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On 3-rainbow Domination Number of Certain Graphs

Research Article*

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Abstract: Given an undirected graph G=(V,E) and a set of k-colors numbered 1,2. . . .k. The 3-rainbow domination defined as $f : V(G) \to \{1, 2, 3\}$ such that for each vertex $v \in V(G)$ with $f(v) = \phi$

$$\bigcup_{u \in N(v)} f(u) = \{1, 2, 3\}$$

Such a function is called a 3-rainbow dominating function (3RDF) and minimum weight of such function is called the 3-rainbow domination number of G and is denoted by $\gamma_{r3}(G)$. In this paper we found the 3-rainbow domination number of certain graphs.

Keywords: Domination number, 3-rainbow domination number, wheel graph, triangular snake graph, double triangular snake graph, n-barbell graph, n-sunlet graph, n-centipede graph, crown graph, clebsch graph.

1. Introduction and Preliminaries

In this paper, we review the notions of domination number, 3-rainbow domination number, wheel graph, Triangular snake graph, double triangular snake graph, n-barbell graph, n-sunlet graph, n-centipede graph, crown graph, clebsch graph [2,3,5,7].

Definition 1.1. *A subset T of vertex set V(G) of a graph G is said to be dominating set if every vertex in (V-T) is adjacent to a vertex in T. The minimum cardinality of dominating set is said to be domination number and is denoted by $\gamma(G)$.*

Definition 1.2. *Consider a graph G with $v\in V(G)$ and the open neighbourhood of v is the set $N(v)=\{u\in V(G)/uv\in E(G)\}$ and it closed neighborhood of v is the set $N[v]=\{v\}\cup N(v)$.let $f{:}v(G)\to P\{1,2,3\ldots\ldots k\}$ be a function that assigns to each vertex of G a set of colors chosen from the power set $\{1,2,3\ldots\ldots k\}$.if $v\in V(G)$ with $f(v)=\phi$ then*

$$\bigcup_{u \in N(v)} f(u) = \{1, 2, ..k\}$$

Then the function f is called k-rainbow domination function(k-RDF) of G.

Definition 1.3. *The weight of the function is defined by $w(f)=\sum_{v\in V(G)} |f(v)|$.The minimum weight of a k-RDF is called the k-rainbow domination number of G and it is denoted by $\gamma_{rk}(G)$.*

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Definition 1.4. *When k=3 then we define a mapping f:V(G)→P{1,2,3} such that for each vertex v∈ V(G) with f(v)=* ϕ *we have* $\bigcup_{u\in N(v)} f(u)$*={1,2,3} such function f is said to be a 3-rainbow dominating function(3-RDF) and the minimum weight of such function is said to be 3-rainbow domination number of G and it is denoted by* $\gamma_{r3}(G)$*.*

Definition 1.5. *A triangular cactus is a connected graph all of whose blocks are triangles. A triangular snake is a triangular cactus whose block-cut point graph is a path. Equivalently it is obtained from a path* $p=v_1,v_2,v_3,\ldots\ldots\ldots.v_{n+1}$ *by joining* v_{i+1} *to a new vertex* $u_1,u_2,\ldots\ldots\ldots.u_n$*..A triangular snake has 2n+1 vertices and 3n edges, where n is the number of blocks in the triangular snake. we denoted by* T_n*.*

Definition 1.6. *A double triangular snake D[*T_n*] consists of two triangular snake that have a common path ,that is a double triangular snake is obtained from a path* $v_1,v_2,\ldots\ldots v_n$ *by joining* v_i *and* v_{i+1} *to a new vertex* w_i *and to a new vertex* u_i *for* $1\leq i\leq n$*.*

Definition 1.7. *The n-sunlet graph* S_n *is a graph with cycle* C_n *and each vertex of the cycle attached to one pendent vertex .Each n-sunlet graph consists 2n nodes and 2n edges.*

Definition 1.8. *The n-centipede graph is a tree on 2n nodes obtained by joining the bottoms of n-copies of the path graph* P_2 *laid in a row with edge and it is denoted by* C_n*.*

