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Fuzzily Determined the Game Value for Decision Making of an Interval Matrix Game on Four Players

Research Article*

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Abstract: In this paper, we consider a solution for Fuzzy matrix game with fuzzy pay offs. The Solution of Fuzzy matrix games with pure strategies with max min min max principle is discussed. A method takes advantage of the relationship between fuzzy sets and fuzzy matrix game theories can be offered for multicriteria decision making. To model such uncertainty in matrix games, we consider interval-valued game matrices In this paper we extend the results of classical strictly-determined matrix games to fuzzily determined interval matrix games.

Keywords: Fuzzy set, fuzzy payoffs, fuzzy matrix games, strategy.

1. Introduction

The mathematical treatment of the Game theory was made available in 1944, when Johnvon Newmann and Oscar Morgenstern published the famous article 'Theory of games and economic Behavior. The problem of Game theory defined as a body ofknowledge that deals with making decisions when two or more intelligent and rational opponents are involved under conditions of conflict and competition. Game theory has played an important role in the fields of decision making theory such as economics,management etc. When we apply the Game theory to model some practical problems which we encounter in real situations, we have to know the values of payoffs exactly.However, it is difficult to know the exact values of payoffs and we could only know the values of payoffs approximately. In such situations, it is useful to model the problems as games with fuzzy payoffs.In a fuzzy game problem, all parameters are fuzzy numbers. Fuzzy numbers may be normal (or) abnormal. Using this ranking, the fuzzy Game problem is converted to a crisp value problem, which can be solved using the traditional method.

Definition 1.1 (Fuzzy set)**.** *Let X be a non empty set, A fuzzy set A in X is characterized by its membership function $A \rightarrow [0, 1]$ and $A(x)$ is interpreted as the degree of membership of element x in fuzzy A for each $x \epsilon X$. The Value zero is used to represent complete non-membership; the value one is used to represent complete membership and values in between are used to represent intermediate degrees of membership. The mapping A is also called the membership function of fuzzy set A.*

Definition 1.2 (Crisp set)**.** *A crisp set is a special case of a fuzzy set, in which the membership function only takes two values, commonly defined as 0 and 1.*

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Definition 1.3 (Fuzzy number)**.** *A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions.*

(1) There exist atleast one $X_0 \epsilon R$ with $\mu_{\tilde{A}}(X_0) = 1$

(2) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

(3) $\tilde{A}$ must be normal and convex.

2. Fuzzy Matrix Games

The games described in this section will now be considered in terms of the theory of the fuzzy sets.The lack of operationalization has not yet allowed them to become practically used. The classical theory of games assumes that interpersonal conflict situations can be precisely described mathematically. The assumption made in this context is that the elements of a particular game can be represented as sharply defined sets. This involves an analysis of given mathematical expressions. For more stringent requirements to modelling the existence of clearly defined sets can not be postulated. The elements of the game are affected by various sources of fuzziness. The Gain or payoff function is not always defined numerically or sharply. The strategies are employed by players are usually marked by different levels of significance and intensity. These and other conditions account for the need to include the theory of fuzzy sets in the solution concept of the theory of games.

2.1. Comparing Intervals

In order to compare strategies and payoffs for an interval game matrix, we need to define a notion of interval inequality (both $\leq$and$\geq$) that corresponds to an intuitive notion of a "better possible"outcome/payoff. Let **x** and **y** be two non-empty intervals. We will consider their relationship in thefollowing different cases:

Case 1: $\mathbf{x} \bigcap \mathbf{y} =$; and x < y (see Figure 1). In this case, every possible payoff value from **y**exceeds all of the possible payoffs from **x**. Therefore, we say that **x** <**y** and **y** >**x** crisply, whichcorresponds to the traditional definition of comparison used in interval computations [4].

Case 2: $\mathbf{x} = \mathbf{y}$. We then define the crisp inequalities $\mathbf{x} \leq \mathbf{y}$ and $\mathbf{Y} \leq \mathbf{X}$, again paralleling commonusage of existing interval inequality comparisons.

Case 3: $\mathbf{x} \bigcap \mathbf{y} \neq \emptyset$;and $\mathbf{x} \neq \mathbf{y}$.

Definition 2.1. *If the value of $\boldsymbol{x} \leq \boldsymbol{y}$ is exactly one or zero, then we say that **x** and **y** are crisply comparable.Otherwise, we say that they are fuzzily comparable.*

3. Crisply Determined Interval Matrix Game

Now we extend the concept of classical strictly determined games to interval matrix gameswhose row and column entries are crisply comparable.

