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Vertex Odd Mean and Even Mean Labeling of Fan Graph, Mongolian Tent and $\bar{K}_2 + C_n$

Research Article*

J.Arockia Aruldoss[1] and S.Pushparaj[2]

1 Department of Mathematics, St. Joseph's College of Arts & Science, Manjakuppam, Cuddalore (Tamil Nadu), India.

Abstract: A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to $\{0, 1, 2, \ldots, q\}$ such that when each edge uv is labeled with $\frac{(f(u)+f(v))}{2}$ if $(f(u) + f(v))$ is even and $\frac{(f(u)+f(v)+1)}{2}$ if $(f(u) + f(v))$ is odd then the resulting edges are distinct. A fan graph Fm,n is defined as the graph join $\bar{K}_m + P_n$ where $\bar{K}_m$ is the empty graph on m nodes and P_n is the path graph on n nodes. The case $m = 1$ corresponds to the usual fan graphs. In this paper we investigate vertex odd and even mean labeling of Fan graph, Mongolian tent and $\bar{K}_2 + \mathcal{C}_n$.

Keywords: Mean labeling, Mongolian tent, Fan graph, vertex odd mean labeling and vertex even mean labeling.

1. Introduction

A graph G with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. Useful Vertex odd Mean and Even Mean Labeling of Some Graphs of Author N. Revathi [1]. In this paper we investigate vertex odd and even mean labeling of Fan graph, Mongolian tent and $\bar{K}_2 + \mathcal{C}_n$.

1.1. Basic Definitions

Definition 1.1. *A graph G with q edges to be an vertex odd mean graph if there is an injective function f from the vertices of G to* $\{1, 3, 5, \ldots, 2q - 1\}$ *such that when each edge uv is labeled with* $\frac{f(u)+f(v)}{2}$ *if* $f\ (u) + f\ (v)$ *is even and* $\frac{f(u)+f(v)+1}{2}$ *if* $f(u) + f(v)$ *is odd then the resulting edges are distinct. Such a function is called a vertex odd mean labeling.*

Definition 1.2. *A graph G with q edges to be an vertex even mean graph if there is an injective function f from the vertices of G to* $\{2, 4, 6 \ldots, 2q\}$ *such that the edge labels are given by* $\frac{f(u)+f(v)}{2}$ *are distinct. Such a function is called a vertex even mean labeling.*

Definition 1.3 (Fan Graph)**.** *A fan graph* $F_{m,n}$ *is defined as the graph join* $\bar{K}_m + P_n$, *where* $\bar{K}_m$ *is the empty graph on m notes and* p_n *is the path graph on notes. The case* $m = 1$ *corresponds to the usual fan graphs, while* $m = 2$ *corresponds to the double fan, etc, ... Precomputed properties of fan graphs are implemented in the wolfram Language as Graph Data* $[\{Fan, \{m, n\}\}]$. *The (r,2)-fan graph is isomorphic to the complete tripartite graph* k_1, 1_r *and the (r,3)-fan graph to* k_1, 2_r.

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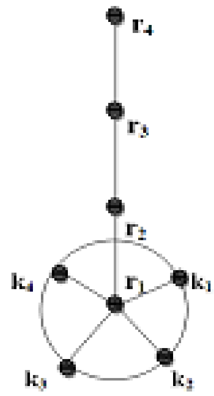
Definition 1.4. *A Mongolian tent as a graph obtained from $P_m \times P_n$ by adding one extra vertex above the grid and joining every other of the top row of $P_m \times P_n$ to the new vertex.*

Definition 1.5. *The join of graphs $\bar{K}_2$ and $\mathcal{C}_n$, $\bar{K}_2 + \mathcal{C}_n$, is obtained by joining a vertex of $\bar{K}_2$ with every vertex of $\mathcal{C}_n$ with an edge.*

2. Main Results

Theorem 2.1. *A fan graph $F_{m,n}$ is defined as the graph join $\bar{K}_m + P_n$. has vertex odd mean labeling.*

Proof. The graph $F_n(\bar{k}_m, r_n)$ has $m + n$ vertices and $k_{2m} + r_{n-2}$ edges. The ordinary labeling for $F_n(4,4)$ is given the figure.



