumber matrix into its membership function by using (2).

Step 3: Set the pentagonal fuzzy number is weight of the FFN

Step 4: Assume the input value 0 and 1.

Step 5: Calculate the weighted sums $m = \sum x_i w_i$

Step 6: The output of a neuron (s) is a function of the weighted sum $S = f(m)$

Step 7: calculated sigmoid function by $f(m) = 1.0/(1.0 + exp(-m))$

Step 8: Determine the minimum value of f(m).

## 5. Numerical Example

Suppose there are three weaving machine $m_1$, $m_2$, $m_3$. Let the possible attributes to the above machines $w = \{a, b, c, d, e\}$ as universal set, where a, b, c, d, e represents the time period, power consumption, spinning & weaving , maintenances

& servicing and dying & finishing respectively. Estimate the pentagonal fuzzy number in three machine $m_1$, $m_2$, $m_3$ by considering to complete work.

Step 1: $W_1 = (0,1,2,3,4,5)$; $W_2 = (2,3,4,5,6)$; $W_3 = (3,4,5,6,7)$.

Step 2: Convert the pentagonal fuzzy number into membership function. $W_1 = (0.0, 0.1, 0.2, 0.3, 0.4)$; $W_2 = (0.2, 0.3, 0.4, 0.5, 0.6)$; $W_3 = (0.3, 0.4, 0.5, 0.6, 0.7)$.

Step 3: Consider the above pentagonal fuzzy number is fuzzy weight $w_{ij}$

$$w_{11} = 0.0,\ w_{12} = 0.1,\ w_{13} = 0.2,\ w_{14} = 0.3,\ w_{15} = 0.4$$
$$w_{21} = 0.2,\ w_{22} = 0.3,\ w_{23} = 0.4,\ w_{24} = 0.5,\ w_{25} = 0.6$$
$$w_{31} = 0.3,\ w_{32} = 0.4,\ w_{33} = 0.5,\ w_{34} = 0.6,\ w_{35} = 0.7$$

Step 4: Assume input value $x = (0,0,1,1,1)$; $x_1 = 0$, $x_2 = 0$, $x_3 = 1$, $x_4 = 1$, $x_5 = 1$.

Step 5: Calculate the weighted sum $m = \sum w_{ij}x_i$ where i, j = 1,2,3,4,5

$$\begin{aligned}
m_1 &= w_{11}x_1 + w_{12}x_2 + w_{13}x_3 + w_{14}x_4 + w_{15}x_5\\
&= (0.0)(0) + (0.1)(0) + (0.2)(1) + (0.3)(1) + (0.4)(1)\\
&= 0 + 0 + 0.2 + 0.3 + 0.4 = 0.9\\
m_2 &= w_{21}x_1 + w_{22}x_2 + w_{23}x_3 + w_{24}x_4 + w_{25}x_5\\
&= (0.2)(0) + (0.3)(0) + (0.4)(1) + (0.5)(1) + (0.6)(1)\\
&= 0 + 0 + 0.4 + 0.5 + 0.6 = 1.5\\
m_3 &= w_{31}x_1 + w_{32}x_2 + w_{33}x_3 + w_{34}x_4 + w_{35}x_5\\
&= (0.3)(0) + (0.4)(0) + (0.5)(1) + (0.6)(1) + (0.7)(1)\\
&= 0 + 0 + 0.5 + 0.6 + 0.7 = 1.8
\end{aligned}$$

Step 6 : The output neuron is

$$s_1 = f(m_1) = 0.9$$
$$s_2 = f(m_2) = 1.5$$
$$s_3 = f(m_3) = 1.8$$

Step 7: Calculate the sigmoid function

$$f(m) = 1.0/(1.0 + exp(-m))$$
$$f(m_1) = 1.0/(1.0 + exp(-m_1))$$
$$f(0.9) = 1.0/(1.0 + exp(-0.9)) = 0.71094$$
$$f(m_2) = 1.0/(1.0 + exp(-m_2))$$
$$f(1.5) = 1.0/(1.0 + exp(-1.5)) = 0.81757$$
$$f(m_3) = 1.0/(1.0 + exp(-m_3))$$
$$f(1.8) = 1.0/(1.0 + exp(-1.8)) = 0.85814$$

Step 8: Determine the minimum value

$$f(m_1) = 0.71094$$

$$f(m_2) = 0.81757$$

$$f(m_3) = 0.85814$$

The minimum value is 0.71094. So, $m_1$ is the best machine.

# 6. Conclusion

NN is interconnected network that resembles human brain. Artificial neural network are widely used as an effective approach for handling non-linear and noisy data, especially in situations where the physical process relationships are not fully understood and they are also particularly well suited to modeling complex systems on a real time basis. With the availability of this methodology, now it will be possible to investigate the approximation solution of other kinds of fuzzy system. In the textile sector present time is the time of modern and new era. Man has invented a lot of modern loom using with modern weft insertion system. So the comparison of conventional loom modified within the passing of time. In this, numerical example $m_1$ loom is best loom.

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