International Journal of Mathematics And its Applications
Volume 4, Issue 4 (2016), 201–206. (Special Issue)
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Fuzzy Systems for Multi-criteria Decision Making with Interval Data

Research Article*

J.Jon Arockiaraj[1] and N.Murali[1]

1 Department of Mathematics, St. Joseph's College of Arts & Science, Manjakuppam, Cuddalore (Tamil Nadu), India.

Abstract: Oceans cover more than 70% of Earth's space and has a vast number of living organisms and underwater vegetation. During early centuries Seas helped us in provision of food and rare medicinal herbs from its vegetation. There are numerous species of fishes which are caught and consumed as food. In this paper the TOPSIS method is used to assess most commonly consumed fishes and evaluate the favorable one corresponding to people's preference.

Keywords: Fuzzy TOPSIS, Fuzzy Matrix, Decision Support Systems, Multi-criteria Decision Making.

1. Introduction

The use of fuzzy logic and of fuzzy sets theory applied to the decision making process was proposed initially in [1]. Since then, several works have been conducted aiming to support the decision making process. Some Decision Support Systems (DSS) are based on classical techniques for decision making as, for example, Decision Tree [2], [3], [4], [5]. Initially proposed by Hwang & Yoon [1] to treat problems with numerical values, currently the technique has many applications and contributions to optimization and manipulation of inaccurate data by Fuzzy TOPSIS [2, 3, 4]. Decision making problem is the process of finding the best option from all of the feasible alternatives. In almost all such problems the multiplicity of criteria for judging the alternatives is pervasive. That is, for many such problems the decision maker wants to solve a multiple criteria decision making (MCDM) problem. In classical, decision theory can be characterized by a set of decision alternatives, a set of nature, a relation assigning to each pair of a decision and state and a result and finally the utility function which orders the results according to their desirability.

The Fuzzy TOPSIS method is based on a procedure that, after the initial steps to calculate the decision matrix, the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) are defined. The first, PIS, aims to maximize the benefits attributes and minimize the costs attributes. However, the NIS minimizes the benefits attributes and maximizes the costs attributes of a possible solution to the problem. Among the alternatives available, the alternative that is closer to the PIS and farther from the NIS will be defined as the best solution. In the Fuzzy TOPSIS method a MCDM (Multi-Criteria Decision-Making) problem with m alternatives (A_1, A_2, … $A_{m)}$ and n attributes (C_1,C_2,… C_n) can be expressed in matrix format as

* *Proceedings : National Conference on* `Recent Trends in Applied Mathematics` *held on 22 & 23.07.2016, organized by Department of Mathematics, St. Joseph's College of Arts & Science, Manjakuppam, Cuddalore (Tamil Nadu), India.*

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \ddots & \vdots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{mn} \end{bmatrix} \end{array}$$

$W = [W_1, W_2, \ldots, W_n]$, where x_{ij}, $i = 1, 2, \ldots m$ and $j = 1, 2, \ldots, n$ are the numerical data of the problem and w_j, $j = 1, 2, \ldots, n$ is the importance degree of each attribute $c_1, c_2, \ldots, c_n$ respectively.

1.1. Main step of MCDM

a) Establishing system evaluation criteria that relate system capabilities to goals.

b) Developing alternative systems for attaining the goals (generating alternatives).

c) Evaluating alternatives in terms of criteria (the values of the criterion functions).

d) Applying a normative multi-criteria analysis method.

e) Accepting one alternative as "optimal" (preferred).

f) If the final solution is not accepted, gather new information and go into the next iteration of multi- criteria optimization.

Steps (i) and (v) are performed at the upper level, where decision makers have the central role, and the other steps are mostly engineering tasks. For step (iv), a decision maker should express his/her performances in terms of the relative importance of criteria, and one approach is to introduce criteria weights. This weights in MCDM do not have a clear economic significance, but their use provides the opportunity to model the actual aspects of decision making(the preference structure). Under many conditions, exact data are inadequate to model real-life situations.

