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A Study on Critical Path Method Using Interval Valued Fuzzy Numbers

Research Article*

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Abstract: In this paper, Fuzzy critical path in project network is studied with the aid of Interval valued fuzzy numbers. Interval valued trapezoidal fuzzy numbers (IVTFNS) are used to represent activity times in the project network. The L_R type interval valued fuzzy numbers(L_RIVFNS) have been studied with new arithmetic operations. A new ranking value of fuzzy number approach on this fuzzy number is discussed to solve the problem. A comparison have been made between sanguansat and chen ranking method and the above discussed ranking approach with a numerical example.

Keywords: Critical path method, Fuzzy Project Network, Fuzzy ranking method, Interval Valued fuzzy numbers.

1. Introduction

A project defines a combination of interrelated activities that must be executed in a certain order before the entire task can be completed. The activities are interrelated in a logic sequence, an activity in a project is usually viewed as a job requiring time and resources for its completion. A fundamental approach to solve these problems is applying fuzzy sets. Introducing the fuzzy set by zadeh in 1965 opened promising new horizons to different scientific area such as project scheduling fuzzy theory, several methods have been developed during the last three decades[5,15]. The first method was proposed by Chanas and Kamburowski [1]. They presented the project completion time in the form of a fuzzy set in the time space. Gazdik [4] developed fuzzy network of an priori unknown project to estimate the activity duration and fuzzy algebraic operator to calculates the duration of the project and its critical path. The fuzzy networking was proposed by Nasution [11] and Lorterapong and Moselhi [8]. Following on this, Mc cahon [9] Chang *et al.* [2] and Lin and Yao [7] presented three methodologies to calculate fuzzy completion project time. Other resources such as Kuchta [6] and Oliveros and Rabinson [12] using fuzzy numbers presented other methods to obtain fuzzy critical paths, critical activities and activity delay.

The operation time for each activity in the fuzzy project network is characterized as an interval valued fuzzy numbers. In accordance with CPM, the forward pass yields the fuzzy earliest-start and earliest-finish times. The backward pass is performed to calculate the fuzzy latest-start and latest finish times. CPM has proved very valuable in evaluating project performance and identifying bottlenecks. Stephen Dinagar and Abirami[18,19] studied the critical path network using topsis ranking procedure. Narayanamoorthy and Maheshwari [10] studied fuzzy critical path method based on sanguansat and chen ranking method. In this paper, we have introduced a new method to find the critical path by using two distinct ranking

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method with the IVFNS. The approach is illustrated by suitable example. The paper is organized as follows, firstly in section 2 we presented the definitions of Interval valued fuzzy numbers. In section 3 , The arithmetic operations of IVFNS and L_RIVFNS have been proposed. The new ranking properties are studied. In section 4, New algorithms are discussed to find the critical path. In section 5, a relevant numerical example is presented to justify the above said notion.The conclusion is also included in section 6.

2. Preliminaries

In this section, some important definitions and results which are useful to this work are presented.

Definition 2.1 ([3]). *A Fuzzy set $\tilde{a}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics.*

(1). $\tilde{a}$ is convex, i.e., $\tilde{a}(\lambda X_1 + (1-\lambda)X_2) = \min\{\tilde{a}(X_1), \tilde{a}(X_2)\}$, for all $X_1,\ X_2 \in R$ and $\lambda \in [0,1]$.

(2). $\tilde{a}$ is normal i.e., there exists an $X_0 \in R$ such that $\tilde{a}(X_0) = 1$

(3). $\tilde{a}$ is piecewise continuous.

Definition 2.2. *An IVFN $\tilde{A}$ on R is given by $\tilde{A} = \{x, (\mu_A^L(x),\ \mu_A^U(x)), x \in R\}$ and $\mu_A^L(x) \leq \mu_A^U(x)$ for all $x \in R$. Denote $\tilde{A} = [\tilde{A}^L,\ \tilde{A}^U]$, where $\tilde{A}^L = (a_1^L, a_2^L, a_3^L, a_4^L)$ and $\tilde{A}^U = (a_1^U, a_2^U, a_3^U, a_4^U)$ are the trapezoidal fuzzy numbers. It is also noted that $a_1^U \leq a_1^L$, $a_2^U \leq a_2^L$, $a_3^L \leq a_3^U$, $a_4^L \leq a_4^U$.*

