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Harmonious Chromatic Number, Bondage Number, Domatic Number of Some Special Graphs

Research Article*

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Abstract: Let $G(V, E)$ be an undirected graph. The Harmonious coloring of a graph G is a proper vertex coloring in which each pair of colors appears on at most one pair of adjacent vertices. The Harmonious chromatic number is minimum number of colors needed for the harmonious coloring of G. In this paper we found the Harmonious chromatic number, Bondage number, and Domatic number of some special graphs.

Keywords: Proper Coloring, Bondage Number, Domatic Number, Moser Spindle Graph, Golden Harary Graph, Tietzes Graph, Soifer Graph.

1. Introduction and Preliminaries

In this paper, we have taken the graphs to be finite, undirected graphs. Every graph has a harmonious coloring ,since it suffices to assign every vertex a distinct color. There trivially exist graphs G with $\chi_H(G) > \chi(G)$. In this paper we discuss about the harmonious chromatic number Bondage number, and Domatic number of Moser Spindle Graph, Golden Harary Graph, Tietze's Graph and Soifer Graph.

Definition 1.1 (Proper Coloring)**.** *A graph G having no two adjacent vertices receive the same color is said to be proper coloring.*

Definition 1.2 (Bondage Number)**.** *The bondage number b(G) of a nonempty graph G is the cardinality of a smallest set of edges whose removal from G results in a graph with domination number greater than* $\gamma(G)$

Definition 1.3 (Domatic Number)**.** *In a dominating set one can partition the vertex set of G into atleast two disjoint dominating sets. The maximum number of dominating sets into which the vertex set of a graph G,can be partitioned is called the Domatic number of G and denoted by dom(G).*

Definition 1.4 (Moser Spindle Graph)**.** *Moser spindle graph is an undirected and planar graph. It as a embedded as a unit distance graph in the plane. It has 7 vertices and 11 edges.*

Example 1.5.

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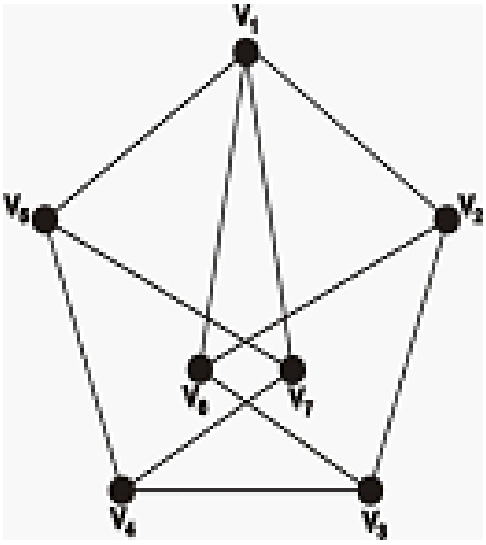
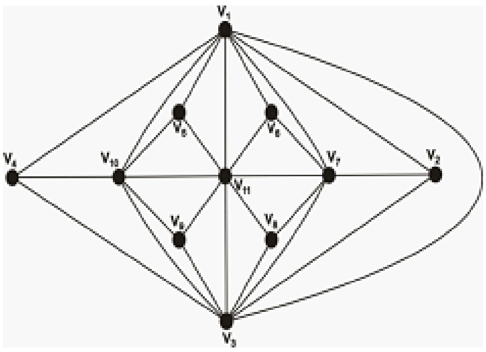


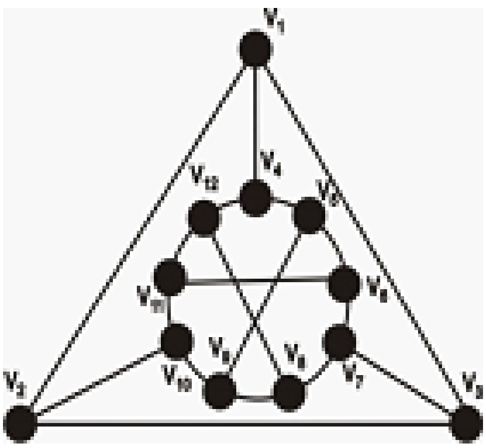
Definition 1.6 (Golden Harary Graph)**.** *The Golden Harary graph is undirected and planar graph with 11 vertices and 27 edges. It is also 3-vertex connected, the removal of any two of its vertices leaves a connected subgraph.*

Example 1.7.