For example, human judgments including preferences are often vague and cannot estimate his preference with an exact numerical data, therefore these data may have some structures such as bounded data, In this paper, by considering the fact that, in some cases, determining precisely the exact value of the attributes is difficult and that, as a result of this, their values are considered as intervals, therefore we extended the concept of TOPSIS to develop a methodology for solving MCDM problems with interval data.

2. TOPSIS Method

TOPSIS method is presented in chen and Hwang, with reference to Hwang and Yoon. TOPSIS is a multiple criteria method to identified solutions from a finite set of alternatives. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and farthest distance from the negative ideal solution.

Suppose A_1, A_2,....,A_m are m possible alternatives among which decision maker have to choose, C_1, C_2,....,C_n are criteria with which alternative performance are measured, x_{ij} is the rating of alternative A_i with respect to criterion C_i and is not known exactly and only we know $x_{ij} \in [x_{ij}^L, x_{ij}^M, x_{ij}^U]$. A MCDM problem with interval data can be concisely expressed in matrix format

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} [X_{11}^L, X_{11}^M, X_{11}^U] & [X_{12}^L, X_{12}^M, X_{12}^U] & \dots & [X_{1n}^L, X_{1n}^M, X_{1n}^U] \\ [X_{21}^L, X_{21}^M, X_{21}^U] & [X_{22}^L, X_{22}^M, X_{22}^U] & \dots & [X_{2n}^L, X_{2n}^M, X_{2n}^U] \\ \vdots & \ddots & \vdots & \vdots \\ [X_{m1}^L, X_{m1}^M, X_{m1}^U] & [X_{m2}^L, X_{m2}^M, X_{m2}^U] & \dots & [X_{mn}^L, X_{mn}^M, X_{mn}^U] \end{bmatrix} \end{array}$$

Since $W = [w_1, w_2, \ldots, w_n]$, where w is in weight of criterion c_j.

2.1. Algorithm

STEP 1: Consider the normalized decision making. The normalized values $\overline{n}_{ij}{}^{L}$ and $\overline{n}_{ij}{}^{M}$ and $\overline{n}_{ij}{}^{U}$ are calculated as

$$\overline{n}_{ij}^{L} = \frac{x_{ij}^{L}}{\sqrt{\sum_{j=1}^{m}\{(x_{ij}^{L})^2 + (x_{ij}^{M})^2 + (x_{ij}^{U})^2\}}}, \quad j = 1, 2, \ldots, m, \quad i = 1, 2, \ldots, n \tag{1}$$

$$\overline{n}_{ij}^{M} = \frac{x_{ij}^{M}}{\sqrt{\sum_{j=1}^{m}\{(x_{ij}^{L})^2 + (x_{ij}^{M})^2 + (x_{ij}^{U})^2\}}}, \quad j = 1, 2, \ldots, m, \quad i = 1, 2, \ldots, n \tag{2}$$

$$\overline{n}_{ij}^{U} = \frac{x_{ij}^{U}}{\sqrt{\sum_{j=1}^{m}\{(x_{ij}^{L})^2 + (x_{ij}^{M})^2 + (x_{ij}^{U})^2\}}}, \quad j = 1, 2, \ldots, m, \quad i = 1, 2, \ldots, n \tag{3}$$

STEP 2: Consider the difference importance of each criterion, construct the weighted normalized interval decision matrix as

$$\overline{v}_{ij}^{L} = w_i \overline{n}_{ij}^{L}, \quad \text{where} \quad j = 1, 2, \ldots, m, \quad i = 1, 2, \ldots, n \tag{4}$$

$$\overline{v}_{ij}^{M} = w_i \overline{n}_{ij}^{M}, \quad \text{where} \quad j = 1, 2, \ldots, m, \quad i = 1, 2, \ldots, n \tag{5}$$

$$\overline{v}_{ij}^{U} = w_i \overline{n}_{ij}^{U}, \quad \text{where} \quad j = 1, 2, \ldots, m, \quad i = 1, 2, \ldots, n \tag{6}$$

Where w_i is the weight of the i^{th} attribute or criterion and $\sum_{i=1}^{n} \mathrm{w_i} = 1$.