Definition 1.9. *The n-barbell graph is the simple graph obtained by connecting two copies of a complete graph* K_n *by a bridge and it is denoted by B(*k_n*,*k_n*). '*

Definition 1.10. *The crown graph* ${S_n}^0$ *for an integer n> 2 is the graph with the vertex set* $\{u_1,u_2,\ldots.u_n,v_1,v_2,\ldots\ldots v_n\}$ *and edge set* $\{(u_i,v_i):1\leq i,j\leq n,i\neq j\}$

Definition 1.11. *The clebsch graph is a strongly regular quintic graph on 16 vertices and 40 edges .It is also known as the Greenwood Gleason graph.*

2. On 3-rainbow Domination Number of Certain Graphs

In this paper we find the 3-rainbow domination number of wheel graph, triangular snake graph, double triangular snake graph, n-sunlet graph, n-barbell graph, n-centipede graph, crown graph, clebsch graph.

Theorem 2.1. *Let* W_n *be the wheel graph then* $\gamma_{r3}(W_n)$ *=3, n≥3.*

Proof. The wheel graph is obtained by joining cycle graphs C_n and complete graph K_1.i.e)$W_n=C_n+K_1$ we prove this theorem by using induction method. When n=3 the graph W_3contains 4 vertices and 6 edges then all the vertices in the cycle are connected to the hub to form W_3. Let S be the dominating set of $W_3$with $|S|=\gamma_{r1}(W_3)$=1and define a function f:V(W_3)→P{1,2,3} such that we assign color class{1,2,3} to the vertex in the hub and assign empty color to the remaining vertices in the wheel graph. The minimum sum of numbers of assigned colors overall vertices of $W_3$is 3. The 3-rainbow domination number of W_3 is 3.(i.e) $\gamma_{r3}(W_3)=3$.

When n=4 the graph W_4contains 5 vertices and 8 edges . Here all the vertices in the cycle C_4 is connected to the hub to form W_4. Let S be the dominating set of $W_4$with $|S|=\gamma_{r1}(W_4)$=1and define a function f:V(W_4)→P{1,2,3} such that we assign color class{1,2,3} to the vertex in the hub and assign empty color to the remaining vertices in the wheel graph. The minimum sum of numbers of assigned colors overall vertices of $W_4$is 3. The 3-rainbow domination number of W_4 is 3.(i.e.) $\gamma_{r3}(W_4)=3$.

By proceeding in this manner we get the general term for n, the graph $W_n$contains n+1 vertices and 2n edges then here all the vertices in the cycle C_n is connected to the hub to form W_n. Let S be the dominating set of $W_n$with $|S| = \gamma_{r1}(W_n)$=1and define a function f:V(W_n)→P{1,2,3} such that we assign color class{1,2,3} to the vertex in the hub and assign empty color to the remaining vertices in the wheel graph. The minimum sum of numbers of assigned colors overall vertices of $W_n$is 3. The 3-rainbow domination number of W_n is 3.(i.e.) $\gamma_{r3}(W_n) = 3$. □

Example 2.2. *The 3- rainbow domination number for wheel graph W_5. Here there is only one dominating set, S={ v_6}, then $\gamma(W_5)$=1 we assign the color set {1,2,3} to the dominating set { v_6} which is in figure for each vertex v in $\{v_1, v_2, v_3, v_4, v_5\}$ then f {v}=ϕ and $\bigcup_{u\in N(V)} f(u)$={1,2,3}. Thus $\gamma_{r3}(W_5)$=3*

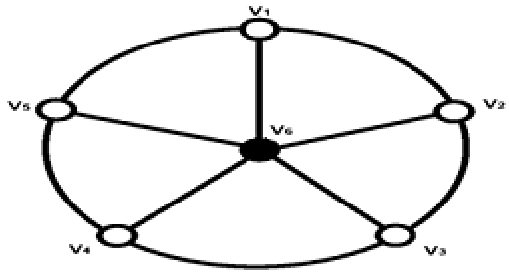


Figure 1. (W_5)

The 3-rainbow domination number of wheel graph of $W_5$is 3. i.e. $\gamma_{r3}(W_5) \leq 3$.