Example 2.3. *Let $\tilde{A} = [(2,4,5,7),(1,3,6,8)]$*

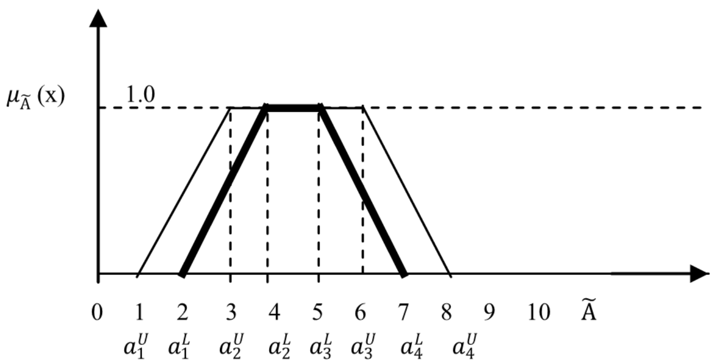

Figure 1. IVFN $\tilde{A}$

$\tilde{A} = [a^L,\ a^U] = [(a_1^L,\ a_2^L,\ a_3^L,\ a_4^L), (a_1^U,\ a_2^U,\ a_3^U,\ a_4^U)]$. *Here* $a_1^U \leq a_1^L$, $a_2^U \leq a_2^L$, $a_3^U \geq a_3^L$, $a_4^U \geq a_4^L$.

Definition 2.4. *An efficient for comparing the fuzzy number is by the use of ranking function defined $R: F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on a set of real numbers, which maps each fuzzy number into a real number where a natural order exists. For $\tilde{A} = (a_1^L,\ a_1^U) \in F(R)$, then the ranking function $R: F(R) \to R$ is defined as:*

$$R(\tilde{A}) = \frac{(a_1^L + a_2^L + a_3^L + a_4^L + a_1^U + a_2^U + a_3^U + a_4^U)}{8}$$

Definition 2.5. *A fuzzy number $\tilde{A}$ is said to be an interval valued fuzzy number if its membership function $\mu_{\tilde{A}} : x \to [0,1]$*

has the following characteristic function:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1^L}{a_2^L-a_1^L}, & a_1^L \le x \le a_2^L \\ \frac{a_4^L-x}{a_4^L-a_3^L}, & a_3^L \le x \le a_4^L \\ 1, & a_2^U \le x \le a_3^U \quad and \quad a_2^L \le x \le a_3^L \\ \frac{x-a_1^U}{a_2^U-a_1^U}, & a_1^U \le x \le a_2^U \\ \frac{a_4^U-x}{a_4^U-a_3^U}, & a_3^U \le x \le a_4^U \\ 0, & otherwise \end{cases}$$

Definition 2.6. *An L-R type Interval valued fuzzy number* $\tilde{A}$ *on R is given by* $\tilde{A}_{LR} = \{x, (\mu^L_{\tilde{A}_{LR}}(x),\ \mu^U_{\tilde{A}_{LR}}(x)), x \in R\}$ *and* $\mu^L_{\tilde{A}_{LR}}(x) \le \mu^U_{\tilde{A}_{LR}}(x)$ *for all* $x \in R$. *And it is denoted by* $\tilde{A}_{LR} = [\tilde{A}^L_{LR},\ \tilde{A}^U_{LR}]$, *where* $\tilde{A}^L_{LR} = (a_1^L, a_2^L, \alpha_1^L, \alpha_2^L)$ *and* $\tilde{A}^U_{LR} = (a_1^U, a_2^U, \alpha_1^U, \alpha_2^U)$ *are the trapezoidal fuzzy numbers. It is also noted that* $a_1^U \le a_1^L$, $a_2^U \le a_2^L$, $\alpha_1^L,\ \alpha_2^L,\ \alpha_1^U,\ \alpha_2^U \ge 0$.

Definition 2.7. *An efficient comparison of fuzzy numbers is by the use of ranking function defined as* $\mathcal{R} : F(R) \to R$, *where F(R) is a set of fuzzy numbers defined on a set of real numbers, which maps each fuzzy number into a real number where a natural order exists. For* $\tilde{A}_{LR} = [\tilde{A}^L_{LR},\ \tilde{A}^U_{LR}] \in F(R)$, *then the ranking function* $R : F(R) \to R$ *is defined as:*

$$\mathcal{R}(\tilde{A}_{LR}) = \left[\frac{2\left(a_1^L + a_2^L + a_1^U + a_2^U\right) + \left(\alpha_2^L + \alpha_2^U\right) - \left(\alpha_1^L + \alpha_1^U\right)}{8}\right]$$

Example 2.8. *Pictorial Representation of L_R IVFN* $\tilde{A}_{LR}$: *Let* $\tilde{A}_{LR} = [(4,5,2,2),(3,6,2,2)]$