STEP 3: Determine the positive ideal solution and negative ideal solution as

$$\overline{A}^{+} = \{\overline{v}_1^{+} \ldots \overline{v}_n^{+}\} = \{(\max_j \overline{v}_{ij}^{U} / i \in I), (median_j \overline{v}_{ij}^{M} / i \in I), \min_j \overline{v}_{ij}^{L} / i \in J) \tag{7}$$

$$\overline{A}^{-} = \{\overline{v}_1^{+} \ldots \overline{v}_n^{+}\} = \{(\min_j \overline{v}_{ij}^{U} / i \in I), (median_j \overline{v}_{ij}^{M} / i \in I), \max_j \overline{v}_{ij}^{U} / i \in J) \tag{8}$$

Where I is associated with benefit criteria and J is associated with cost criteria.

STEP 4: Calculate the separation of each alternative from the positive ideal solution, using the n-dimensional Euclidean distance as

$$\overline{d}_j^{+} = \{\sum_{i \in I} (\overline{v}_{ij}^{L} - \overline{v}_i^{+})^2 + \sum_{i \in J} (\overline{v}_{ij}^{M} - \overline{v}_i^{+})^2 + \sum_{i ? J} (\overline{v}_{ij}^{U} - \overline{v}_i^{+})^2\}^{1/2}, \quad j = 1, 2, \ldots, m \tag{9}$$

Similarly, the separation from the negative ideal be calculated as

$$\overline{d}_j^{-} = \{\sum_{i \in I} (\overline{v}_{ij}^{U} - \overline{v}_i^{-})^2 + \sum_{i \in J} (\overline{v}_{ij}^{M} - \overline{v}_i^{-})^2 + \sum_{i \in J} (\overline{v}_{ij}^{L} - \overline{v}_i^{-})^2\}^{1/2}, \quad j = 1, 2, \ldots, m \tag{10}$$

STEP 5: Calculate the relative closeness coefficient to determine the ranking order of all alternatives. once the $\overline{d}_j^{+}$ and $\overline{d}_j^{-}$ of each alternatives A_j has been calculated. The relative closeness of the alternative Aj with respect to $\overline{A}^{+}$ is defined as

$$\overline{R}_j = \frac{\overline{d}_j^{-}}{(\overline{d}_j^{+} + \overline{d}_j^{-})}, \quad j = 1, 2, \ldots, m \tag{11}$$

Obviously an alternative A_j is closer to the $\overline{A}^{+}$ and farther from $\overline{A}^{-}$ as $\overline{R}_j$ approaches to 1. Therefore according to the closeness coefficient, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives.

2.2. The Presented Algorithm

In sum, an algorithm to determine the most preferable choice among all possible choices when data is interval with extended TOPSIS approach is given in the following

Step 1: establishing system evaluation criteria that relate system capabilities to goals (identification evaluation criteria)

Step 2: Developing alternative system for attaining the goals (generating alternatives).

Step 3: Evaluating alternatives in terms of criteria (values of the criterion functions which are intervals).

Step 4: Identifying the weight of criteria.

Step 5: construct the internal decision matrix and the interval normalized decision matrix (using formulas (1), (2) and (3)).

Step 6: Construct the interval weighted normalized decision matrix (using formulas (4), (5) and (6)).

Step 7: Determine positive ideal $\overline{A}^{+}$ and negative ideal $\overline{A}^{-}$ solutions (using formulas (7) and (8)).

Step 8: Determine separation of each alternative from positive ideal and negative ideal solutions respectively. (identification of $\overline{d}_j^{+}$ and $\overline{d}_j^{-}$, using formulas (9) and (10)).