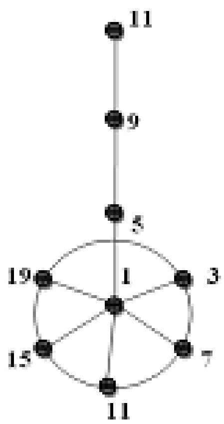
Define a vertex labeling $f : v(F_n(\bar{k}_m, r_n)) \to \{1, 3, 5, \ldots, 2_{q-1}\}$ by

$$F(\bar{k}_m) = 4i - 1, \quad i = 1, 2, \ldots, m$$

$$F(r_n) = 4i - 3, \quad i = 1, 2, \ldots, n$$

Clearly labels of the edges received by the mean of the labels on end vertices are all distinct. Hence the Fan graph has an odd mean labeling. □

Example 2.2. *In the following figure we exhibit vertex odd mean labeling. For $F_n(5,4)$.*



Theorem 2.3. *A fan graph $F_{m,n}$ is defined as the graph join $\bar{K}_m + P_n$. has vertex Even mean labeling.*

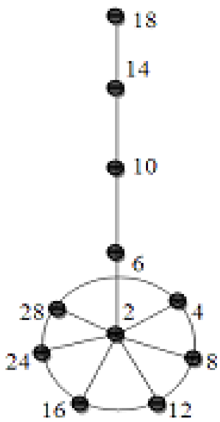
Proof. The graph $F_n(\bar{k}_m, r_n)$ has $m+n$ vertices and $\bar{k}_{2m} + r_{n-2}$ edges. Define a vertex labeling $f : v(F_n(\bar{k}_m, r_n)) \to \{2, 4, 6, \ldots, 2_q\}$ by

$$f(\bar{k}_m) = 4i, \quad i = 1, 2, \ldots, m$$

$$f(r_n) = 4i - 2, \quad i = 1, 2, \ldots, n$$

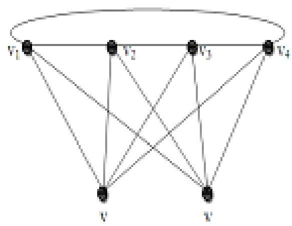
clearly labels of the edges received by the mean of the labels on end vertices are all distinct. Hence the Fan graph has an vertex Even mean labeling. □

Example 2.4. *In the following figure we exhibit vertex Even mean labeling for* $F_n(6, 5)$.



Theorem 2.5. *The graph* $\bar{K}_2 + \mathcal{C}_n$ *has vertex odd mean labeling.*

Proof. The graph $\bar{K}_2 + \mathcal{C}_n$ has $n+1$ vertices and 2n edges. Let v be vertex of $\bar{K}_2$ and $v_1, v_2, \ldots, v_n$ be the vertices of the cycle. The ordinary of $\bar{K}_2 + \mathcal{C}_4$ is given in figure.



Define a vertex labeling $f : v(\bar{k}_2 + c_n) \to \{1, 3, 5, \ldots, 2q-1\}$ by

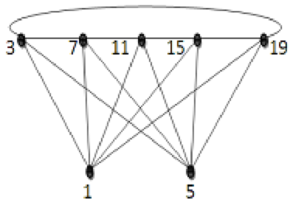
$$f(u) = 2$$

$$f(v_i) = 4i - 1, \quad 1 \le i \le n \text{ if n is odd}$$

$$f(v_i) = \begin{cases} 3, & i = 1 \\ 2i + 3, & \text{i is even if n is odd} \\ q + 2i + 1, & \text{i is odd.} \end{cases}$$

Clearly labels of the edges received by the mean of the labels on end vertices are old distinct. Hence the graph $\bar{K}_2 + \mathcal{C}_n$, has vertex odd mean labeling. □

Example 2.6. *The vertex odd mean labeling of* $\bar{K}_2 + V_5$.