Theorem 2.3. *Let $T_n$be the triangular snake graph then* $\gamma_{r3}(T_n)=\begin{cases} 3\left\lfloor \frac{n}{2} \right\rfloor + 1 & \text{when n is odd} \\ 3\left(\frac{n}{2}\right) & \text{when n is even} \end{cases}$

Proof. Let $T_n$be the triangular snake graph has n +1 vertices and 3n edges and it is obtained from a path p=v_1,v_2,v_3,..........v_{n+1} by joining v_i andv_{i+1} to a new vertex u_1,u_2,..........u_n..

Case:1 when n is odd Subcase:1.1 when n=1 We have triangular snake graph T_1which contains 3 vertices and 3 edges. Let S be the dominating set of $T_1$with $|S| = \gamma_{r1}(T_1)$=1and define a function f:V(T_1)→P{1,2,3}.such that we assign color class{1,2,3} to the vertex $v_1$in the set S and assign empty color to the remaining vertices v_2,v_3.The minimum sum of number of assigned colors overall vertices is 1.clearly f is a 3-rainbow domination function and $\gamma_{r3}(T_1)$=3.

Subcase:1.2 when n =3.

The triangular snake graph T_3contains 7 vertices and 9 edges. Let S be the dominating set of $T_3$with $|S| = \gamma_{r1}(T_3)$=2and define a function f:V(T_3)→P{1,2,3} such that we assign color class{1,2,3} to the vertices in the set S and assign empty color to the remaining vertices . The minimum sum of numbers of assigned colors overall vertices of $T_3$is 2. clearly f is a 3-rainbow domination function and $\gamma_{r3}(T_3)$=6. Repeating in this manner for order n, we get $\gamma_{r3}(T_n)$= $3\left\lfloor \frac{n}{2} \right\rfloor + 1$ ∀ n is odd.

Case:2 when n is even

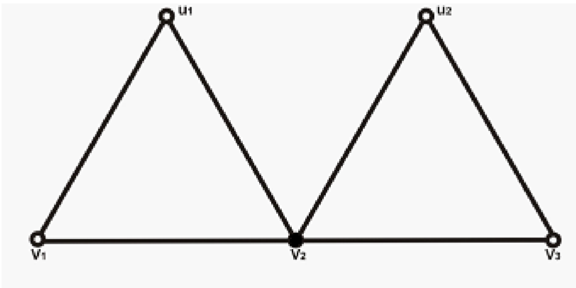
Subcase:2.1 when n=2

The triangular snake graph T_2contains 5 vertices and 6 edges. Let S be the dominating set of $T_2$with $|S| = \gamma_{r1}(T_2)$=1and define a function f:V(T_2)→P{1,2,3}.such that we assign color class{1,2,3} to the vertices in the set S and assign empty color to the remaining vertices .The minimum sum of number of assigned colors overall vertices is 1.clearly f is a 3-rainbow domination function and $\gamma_{r3}(T_2)$=3.

Subcase:2.2 when n =4

The triangular snake graph T_4contains 9 vertices and 12 edges. Let S be the dominating set of $T_4$with $|S| = \gamma_{r1}(T_4)$=2and define a function f :V(T_4)→P{1,2,3} such that we assign color class{1,2,3} to the vertices in the set S and assign empty color to the remaining vertices .The minimum sum of numbers of assigned colors overall vertices of $T_4$is 2 . clearly f is a 3-rainbow domination function and $\gamma_{r3}(T_4)$=6. Repeating in this manner for order n, we get $\gamma_{r3}(T_n)=3\left(\frac{n}{2}\right)$ ∀ n is even. This function is a 3-rainbow dominating function of T_n and we have $\gamma_{r3}(T_n) = \begin{cases} 3\left\lfloor\frac{n}{2}\right\rfloor+1 & \textit{when n is odd} \\ 3\left(\frac{n}{2}\right) & \textit{when n is even}\end{cases}$ □

Example 2.4.



The 3-rainbow domination of triangular snake graph of T_2 is 3. (i.e.) $\gamma_{r3}(T_2)$=3. we assign color set {1,2,3} to the vertex {v_2}and remaining vertices are assign empty color.