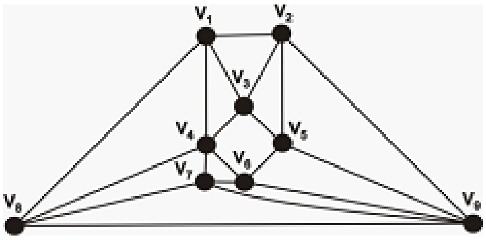
Definition 1.8 (Tietze's Graph)**.** *A cubic graph on 12 nodes and 18 edges. The Tietze's is the unique almost Hamiltonian cubic graph on 12 vertices.*

Example 1.9.



Definition 1.10 (Soifer Graph)**.** *The Soifer graph is a planar graphs on 9 nodes that triangle.*

Example 1.11.



2. Harmonious Chromatic Number, Bondage Number, Domatic Number of Some Graph

In this section we have obtained Harmonious chromatic number, Bondage number, and Domatic number of Moser Spindle Graph , Golden Harary Graph, Tietze's Graph and Soifer Graph.

Theorem 2.1. *Let G be a Moser spindle graph, then*

(1) The harmonious chromatic number of Moser spindle is 'n-2'.

(2) The bondage number of Moser spindle graph is 1.

(3) The domatic number of Moser spindle graph is 2.

Proof. Let G be a Moser spindle graph with 9 vertices and 11 edges. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$. Let us determine the harmonious chromatic number, bondage number and domatic number.

Case (i) The Harmonious chromatic number Moser spindle of graph. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$ $\forall\, n = 7$. Assign the color 'i' to the vertex v_i $\forall$ $1 \leq i \leq n-2$ randomly. By the definition of harmonious coloring [5] each pair of colors appears at most one pair of vertices. So this is a valid proper coloring and here all the edges are different pair of colors and no color pair of colors is repeated. Hence this is valid for harmonious coloring. Therefore, the Harmonious chromatic number of Moser spindle graph is $n-2$.

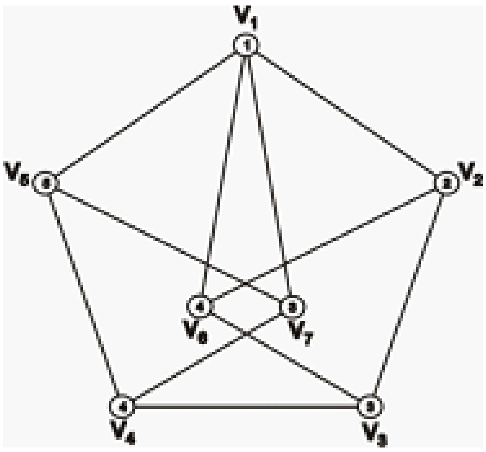
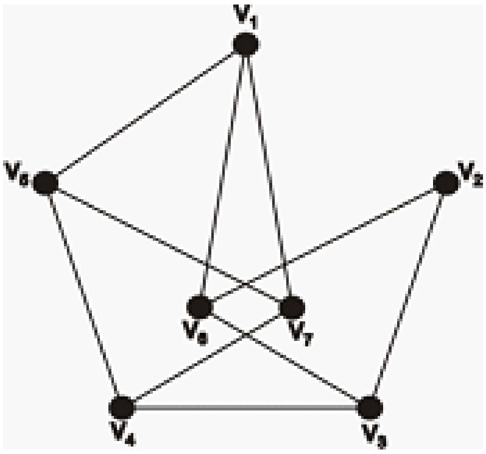


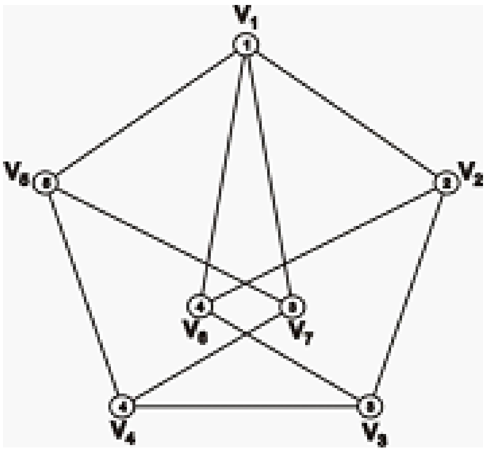
Figure 1. $\chi_H(G) = 5$

Case (ii) Let determine the bondage number of G. By the definition of bondage number [2], we know that the cardinality of the smallest set edges E such that the domination of the graph with edge removed is greater than the domination of the original graphs. First we found the domination number for graph. We know that, A dominator coloring of a graph G is a proper coloring of a graph such that every vertex of V dominates all the vertices of at least one color class. Here domination number of Moser spindle graph is 2. We need to remove one edges to make the domination number of this graph to become greater than that the domination of $G - 1e$. Therefore bondage number of Golden Harary graph is 1.