Definition 3.1. *Let **G** be a m×n interval game matrix such that all intervals in the same row (or column) of **G** are crisply comparable. If there exists a **g**ij2 **G** such that **g**ijis simultaneously crisply less than or equal to **g**ik for all k ϵ {1, 2,..., n} and crisply greater than or equal to **g**lj for all l ϵ{1, 2,...,m} then the interval **g**ij is called a saddle interval of the game. An interval matrix game is crisply determined if it has a saddle interval. By the definition above, to determine whether an interval game matrix is crisply determined, one needs only to do the following:*

*1. For each row (1 ≤i≤ m), find the entry **g**ij that is crisply less than or equal to all other entries in the ith row.*

*2. For each column (1 ≤j ≤n), find the entry **g**ij that is crisply greater than or equal to all other entries in the jth column.*

*3. Determine if there is an entry **g**ij that is simultaneously the minimum of the ith row and the maximum of the jth column.*

4. If any of the above values cannot be found the game is not crisply determined. Otherwise, it is a crisply determined interval matrix game.

Definition 3.2. *Let* ***G*** *be an m × n interval game matrix. If there is a* ***g****ij*ϵ***G*** *such that* ***g****ij is simultaneously a least and a greatest interval for the ith row and the jth column of* ***G****, respectively, then* ***G*** *is a fuzzily determined interval game. We also call such* ***g****ij a fuzzy saddle interval of the game with its membership as min*{μ*(****g****ij),* γ*(****g****ij)*}*. It is obvious that the crisply determined interval game defined in just a special case of fuzzily determined interval game with 1 as its membership. The game value of a fuzzily determined interval game can be reasonably defined as its fuzzy saddle interval with respect to its membership. For the convenience of computer implementations, we summarize our discussion as the algorithm below.*

Algorithm

Step-1 : Choose the strategy of the players.

Step-2 : Construct the interval game Matrix.

Step-3 : Compute the crisp game value.

Step-4 : Computing the minimum interval and maximum interval of fuzzy matrix game.

Step-5 : Find the decision with highest strategical value.

The matrix game is given as follows:

	B_1	B_2	B_3	B_4
A_1	(a_1,b_1)	(a_2,b_2)	(a_3,b_3)	(a_4,b_4)
A_2	(a_5,b_5)	(a_6,b_6)	(a_7,b_7)	(a_8,b_8)
A_3	(a_9,b_9)	(a_{10},b_{10})	(a_{11},b_{11})	$(a_{12},b_{12)})$
A_4	(a_{13},b_{13})	(a_{14},b_{14})	(a_{15},b_{15})	(a_{16},b_{16})

Table 1. **Four players table**

The players A and B have strategies [A_1,A_2] and [B_1,B_2]. In the first case, We assume the player C chooses A_1 and in the second case he chooses A_2 and in third case he choose A_3 and finally the fourth case the player chooses the strstegy A_4.

	B_1	B_2	B_3	B_4
A_1	[0,1]	[4,5]	[-2,0]	[1,5]
A_2	[5,6]	[2,3]	[1,4]	[1,3]
A_3	[-8,-6]	[-1,0]	[-4,-2]	[-3,-1]
A_4	[-2,0]	[-2,1]	[3,4]	[-3,-2]

Example 3.3.

Minimum Interval for A_1

A_{11}

[0,1]<[4,5]=1 , [0,1]<[-2,0]=0

[0,1]<[1,5]=1 , [0,1]<[5,6]=1

[0,1]<[2,3]=1 , [0,1]<[1,4]=1

[0,1]<[1,3]=1 , [0,1]<[-8,-6]=0

[0,1]<[-1,0]=0 , [0,1]<[-4,-2]=0

[0,1]<[-3,-1]=0, [0,1]<[-2,0]=0

[0,1]<[-2,1]=0 , [0,1]<[3,4]=1

[0,1]<[-3,-2]=0.

Min{ 1,0,1,1,1,1,1,0,0,0,0,0,0,,1,0 }=0

A_{12}

[4,5]<[0,1]=0 , [4,5]<[-2,0]=0

[4,5]<[1,5]=0 , [4,5] <[5,6]=1

[4,5]<[2,3]=0 , [4,5]<[1,4]=0

[4,5]<[1,3]=0 , [4,5] <[-8,-6]=0

[4,5]<[-1,0]=0 , [4,5]<[-4,-2]=0

[4,5]<[-3,-1]=0 , [4,5]<[-2,0]=0

[4,5]<[-2,1]=0 , [4,5]<[3,4]=0

[4,5]<[-3,-2]=0

Min{ 0,0,0,1,0,0,0,0,0,0,0,0,0,0,0 }=0

A_{13}

[-2,0]<[0,1]=1 , [-2,0]<[4,5]=1

[-2,0]<[1,5]=1 , [-2,0]<[5,6]=1

[-2,0]<[2,3]=1 , [-2,0]<[1,4]=1

[-2,0]<[1,3]=1 , [-2,0]<[-8,-6]=0

[-2,0]<[-1,0]=0.5 , [-2,0]<[-4,-2]=0

[-2,0]<[-3,-1]=0 , [-2,0]<[-2,0]=1

[-2,0]<[-2,1]=0.3 , [-2,0]<[3,4]=1

[-2,0]<[-3,-2]=0.