Theorem 2.5. *Let $D(T_n)$ be the double triangular snake graph then* $\gamma_{r3}(D(T_n))= \begin{cases} 3\left\lfloor\frac{n}{2}\right\rfloor+1 & \textit{when n is odd} \\ 3\left(\frac{n}{2}\right) & \textit{when n is even}\end{cases}$

Proof. Let D(T_n) be the double triangular snake graph with 3n+1 vertices and 5n edges.Let{v_1,v_2,. . . . . . v_{n+1},u_1. . . . .u_n,w_1,w_2,.......wn}be the vertices of the double triangular snake graph D(T_n).

Case:1 when n is odd

Subcase:1.1 when n=1

The double triangular snake graph D(T_1)it contains 4 vertices and 5 edges. Let S be the dominating set of D(T_1)with $|S| = \gamma_{r1}$(D(T_1)=1and define a function f:V(D(T_1))→P{1,2,3}.such that we assign color class{1,2,3} to the vertex $v_1$in the set S and assign empty color to the remaining vertices v_2,v_3,v_4.The minimum sum of number of assigned colors overall vertices is 3.clearly f is a 3-rainbow domination function and γ_{r3}(D(T_1) =3.

Subcase:1.2 when n =3

The double triangular snake graph D(T_3) contains 7 vertices and 9 edges. Let S be the dominating set of $T_3$with $|S| = \gamma_{r1}$ (D(T_3))=2and define a function f:V(D(T_3))→P{1,2,3} such that we assign color class{1,2,3} to the vertices in the set S and assign empty color to the remaining vertices . The minimum sum of numbers of assigned colors overall vertices of D(T_3) is 6. Clearly f is a 3-rainbow domination function and we have γ_{r3}(D(T_3))=6. Repeating this process for n times we get γ_{r3}(D(T_n))=$3\left\lfloor\frac{n}{2}\right\rfloor+1$ ∀ n is odd.

Case:2 when n is even

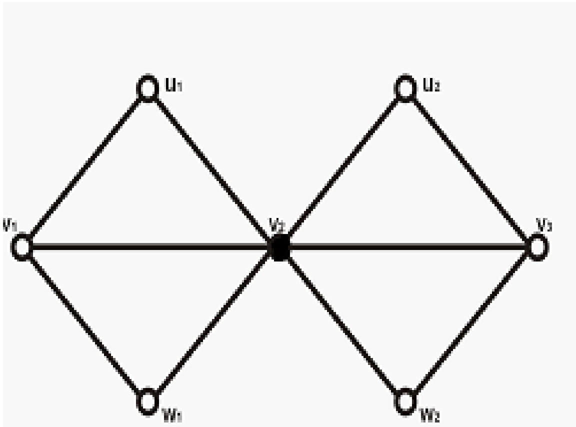
Subcase:2.1 when n=2

The double triangular snake graph T_2contains 7 vertices and 10 edges. Let S be the dominating set of D(T_2)with $|S| = \gamma_{r1}$(D(T_2))=1and define a function f:V(D(T_2))→P{1,2,3} such that we assign color class{1,2,3} to the vertices in the set S and assign empty color to the remaining vertices .The minimum sum of number of assigned colors overall vertices is 3.clearly f is a 3-rainbow domination function and γ_{r3}(D(T_2))=3.

Subcase:2.2 when n =4

The double triangular snake graph D(T_4)contains 14 vertices and 20 edges. Let S be the dominating set of $T_4$with $|S| = \gamma_{r1}$(D(T_4))=2and define a function f:V(D(T_4))→P{1,2,3} such that we assign color class{1,2,3} to the vertices in the set S and assign empty color to the remaining vertices .The minimum sum of numbers of assigned colors overall vertices of $T_4$is 2 . clearly f is a 3-rainbow domination function and γ_{r3}(T_4)=6. Repeating in this manner we get $\gamma_{r3}(T_n)= 3\left(\frac{n}{2}\right)$ ∀ n is even. This function is a 3-rainbow dominating function of D(T_n) and we have $\gamma_{r3}(D(T_n)) = \begin{cases} 3\left\lfloor \frac{n}{2} \right\rfloor + 1 & \textit{when n is odd} \\ 3\left(\frac{n}{2}\right) & \textit{when n is even} \end{cases}$ □

Example 2.6.