Here the edge of (v_1, v_2) is removed. Therefore $\chi_b(G) = 1$.

Case (iii) Let determine the domatic number of G. By the definition of domatic number [3] is the maximum size of a domatic partion that is the maximum disjoint dominating sets. In this graph $S = \{v_1, v_4\}$ be the dominating set are the minimum cardinality which dominates the whole graph G, then the domination number of Soifer. Then take another set of vertices which dominates the group G which is disjoint from S. $S_1 = \{v_2, v_5\}$ Continuing this process until we get disjoint set of vertices. By the definition of domatic number is the maximum number of disjoint dominating sets. So in this graph have 2 maximum disjoint dominating sets. Therefore, the domatic number of Moser spindle graph is 2.



Here $S = \{v_1, v_4\}$, $S_1 = \{v_2, v_5\}$ are two disjoint dominating sets. Therefore $d(G) = 2$. □

Theorem 2.2. *Let G be a Golden Harary graph, then*

1. *The harmonious chromatic number of Golden Harary graph is 'n'.*
2. *The bondage number of Golden Harary graph is 3.*
3. *The domatic number of Golden Harary graph is 3.*

Proof. Let G be a Golden Harary graph with 9 vertices and 18 edges. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$. Let us determine the harmonious chromatic number, bondage number and domatic number.

Case (i) The Harmonious chromatic number of Golden Harary graph. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$ $\forall$ $n = 11$. Assign the color 'i' to the vertex v_i $\forall$ $1 \le i \le n$, randomly. By the definition of harmonious coloring each pair of colors appears at most one pair of vertices. So this is a valid proper coloring and here all the edges are different pair of colors and no color pair of colors is repeated. Hence this is valid for harmonious coloring. Therefore, the Harmonious chromatic number of Golden Harary graph is n.

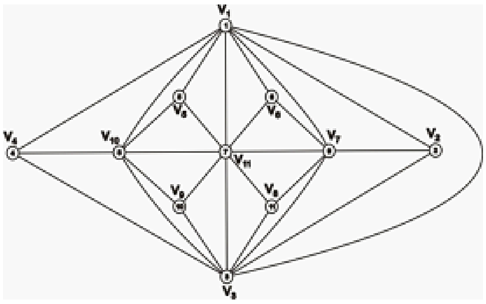
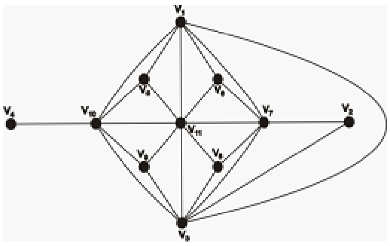


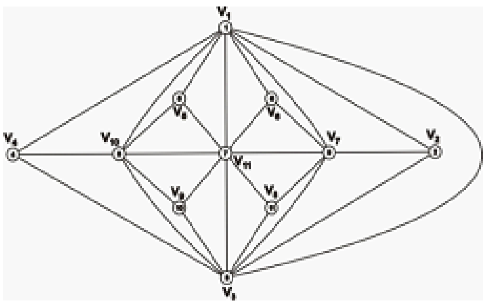
Figure 2. $\chi_H(G) = 11$

Case (ii) Let determine the bondage number of G. By the definition of bondage number, we know that the cardinality of the smallest set egdes E such that the domination of the graph with edge removed is greater than the domination of the original graphs. First we found the domination number for graph. We know that, A dominator coloring of a graph G is a proper coloring of a graph such that every vetex of V dominates all the vertices of at least one color class. Here domination number of Golden Harary is 3. We need to remove three edges to make the domination number of this graph to become greater than that the domination of $G - 3e$. Therefore bondage number of Golden Harary graph is 3.



Here the edges of $(v_1, v_2), (v_1, v_4), (v_3, v_4)$ are removed. Therefore $\chi_b(G) = 3$.

Case (iii) Let determine the domatic number of G. By the definition of domatic number is the maximum size of a domatic partion that is the maximum disjoint dominating sets. In this graph $S = \{v_1, v_3\}$ be the dominating set are the minimum cardinality which dominates the whole graph G, then the domination number of Soifer. Then take another set of vertices which dominates the group G which is disjoint from S. $S_1 = \{v_2, v_4, v_{11}\}$; $S_2 = \{v_7, v_{10}\}$. Continuing this process until we get disjoint set of vertices. By the definition of domatic number is the maximum number of disjoint dominating sets. So in this graph have 3 maximum disjoint dominating sets. Therefore, the domatic number of Golden Harary graph is 3.



