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Harmonious Coloring of Middle and Central Graph of Some Special Graphs

Research Article*

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Abstract: Let $G(V,E)$ be an undirected graph. The Harmonious coloring of a graph G is a proper vertex coloring in which each pair of colors appears on at most one pair of adjacent vertices. The Harmonious chromatic number is minimum number of colors needed for the harmonious coloring of G. In this paper we investigate the harmonious chromatic number of middle and central graph of some special graphs.

Keywords: Proper coloring, Middle graph, Central graph, Tadpole graph Web graphs.

1. Introduction

In this paper, we have taken the graphs to be finite, undirected graphs. Every graph has a harmonious coloring, since it suffices to assign every vertex a distinct color. There trivially exist graphs G with $\chi_H(G) > \chi(G)$. Then Akhalak Masuri, R.S.Chandel Vijay Gupta published a paper On Harmonious Coloring of $M[Y_n]$ and $C[Y_n]$. In this paper we discuss about the harmonious chromatic number of central and middle graph of tadpole graph and web graph.

2. Preliminaries

Definition 2.1 (Proper Coloring)**.** *A graph G having no two adjacent vertices receive the same color is said to be proper coloring.*

Definition 2.2 (Middle Graph)**.** *The M iddle graph of G denoted by $M(G)$. The Vertex set of M (G) is $V(G) \cup E(G)$ in which two elements are adjacent in $M(G)$ if the following conditions holds.*
(1) $x, y \in E(G)$ and x, y are adjacent in G.
(2) $x \in V(G)$, $y \in E(G)$ and they are incident in G Or A graph G is obtained by subdividing each edge of G exactly once and join all the newly middle vertices of adjacent edges of G is called the Middle graph.

Definition 2.3 (Central Graph)**.** *The central graph of G, denoted by C(G) is obtained by subdividing each edge of G exactly once and joining all the non adjacent vertices G in C(G).*

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Definition 2.4 (Tadpole Graph)**.** *The (m,n)-tadpole graph is the graph is obtained by joining a cycle graph* $\mathrm{C_n}$ *to a path graph* $\mathrm{P_n}$ *with a bridge.*

Example 2.5.

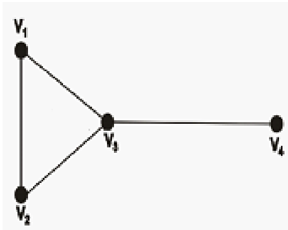
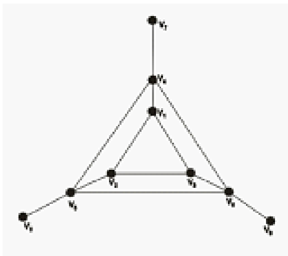


Figure 1: $T_{(3,1)}$

Definition 2.6 (Web Graph)**.** *Kohetal (1980) and Gallian (2007) define a web graph as generalized prism graph* Y_{n+13}*. With edges of outer cycle removed*

Example 2.7.



3. Harmonious Chromatic Number of Tadpole Graph and Web Graph

In this section we have obtained Harmonious chromatic number of middle and central graph of Tadpole graph and web graph.

Theorem 3.1. *For Tadpole graph* $T_{m,n}$ *where* $n = 1$ *and* $m = 3$ *then,*

$$\chi_H\left[M\left(T_{m,1}\right)\right] = \begin{cases} \left\lceil \frac{m}{2} \right\rceil + 5 & \textit{when m is odd} \\ m+4 & \textit{when m is even} \end{cases}$$

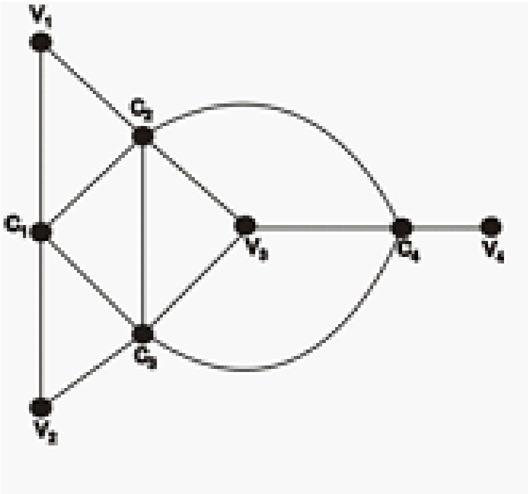
Proof. Let $[\mathrm{T_{m,1}}]$ the tadpole graph of order $m \geq 3$ and $n = 1$. Let $V\ \ [T_{m,1}] = \{v_1, v_2, v_3 \ldots\ldots v_n\}$. Now by the definition of middle graph[1], each edge of the Tadpole graph is subdivided by a new vertex. Therefore assume that each edge of (v_i,v_{i+1}) for $1 = i = n-1$ is subdivided by the vertex v_i,v_{i+1} and naming the new vertices c_j for $1 = j = n$, where c_j be the new middle vertices. Clearly $V\,[M\,[T_{m,1}]]= v\ _i \cup v\ c\ _j \cup v\ _{i+1}$ for $1 = i = n-1$ and $1 = j = n$. Now we assign the coloring to the vertices of $M\,[T_{m,1}]$ as follows:

Consider the color class C=1, 2, 3, 4, 5, 6 etc. For $1 = i = n$,assign the color 1,2,3,4. . . .to v_i. To color remaining the vertices, assign the color randomly. Because by the definition of harmonious coloring is every pair of colors appears on at most one edge. If we replace any color which is minimum in number by a color already used then the resulting coloring will be not harmonious. Therefore,

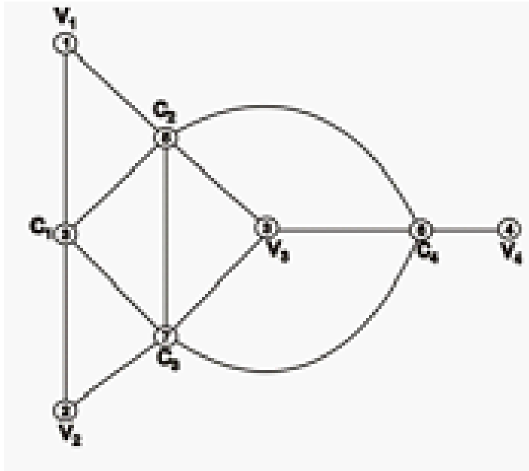
$$\mathrm{\chi_H\left[M\left(T_{m,1}\right)\right] = \begin{cases} \left\lceil \frac{m}{2} \right\rceil +5 & \text{when m is odd} \\ m+4 & \text{when m is even} \end{cases}}$$

□

Example 3.2.



(a) $M\,[T_{3,1}]$



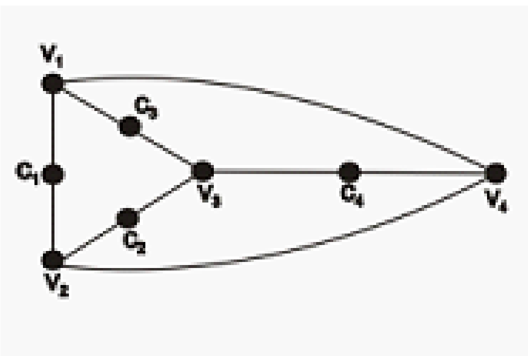
(b) $\chi_H[M(T_{3,1})]$

Hence, $\chi_H[M(T_{3,1})]$*=7.*

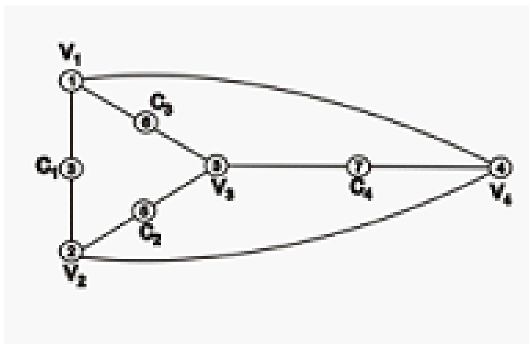
Theorem 3.3. *For Tadpole graph of* $T_{m,n}$ *where* $n = 1$ *and* $m{\geq}3$*, then* $\chi_H[C(T_{m,1})] = n + 3$.

Proof. Let $[\mathrm{T}_{\mathrm{m},1}]$ the tadpole graph, of order $m \geq 3$ and $n = 1$. Let $V\,[T_{m,1}] = \{v_1, v_2, v_3, \ldots, v_n\}$. Now by the definition of central graph[1], each edge of the Tadpole graph is subdivided by a new vertex and joining all non adjacent vertices. Therefore assume that each edge of (v_i,v_{i+1}) for $1 = i = n-1$ is subdivided by the vertex v_i,v_{i+1} and naming the new vertices c_j for $1 = j = n$, where c_j be the central vertices. Clearly $V\,[C\ \ [T_{m,1}]]$=$v\ _i \cup c\ _j \cup v\ _{i+1}$ for $1 \leq i \leq n-1$ and $1 \leq j \leq n$. Now we assign the coloring to the vertices of $C\,[T_{m,1}]$ as follows: Consider the color class $C = 1, 2, 3, 4, 5, 6$ etc. For $1 \leq i \leq n$, assign the color 1,2,3,4. . . .to v_i. To color remaining the vertices, assign the color randomly. Because by the definition of harmonious coloring is every pair of colors appears on at most one edge. If we replace any color which is minimum in number by a color already used then the resulting coloring will be not harmonious. Therefore, $\chi_H[C(T_{m,1})] = n + 3$. □

Example 3.4.