The 3-rainbow domination number of double triangular snake graph $T_2$is 3. (i.e.) γ_{r3}(D(T_2))=3.we assign color set {1,2,3} to the vertex {v_2}and remaining vertices are assign empty color.

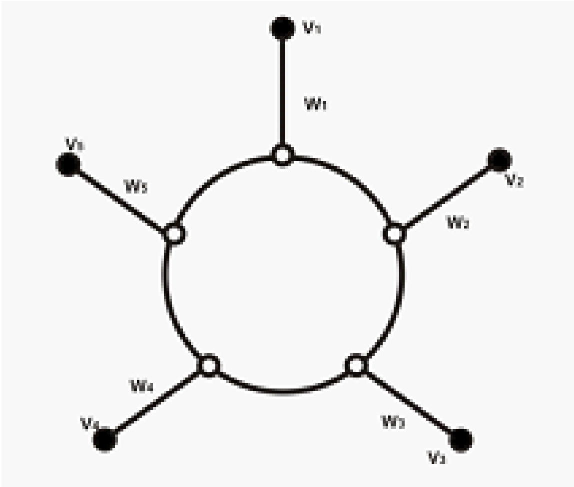
Theorem 2.7. *Let S_n be the n-sunlet graph then $\gamma_{r3}(S_{n)}) \leq 3n$, when $n \geq 3$.*

Proof. Let $S_n$be the n-sunlet graph on 2n vertices is obtained by attaching n-pendent edges to the cycle C_n. let the pendent vertices be defined by V={v_1,v_2,v_3.................v_n}and the vertices in the cycle be defined by W={w_1,w_2,........w_n}. This theorem is proved by using induction method, When n =3 let D be a dominating set of $S_3$with $|D| = \gamma_{r1}(S_3)$=3and define f:V(S_3)→P{1,2,3} such that we assign color class{1,2,3}to the vertices in the set D and assign the empty color to the remaining vertices. The minimum sum of numbers of assigned colors overall vertices of S_3 is 9. This function is a 3-rainbow dominating function of $S_3$and we have $\gamma_{r3}(S_3) \leq 9$.
By proceeding in this manner for order n, Let D be a dominating set of $S_n$with $|D| = \gamma_{r1}(S_n)$=nand define f:V(S_n)→P{1,2,3} such that we assign color class{1,2,3}to the vertices in the set D and assign the empty color to the remaining vertices. The minimum sum of numbers of assigned colors overall vertices of S_n is 3n. This function is a 3-rainbow dominating function of $S_n$and we have $\gamma_{r3}(S_n) \leq$ 3n. □

Example 2.8.
The 3-rainbow domination number of n-sunlet graph is 5 (i.e.) γ_{r3}(S_5)=5.we assign color set {1,2,3} to the pendent vertices and remaining vertices are assign empty color

Theorem 2.9. *Let G be the n-centipede graph then $\gamma_{r3}(G) \leq 3n$.*

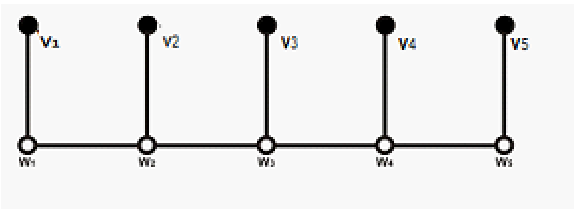
Proof. The n-centipede graph is a tree on 2n vertices obtained by joining the bottoms of n-copies of the path graph p_2 laid in a row with edge. Let the end vertices be the defined by V={v_1,v_2,v_3.................v_n}and the support vertices be W={w_1,w_2,........w_n}. We prove this theorem by using induction method, When n=3 the n-centipede graph contains 6 vertices and 3 edges .Let D be a dominating set of G with $|D| = \gamma(G)$=nand define f:V(G)→P{1,2,3} such that we assign color class{1,2,3}to the vertices in the set D and assign the empty color to the remaining vertices. The minimum sum of



numbers of assigned colors overall vertices of G is 9. This function is a 3-rainbow dominating function of G and we have γ_{r3}(G) $\leq$ 9.