Min{ 1,1,1,1,1,1,1,0,0.5,0,0,1,0.3,1,0 }=0

A_{14}

[1,5]<[0,1]=0 , [1,5]<[4,5]=0.8

[1,5]<[-2,0]=0 , [1,5]<[5,6]=1

[1,5]<[2,3]=0.3 , [1,5]<[1,4]=0

[1,5]<[1,3]=0 , [1,5]<[-8,-6]=0

[1,5]<[-1,0]=0 , [1,5]<[-4,-2]=0

[1,5]<[-3,-1]=0 , [1,5]<[-2,0]=0

[1,5]<[-2,1]=0 , [1,5]<[3,4]=0.5

[1,5]<[-3,-2]=0

Min{ 0,0.8,0,1,0.3,0,0,0,0,0,0,0,0,0.5,0 }=0.

Hence Max{0,0,0,0}=0

Maximum Interval for A_1

A_{11}

[0,1]> [4,5]=0 , [0,1]>[-2,0]=1

[0,1]>[1,5]=0 , [0,1]<[5,6]=0

[0,1]>[2,3]=0 , [0,1]>[1,4]=0

[0,1]>[1,3]=0 , [0,1]>[-8,-6]=1

[0,1]>[-1,0]=1 , [0,1]>[-4,-2]=-1

[0,1]>[-3,-1]=1, [0,1]>[-2,0]=1

[0,1]>[-2,1]=0.7 , [0,1]>[3,4]=0

[0,1]>[-3,-2]=1.

Max { 0,1,0,0,0,0,0,1,1,1,1,1,0.7,0,1 }=1

A_{12}

[4,5]>[0,1]=1 , [4,5]>[-2,0]=1

[4,5]>[1,5]=0.8 , [4,5] >[5,6]=0

[4,5]>[2,3]=1 , [4,5]>[1,4]=1

[4,5]>[1,3]=1 , [4,5]>[-8,-6]=1

[4,5]>[-1,0]=1 , [4,5]>[-4,-2]=1

[4,5]>[-3,-1]=1 , [4,5]>[-2,0]=1

[4,5]>[-2,1]=1 , [4,5]>[3,4]=1

[4,5]>[-3,-2]=1

Max { 1,1,0.8,0,1,1,1,1,1,1,1,1,1,1,1 }=1

A_{13}

[-2,0]>[0,1]=0 , [-2,0]>[4,5]=0

[-2,0]>[1,5]=0 , [-2,0]>[2,3]=0 ,

[-2,0]>[1,4]=0 , [-2,0]>[1,3]=0 ,

[-2,0]>[-8,-6]=1 , [-2,0]>[-1,0]=0 ,

[-2,0]>[-4,-2]=1 , [-2,0]>[-3,-1]=0 ,

[-2,0]>[-2,0]=1 , [-2,0]>[-2,1]=0 ,

[-2,0]>[3,4]=0 , [-2,0]>[-3,-2]=2.

[-2,0]>[-1,0]=0

Max { 0,0,0,0,0,0,1,0,1,0,1,0,0,1,0 }=1

A_{14}

[1,5]>[0,1]=1 , [1,5]>[4,5]=0

[1,5]>[-2,0]=1 , [1,5]>[5,6]=0

[1,5]>[2,3]=0.5 , [1,5]>[1,4]=0.3

[1,5]>[1,3]=0.5 , [1,5]>[-8,-6]=1

[1,5]>[-1,0]=1 , [1,5]>[-4,-2]=1

[1,5]>[-3,-1]=1 , [1,5]>[-2,0]=1

[1,5]>[-2,1]=1 , [1,5]>[3,4]=0.3

[1,5]>[-3,-2]=1

Max { 1,0,1,0,0.5,0.3,0.5,1,1,1,1,1,1,0.3,1 }=1. Hence Min{1,1,1,1}=1.This corresponds to the interval [4,5] and [1,5], if the third player c chooses A_1 Who wins. If he chooses others strategies he loses the game.

4. Conclusion and Future Work

This paper develops a static model of a fuzzy game by extending the fuzzy decision theory of Bellman and Zadeh (1970). The model developed is simple and exploratory in nature. We identify conditions that guarantee theexistence of equilibrium as well as how to attain a certain minimum payoff in the game.In this paper, we have introduced interval valued matrix games. By defining fuzzy comparison relations, we extended the strategies for classical strictly determined matrix games into fuzzily determined interval matrix games. This extension provides a method of handling uncertainty in decision making modelled by matrix games. This clearly does not cover all possible cases for an interval matrix game as in the classical case. Some interval matrix game may be neither crisply nor fuzzily determined. One approach that we are investigating for such non-determined games is a consideration of a combination of the μ and as a measurement of fuzzy saddle intervals. Fuzzy matrix games provide numerous new possibilities of handling practical engineering,economic, investment planning, and other problems. The resolution of fuzzy matrix games constitutes a new quality of decisions representing a high degree of complexity The concept of an interval valued four-person fuzzy game can be extended to a multi-player one.

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