(a) $C\,[T_{3,1}]$



(b) χ_H[C $(T_{3,1})$]

Hence $\chi_H[C(T_{3,1})] = 7$.

Theorem 3.5. *For Web graph, the Harmonious Chromatic number is,* $\chi_H[M\,\{''web'', n\}] = n + 3$.

Proof. Let $\{''web'', n\}$ the Web graph, of order $n = 3$. Let $V\,[\{''web'', n\}] = \{v_1, v_2, v_3 \ldots\ldots v_n\}$. Now by the definition of middle graph[2], each edge of the web graph is subdivided by a new vertex. Therefore assume that each edge of (v_i,v_{i+1})

for $1 = i = n - 1$ is subdivided by the vertex v_i,v_{i+1} and naming the new vertices c_j for $1 \leq j \leq n$, where c_j be the new middle vertices. Clearly, $V[M[\{"web", n\}]] = v_i \cup vc_j \cup v_{i+1}$ for $1 \leq i \leq n-1$ and $1 \leq j \leq n$. Now we assign the coloring to the vertices of $M[\{"web", n\}]$ as follows: Consider the color class $C = 1, 2, 3, 4, 5, 6$ etc. For $1 \leq i \leq n$, assign the color 1,2,3,4. . . .to v_i. To color remaining the vertices, assign the color randomly. Because by the definition of harmonious coloring is every pair of colors appears on at most one edge. In case we replace any color which is minimum in number by a color already used then the resulting coloring will be not harmonious. Therefore, $\chi_H[M\{"web", n\}] = n + 3$. □

Example 3.6.

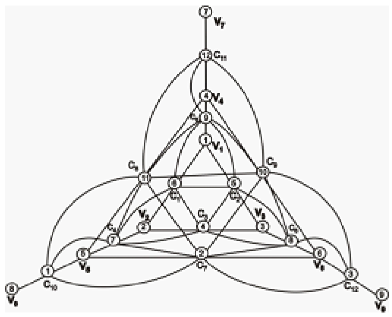


Figure 2: $\chi_H[M\{"web", 3\}]$

Hence, $\chi_H[M\{"web", 3\}] = 12$.

Theorem 3.7. *For Web graph, the Harmonious Chromatic number,* $\chi_H[C\{"web", n\}] = n + 4$.

Proof. Let $[\{"web", n\}]$ the Web graph. Let $V[\{"web", n\}] = \{v_1, v_2, v_3 \ldots\ldots v_n\}$. Now by the definition of central graph [4], each edge of the Web graph is subdivided by a new vertex, and joining all the non adjacent vertices. Therefore assume that each edge of $(v_{i,}v_{i+1})$ for $1 \leq i \leq n-1$ is subdivided by the vertex v_i, v_{i+1} and naming the new vertices c_j for $1 \leq j \leq n$, where c_j be the central vertices. Clearly, $V[C[\{"web", n\}]] = v_i \cup c_j \cup v_{i+1}$ for $1 \leq i \leq n-1$ and $1 \leq j \leq n$. Now we assign the coloring to the vertices of $C[\{"web", n\}]$ as follows: Consider the color class $C = 1, 2, 3, 4, 5, 6$ etc. For $1 \leq i \leq n$, assign the color 1,2,3,4. . . .to v_i. To color remaining the vertices, assign the color randomly. Because by the definition of harmonious coloring [1] is every pair of colors appears on at most one edge. In case we replace any color which is minimum in number by a color already used then the resulting coloring will be not harmonious. Therefore, $\chi_H[C\{"web", n\}] = n + 4$. □

Example 3.8.

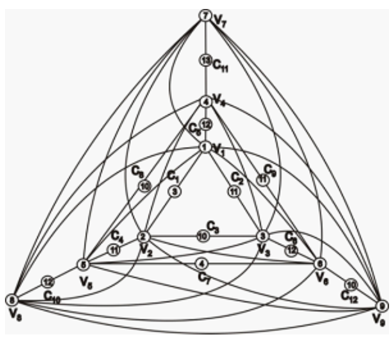


Figure 3: $C[\{"web", 4\}]$

Hence, $\chi_H[C\{"web", 4\}] = 13$.

4. Conclusion

In this paper, we have tried to obtain the middle and central graph of Tadpole and Web graph, evaluated the harmonious coloring of that graph and also found the number of that graph.

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