Theorem 2.7. *The graph $\bar{K}_2 + \mathcal{C}_n$ has vertex Even mean labeling.*

Proof. Define a vertex labeling $f : v(\bar{K}_2 + \mathcal{C}_n) \to \{2, 4, 6, \ldots, 2q\}$ by

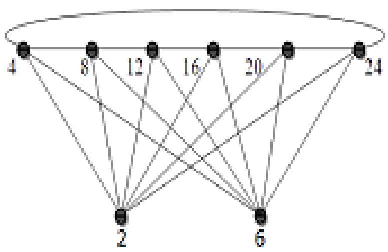
$$f(u) = 2$$

$$f(v_i) = 4i, \quad 1 \leq i \leq n \text{ if n is odd}$$

$$f(v_i) = \begin{cases} 4, & i = 1 \\ 2i + 4, & \text{i is even, if n is odd} \\ q + 2i + 2, & \text{i is odd.} \end{cases}$$

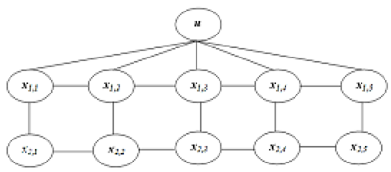
Clearly labels of the edges received by the mean of the labels on end vertices are old distinct. Hence the graph $\bar{K}_2 + \mathcal{C}_n$, has vertex Even mean labeling. □

Example 2.8. *The vertex odd mean labeling of $\bar{K}_2 + V_6$.*



Theorem 2.9. *For any integer $m > 2$ and $n = 2$, the Mongolian tent has vertex odd mean labeling.*

Proof. Consider $m(m, n)$ with the vertex set $\{u_1, x_{1,1}, x_{1,2}, \ldots, x_{1,m}, x_{2,1}, x_{2,2}, \ldots, x_{2,m}\}$. Where u is the pendent vertex the ordinary labeling for $m(5, 2)$ is given in figure.



Define a vertex labeling $f : v(m(m, n)) \to \{1, 3, 5, \ldots, 2q - 1\}$ by

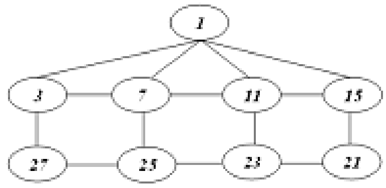
$$f(u) = 1$$

$$f(x_1, i) = 4i - 1, \quad i = 1, 2, \ldots, m$$

$$f(x_2, i) = 2q - (2i - 1), \quad i = 1, 2, \ldots, m$$

Clearly labels of the edges received by the mean of the labels on end vertices are all distinct. Hence the Mongolian tent has vertex odd mean labeling. □

Example 2.10. *The vertex odd mean labeling of the graph $m(4,2)$.*



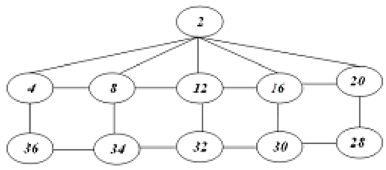
Theorem 2.11. *For any integer $m > 2$ and $n = 2$, the Mongolian tent has vertex even mean labeling.*

Proof. Define a vertex labeling $f : v(m(m,n)) \to \{2,4,6,\ldots,2q\}$ by

$$f(u) = 2$$
$$f(x_1,i) = 4i, \quad i = 1,2,\ldots,m$$
$$f(x_2,i) = 2q - (2i-2), \quad i = 1,2,\ldots,m$$

Clearly labels of the edges received by the mean of the labels on end vertices are all distinct. Hence the Mongolian tent has vertex even mean labeling. □

Example 2.12. *The vertex odd mean labeling of the graph $m(5,2)$.*



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