Here $S = \{v_1, v_3\}$, $S_1 = \{v_2, v_4, v_{11}\}$, $S_2 = \{v_7, v_{10}\}$ are three disjoint dominating sets. Therefore $d(G) = 3$. □

Theorem 2.3. *Let G be a Tietze's graph, then*

1. *The harmonious chromatic number of Tietze's graph is 'n-2'.*

2. *The bondage number of Tietze's graph is 2.*

3. *The domatic number of Tietze's graph is 3.*

Proof. Let G be a Tietze's graph with 9 vertices and 18 edges. Let V={v_1,v_2,v_3,....v_n}. Let us determine the harmonious chromatic number, bondage number and domatic number.

Case (i) The Harmonious chromatic number of Tietze's graph. Let $V = \{v_1, v_2, v_3, \ldots, v_n\}$ $\forall\, n = 12$. Assign the color 'i' to the vertex v_i $\forall$ $1 \leq i \leq n-2$, randomly. By the definition of harmonious coloring [5] each pair of colors appears at most one pair of vertices. So this is a valid proper coloring and here all the edges are different pair of colors and no color pair of colors is repeated. Hence this is valid for harmonious coloring. Therefore, the Harmonious chromatic number of Tietze's graph is $n-2$.

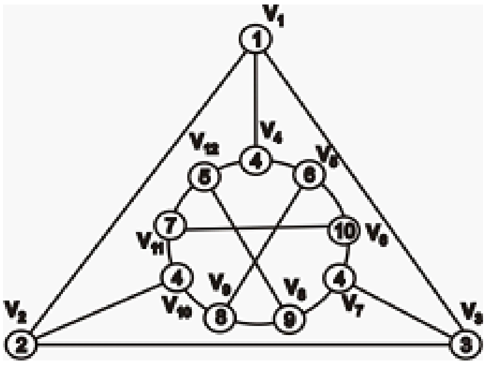
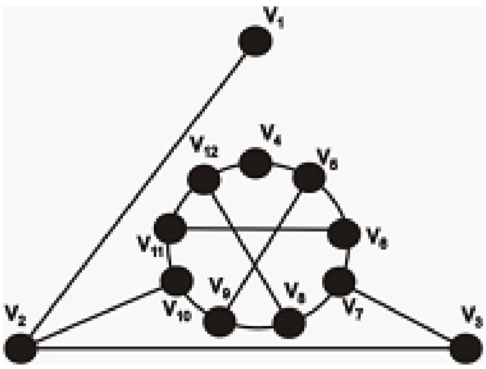


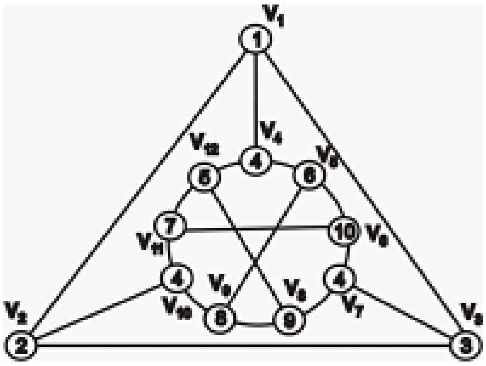
Figure 3. $\chi_H(G) = 10$

Case (ii) Let determine the bondage number of G. By the definition of bondage number [2], we know that the cardinality of the smallest set edges E such that the domination of the graph with edge removed is greater than the domination of the original graphs. First we found the domination number for graph. We know that, A dominator coloring of a graph G is a proper coloring of a graph such that every vertex of V dominates all the vertices of at least one color class. Here domination number of Tietze's graph is 3. We need to remove two edges to make the domination number of this graph to become greater than that the domination of $G - 2e$. Therefore bondage number of Tietze's graph is 2.



Here the edges of (v_1, v_3), (v_1, v_4) are removed. Therefore $\chi_b(G) = 2$.

Case (iii) Let determine the domatic number of G. By the definition of domatic number [3] is the maximum size of a domatic partion that is the maximum disjoint dominating sets. In this graph $S = \{v_4, v_7, v_{10}\}$ be the dominating set are the minimum cardinality which dominates the whole graph G, then the domination number of Soifer. Then take another set of vertices which dominates the group G which is disjoint from S. $S_1 = \{v_2, v_{5,}, v_8, v_{11}\}$, $S_2 = \{v_1, v_6, v_9, v_{12}\}$. Continuing this process until we get disjoint set of vertices. By the definition of domatic number is the maximum number of disjoint dominating sets. So in this graph have 3 maximum disjoint dominating sets. Therefore, the domatic number of Tietze's graph is 3.