Proceeding in this way for order n. The n-centipede graph contains 2n vertices and n edges .Let D be a dominating set of G with $|D| = \gamma(G)$=nand define f:V(G)→P{1,2,3} such that we assign color class{1,2,3}to the vertices in the set D and assign the empty color to the remaining vertices. The minimum sum of numbers of assigned colors overall vertices of G is 3n . This function is a 3-rainbow dominating function of G and we have γ_{r3}(G) $\leq$ 3n. □

Example 2.10.



The 3-rainbow domination of n-centipede graph is 5 i.e. $\gamma_{r3}(G)$*=5.we assign color set* {*1,2,3*} *to the pendent vertices and assign empty color to the remaining vertices.*

Theorem 2.11. *Let* $B(k_n,k_n)$ *be the n-barbell graph then* $\gamma_{r3}(B(k_n,k_n))$*=6,when* $n \geq 3$*.*

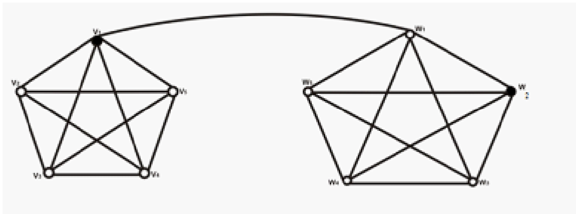
Proof. Let the B(k_n,k_n) be the n-barbell graph is obtained by connecting two copies of a complete graph K_n by a bridge. Let V={v_1,v_2,v_3................v_n} be the vertex set of copy A and W={w_1,w_2,........w_n} be the vertex set copy B. we prove this theorem by using induction method,

When n=3 the B(k_3,k_3) be the n-barbell graph is the simple graph obtained by connecting two copies of a complete graph K_3 by a bridge let D be a dominating set of B(k_3,k_3) with $|D| = \gamma_{r1}$(B(k_3,k_3))=2and define a function f:V(B(k_3,k_3))→ P{1,2,3}Such that we assign color class{1,2,3}to the vertices in the set D and assign the empty color to the remaining vertices. The minimum sum of numbers of assigned colors overall vertices of B(k_3,k_3) is 6.This function is a 3-rainbow dominating function of B(k_3,k_3) and we have γ_{r3} (B(k_3,k_3))=6.

When n=4 let D be a dominating set of B(k_4,k_4) with $|D| = \gamma_{r1}$(B(k_4,k_4)) =2and define a function f:V(B(k_4,k_4))→ P{1,2,3}Such that we assign color class{1,2,3}to the vertices in the set D and assign the empty color to the remaining vertices. The minimum sum of numbers of assigned colors overall vertices of B(k_4,k_4) is 6.This function is a 3-rainbow dominating function of B(k_4,k_4) and we have γ_{r3} (B(k_4,k_4))=6. By proceeding in this manner for order n.

Let D be a dominating set of B(k_n,k_n) with $|D| = \gamma_{r1}$ B(k_n,k_n)) =2and define a function f:V(B(k_n,k_n))→ P{1,2,3}Such that we assign color class{1,2,3}to the vertices in the set D and assign the empty color to the remaining vertices. The minimum sum of numbers of assigned colors overall vertices of B(k_n,k_n) is 6.This function is a 3-rainbow dominating function of B(k_n,k_n) and we have γ_{r3} (B(k_n,k_n))=6. □

Example 2.12.

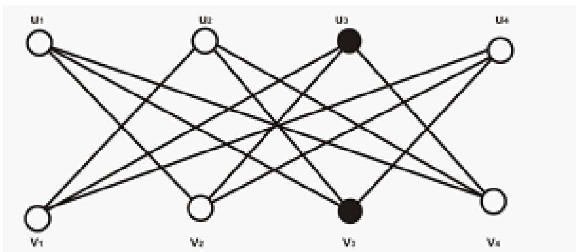


The 3-rainbow domination of barbell graph is i.e. $\gamma_{r3}(B(k_5,k_5))$=6,we assign color set {1,2,3} to vertex set $\{v_1, w_2\}$and assign empty color to the remaining vertices.