Step 9: Determine the relative closeness of each alternative to positive ideal solution (identification of $\overline{R}_j$, using formula (11).

Step 10: rank the preference order of all alternatives according to the closeness coefficient.

3. Numerical Problem

In this section, we work out a numerical example to illustrate the TOPSIS method for decision making problems with interval data. The aim of this study is to analyze the sea food eating habits of the peoples based on the data collected from them and to find out their preference in eating the following choices of the sea foods.

1. Crab (or) lobster, 2. Prawn, 3. Snail, 4. Small Fish (matthi, kavalai, kanamkathhai, etc,) 5. Big Fish (vanjuram, paarai, etc,) and 6. Octobus. The following three criteria were identified as the evaluation criteria to find out the eating choice of peoples. That is c_1-price, c_2-taste and c_3-availability. Note that steps 1, 2 and 3 are done.

Step 4: Suppose that the vector of corresponding weight of each criteria is $W = [0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1]$

Step 5: The interval decision matrix and interval normalized decision matrix are shown in tables 1 and 2 respectively.

A_i	C_1			C_2			C_3		
	$x^L{}_{1j}$	$x^M{}_{1j}$	$x^U{}_{1j}$	$x^L{}_{2j}$	$x^M{}_{2j}$	$x^U{}_{2j}$	$x^L{}_{3j}$	$x^M{}_{3j}$	$x^U{}_{3j}$
A_1	18	21.5	25	17	18.5	20	12	15	18
A_2	14	21.5	29	13	21.5	30	9	17.5	26
A_3	30	40	50	33	37	41	3	7.5	12
A_4	2	4.5	7	16	17.5	19	19	28.5	38
A_5	4	7.5	11	9	12.5	16	23	31.5	40
A_6	20	34	48	5	7.5	10	13	1.85	24

Table 1. The interval decision matrix

A_i	C_1			C_2			C_3		
	$\overline{n}^L{}_{1j}$	$\overline{n}^M{}_{1j}$	$\overline{n}^U{}_{1j}$	$\overline{n}^L{}_{2j}$	$\overline{n}^M{}_{2j}$	$\overline{n}^U{}_{2j}$	$\overline{n}^L{}_{3j}$	$\overline{n}^M{}_{3j}$	$\overline{n}^U{}_{3j}$
A_1	0.1641	0.1960	0.2279	0.1866	0.2031	0.2195	0.1280	0.1600	0.1920
A_2	0.1276	0.1960	0.2643	0.1427	0.2360	0.3293	0.0960	0.1866	0.2773
A_3	0.2734	0.3646	0.4557	0.3642	0.4061	0.4501	0.0320	0.0800	0.1280
A_4	0.0183	0.0410	0.0641	0.1758	0.1921	0.2086	0.2026	0.3040	0.4053
A_5	0.0365	0.0684	0.1003	0.0988	0.1372	0.1756	0.2453	0.3360	0.4266
A_6	0.1823	0.3099	0.4375	0.0549	0.0823	0.1098	0.1386	0.1973	0.2560

Table 2. **The interval normalized decision matrix**

A_i	C_1			C_2			C_3		
	$\overline{v}^L{}_{1j}$	$\overline{v}^M{}_{1j}$	$\overline{v}^U{}_{1j}$	$\overline{v}^L{}_{2j}$	$\overline{v}^M{}_{2j}$	$\overline{v}^U{}_{2j}$	$\overline{v}^L{}_{3j}$	$\overline{v}^M{}_{3j}$	$\overline{v}^U{}_{3j}$
A_1	0.03282	0.03920	0.04558	0.01866	0.02031	0.02195	0.01280	0.01600	0.01920
A_2	0.02552	0.03920	0.05286	0.01427	0.02360	0.03293	0.00960	0.01866	0.02773
A_3	0.05468	0.07292	0.09114	0.03642	0.04061	0.04501	0.00320	0.00800	0.01280
A_4	0.00366	0.00820	0.01282	0.01758	0.01921	0.02086	0.02026	0.03040	0.04053
A_5	0.00730	0.01368	0.02006	0.00988	0.01372	0.01756	0.02453	0.03360	0.04266
A_6	0.03646	0.06198	0.08750	0.00549	0.00823	0.01098	0.01386	0.01973	0.02560