Here $S = \{v_4, v_7, v_{10}\}$, $S_1 = \{v_2, v_5, v_8, v_{11}\}$, $S_2 = \{v_1, v_6, v_9, v_{12}\}$ are three disjoint dominating sets. Therefore $d(G) = 3$. □

Theorem 2.4. *Let G be a Soifer graph, then*

1. *The harmonious chromatic number of Soifer graph is 'n'.*
2. *The bondage number of Soifer graph is 3.*
3. *The domatic number of Soifer graph is 4.*

Proof. Let G be a Soifer graph with 9 vertices and 18 edges. Let $V = \{v_1, v_2, v_3, \ldots .v_n\}$. Let us determine the harmonious chromatic number, bondage number and domatic number.

Case (i) The Harmonious chromatic number of Soifer graph. Let $V = \{v_1, v_2, v_3, \ldots .v_n\}$ $\forall$ $n = 9$. Assign the color 'i' to the vertex v_i $\forall$ $1 \leq i \leq 9$. By the definition of harmonious coloring [5] each pair of colors appears at most one pair of vertices. So this is a valid proper coloring and here all the edges are different pair of colors and no color pair of colors is repeated. Hence this is valid for harmonious coloring. Therefore, the Harmonious chromatic number of Soifer graph is n.

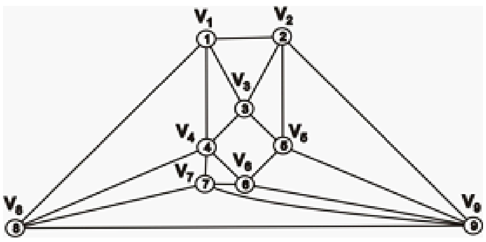
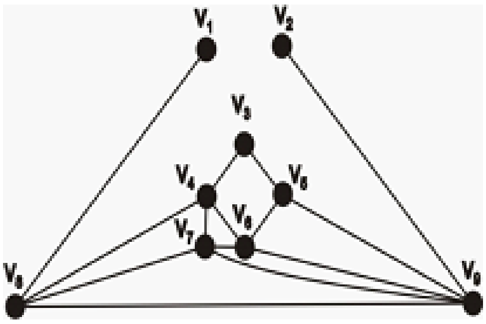


Figure 4. $\chi_H(G) = 9$

Case (ii) Let determine the bondage number of G. By the definition of bondage number [2], we know that the cardinality of the smallest set egdes E such that the domination of the graph with edge removed is greater than the domination of the original graphs. First we found the domination number for Soifer graph. We know that, A dominator coloring of a graph G is a proper coloring of a graph such that every vetex of V dominates all the vertices of at lest one color class. Here domination number of Soifer graph is 2. We need to remove five edges to make the domination number of this graph to become greater than that the domination of G-5e. Therefore, the bondage number of Soifer graph is 5. (ie) $\gamma_b(G) = 5$.
Here the edges of $(v_1.v_2), (v_1, v_3), (v_1, v_4), (v_2, v_3), (v_2, v_5)$ are removed. Therefore $\gamma_b(G) = 5$.
Case (iii) Let determine the domatic number of G. By the definition of domatic number [3] is the maximum size of a domatic partion that is the maximum disjoint dominating sets. In this graph $S = \{v_1, v_3\}$ be the dominating set are the minimum



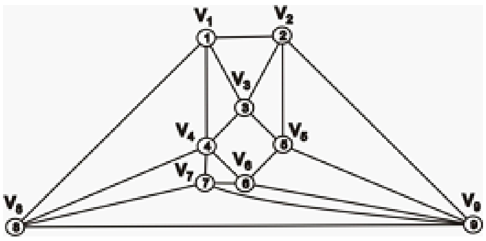
cardinality which dominates the whole graph G,then the domination number of Soifer 2. Then take another set of vertices which dominates the group G which is disjoint from S.

$$S_1 = \{v_1, v_9\}$$

$$S_2 = \{v_5, v_4\}$$

$$S_3 = \{v_7, v_6, v_2\}$$

Continuing this process until we get disjoint set of vertices. By the definition of domatic number is the maximum number of disjoint dominating sets. So in this graph have 4 maximum disjoint dominating sets. Therefore, the domatic number of Soifer graph is 4.



Here $S = \{v_1, v_3\}$, $S_1 = \{v_1, v_9\}$, $S_2 = \{v_5, v_4\}$, $S_3 = \{v_7, v_6, v_2\}$. Therefore $d(G) = 4$. □

3. Conclusion

In this paper, we have found the harmonious chromatic number Bondage number, and Domatic number of Moser Spindle Graph, Golden Harary Graph, Tietze's Graph and Soifer Graph.

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