Theorem 2.13. *Let $S_n{}^0$ be the crown graph then $\gamma_{r3}(S_n{}^0)$ =6 when n≥3.*

Proof. Consider a crown graph $S_n{}^0$with the vertex set {u_1,u_2,. . . .u_n,v_1,v_2,. . . . . . v_n} and edge set {(u_i,v_i):1≤ $i, j \leq n$,i≠j},to prove this theorem we use induction method.
When n=3 the crown graph contains 6 vertices and 6 edges. Let D be a dominating set of $S_3{}^0$ with $|D| = \gamma_{r1}(S_3{}^0)$ =2and define f:V($S_3{}^0$)→P{1,2,3} such that we assign color class{1,2,3}to the vertices in the set D and assign the empty color to the remaining vertices. The minimum sum of numbers of assigned colors overall vertices of $S_3{}^0$ is 6 . This function is a 3-rainbow dominating function of $S_3{}^0$and we have γ_{r3}($S_3{}^0$) =6.
When n=4 the graph contains 4 vertices and 4 edges. Let D be a dominating set of $S_4{}^0$ with $|D| = \gamma_{r1}(S_4{}^0)$ =2and define f:V($S_4{}^0$)→P{1,2,3} such that we assign color class{1,2,3}to the vertices in the set D and assign the empty color to the remaining vertices. The minimum sum of numbers of assigned colors overall vertices of $S_4{}^0$ is 6 . This function is a 3-rainbow dominating function of $S_4{}^0$and we have γ_{r3}($S_4{}^0$) =6.
By proceeding this way for order n ,the crown graph contains 2n vertices and 2n edges. Let D be a dominating set of $S_n{}^0$ with $|D| = \gamma_{r1}(S_n{}^0)$ =2and define f:V($S_n{}^0$)→P{1,2,3} such that we assign color class{1,2,3}to the vertices in the set D and assign the empty color to the remaining vertices. The minimum sum of numbers of assigned colors overall vertices of $S_n{}^0$ is 6. This function is a 3-rainbow dominating function of $S_n{}^0$and we have γ_{r3}($S_n{}^0$) =6. □

Example 2.14.

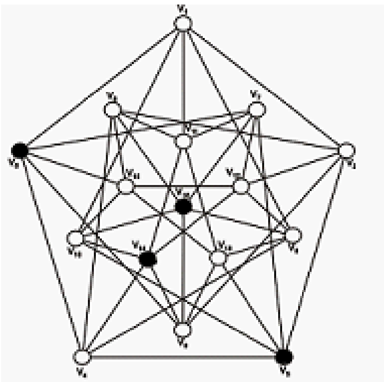


The 3-rainbow domination number of crown graph is 6 (i.e) $\gamma_{r3}(S_4{}^0)$ =6.And we assign color set {1,2,3} to the vertices{ u_3, v_3}and remaining vertices are assign empty color.

Theorem 2.15. *Let G be the clebsch graph then $\gamma_{r3}(G)$ =12.*

Proof. Let G be the clebsch graph is strongly regular quintic graph on 16 vertices and 40 edges. Let S be the dominating set of G with $|S|=\gamma_{r1}$(G)=4and define a function f:V(G)→P{1,2,3} such that we assign color class{1,2,3} to the vertices in the set S and assign empty color to the remaining vertices. The minimum sum of numbers of assigned colors overall vertices of G is 12 . This function is a 3-rainbow dominating function of Gand we have γ_{r3}(G) =12. □

Example 2.16.



Clebsch Graph

The 3-rainbow domination number of clebsch graph is 12. (i.e) γ_{r3}(G)=12.And we assign color set {1,2,3} to vertices $\{v_3, v_5, v_{14}, v_{16}\}$*and remaining vertices are assign empty color.*

3. Conclusion

In this paper we established 3-rainbow domination number of wheel graph, triangular snake graph, double triangular snake graph ,n-barbell graph, n-sunlet graph, n-centipede graph, crown graph, clebsch graph. This work could be further extended to other classes of graphs also.

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