Table 3. **The interval weighted normalized decision matrix**

$\overline{d}_1^+$	$\overline{d}_2^+$	$\overline{d}_3^+$	$\overline{d}_4^+$	$\overline{d}_5^+$	$\overline{d}_6^+$
0.07810	0.07906	0.14107	0.02792	0.03130	0.11683

Table 4. **Distance of each alternative from positive ideal solution**

$\overline{d}_1^-$	$\overline{d}_2^-$	$\overline{d}_3^-$	$\overline{d}_4^-$	$\overline{d}_5^-$	$\overline{d}_6^-$
0.09322	0.09772	0.05459	0.13058	0.14560	0.07239

Table 5. **Distance of each alternative from Negative ideal solution**

Alternatives	$\overline{R}_j$	Rank
A_1	0.54413	4
A_2	0.55278	3
A_3	0.27900	6
A_4	0.82385	1
A_5	0.82306	2
A_6	0.38256	5

Table 6. **closeness coefficient and ranking**

From table 6 we conclude that people choice of sea food eat habit is as below: 1. Small Fish 2. Big fish 3. Prawn 4. Crab (or) Lobster 5. Octobus and 6. Snail. Thus from the above calculation, we infer the peoples are giving high priority to eat the small fish variety (mathi, kavalai, etc..) and their next preference goes to eat the big fish (vanjiram, paarai, etc), etc.

4. Conclusion

Since multi criteria decision making problem generally involves uncertainty, it is important to incorporate different type of uncertainty in any proposed solution. Under many conditions, exact value of the attributes and exact data are inadequate to real life situations. In this paper TOPSIS for interval data has been extended. We have applied the extended TOPSIS method to developing for solving multi criteria decision making problems and its effective in tacking complex, ill-defined and human oriented decision making problems.

References

[1] C.L.Hwang and K.Yoon, *Multiple Attribute Decision Making: Methods and Application*, Springer, New York, (1981).

[2] T.Yang and P.Chou, *Solving a multi-response simulation–optimization problem with discrete variables using a multiple-attribute decision making method*, Math Comput Simulat, 68(2005), 9-21.

[3] K.Yoon and C.Hwang, *Multiple attribute decision making*, Thousand Oaks, CA, Sage Publication, (1995).

[4] T.Yang and C.Hung, *Multiple-attribute decision making methods for plant layout design problem*, Robotics and Computer-Integrated Manufacturing, 23(2005), 126-137.

[5] M.Salehi and R.Tavakkoli-Moghaddam, *Project Selection by Using a Fuzzy TOPSIS Technique*, International Journal of Computer, Information, and Systems Science and Engineering, 2(2)(2008), 99-104.

[6] M.A Abo-sinna and A.H.Amer, *Extension of TOPSIS for multi-objective large scale non-linear programming problems*, Applied Mathematics and Computation, 162(2005), 243-256.

[7] C.T Chen, *Extensions of the TOPSIS for group decision making under fuzzy environment*, Fuzzy sets and Systems, 114(2000), 1-9.

[8] R.Kokila, *Design and analysis of a fuzzy matrix model for decision making*, International Journal of Science and Research.

[9] G.R.Jahanshahloo, F.Hosseinzadeh Lotft and M.Izadikhah, *An algorithmic method to extend TOPSIS for decision making problems with interval data*, Applied Mathematics and Computation, 175(2006), 1375-1384.

[10] M.Zeleny, *Multiple Criteria Decision Making*, McGraw-Hill, New York, (1982).