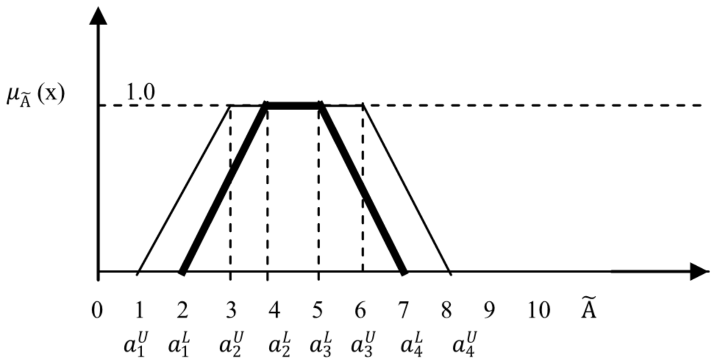

Figure 2. LRIVFN $\tilde{A}_{LR}$

Definition 2.9. *A fuzzy number* $\tilde{A}_{LR}$ *is said to be a L_R interval valued fuzzy number if its membership function* $\mu_{\tilde{A}_{LR}} : x \to [0,1]$ *has the following membership function:*

$$\mu_{\tilde{A}_{LR}}(x) = \begin{cases} 0, & if\ -\infty < x \le a_1^L - \alpha \ and\ -\infty < x \le a_1^U - \alpha \\ 1 - \frac{a_1^L - x}{\alpha_1^L}, & if\ a_1^L - \alpha < x \le a_1^L \\ 1 - \frac{a_1^U - x}{\alpha_1^U}, & if\ a_1^U - \alpha < x \le a_1^U \\ 1, & if\ a_1^L < x \le a_2^L \ and\ a_1^U < x \le a_2^U \\ \frac{a_2^L - x}{\alpha_2^L} + 1, & if\ a_2^L < x \le a_2^L + \alpha_2^L \\ \frac{a_2^U - x}{\alpha_2^U} + 1, & if\ a_2^U < x \le a_2^U + \alpha_2^U \\ 0, & if\ a_2^L + \alpha_2^L < x < \infty \ and\ a_2^U + \alpha_2^U < x < \infty \end{cases}$$

3. Arithmetic Operations [6]

3.1. Parametric Representation of IVFNS

Let $\tilde{A} = [(a_1^L, a_2^L, a_3^L, a_4^L), (a_1^U, a_2^U, a_3^U, a_4^U)]$ and $\tilde{B} = [(b_1^L, b_2^L, b_3^L, b_4^L), (b_1^U, b_2^U, b_3^U, b_4^U)]$

1. Addition for IVFNS

$$\tilde{A} \oplus \tilde{B} = \left[\left(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L\right), \left(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U\right)\right]$$

2. Subtraction for IVFNS

$$\tilde{A} \ominus \tilde{B} = \left[\left(a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L\right), \left(a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U\right)\right]$$

3. Multiplication for IVFNS

$$\tilde{A} \otimes \tilde{B} = \left[\left(a_1^L.\mathcal{R}(\tilde{B}),\ a_2^L.\mathcal{R}(\tilde{B}), a_3^L.\mathcal{R}(\tilde{B}),\ a_4^L.\mathcal{R}(\tilde{B})), (a_1^U.\mathcal{R}(\tilde{B}),\ a_2^U.\mathcal{R}(\tilde{B}), a_3^U.\mathcal{R}(\tilde{B}),\ a_4^U.\mathcal{R}(\tilde{B})\right)\right]$$

   If $R(\tilde{B}) = (b_1^L + b_2^L + b_3^L + b_4^L + b_1^U + b_2^U + b_3^U + b_4^U/8)$ when $\mathcal{R}(\tilde{B}) \geq 0$

$$\tilde{A} \otimes \tilde{B} = \left[\left(a_4^L.\mathcal{R}(\tilde{B}),\ a_3^L.\mathcal{R}(\tilde{B}), a_2^L.\mathcal{R}(\tilde{B}),\ a_1^L.\mathcal{R}(\tilde{B})), (a_4^U.\mathcal{R}(\tilde{B}),\ a_3^U.\mathcal{R}(\tilde{B}), a_2^U.\mathcal{R}(\tilde{B}),\ a_1^U.\mathcal{R}(\tilde{B})\right)\right]$$

   When $\mathcal{R}(\tilde{B}) < 0$

4. Division for IVFNS

$$\frac{\tilde{A}}{\tilde{B}} = \left[\left(\frac{a_1^L}{\mathcal{R}(\tilde{B})}, \frac{a_2^L}{\mathcal{R}(\tilde{B})}, \frac{a_3^L}{\mathcal{R}(\tilde{B})}, \frac{a_4^L}{\mathcal{R}(\tilde{B})}\right), \left(\frac{a_1^U}{\mathcal{R}(\tilde{B})}, \frac{a_2^U}{\mathcal{R}(\tilde{B})}, \frac{a_3^U}{\mathcal{R}(\tilde{B})}, \frac{a_4^U}{\mathcal{R}(\tilde{B})}\right)\right], \quad \text{if } \mathcal{R}(\tilde{B}) > 0$$
$$= \left[\left(\frac{a_4^L}{\mathcal{R}(\tilde{B})}, \frac{a_3^L}{\mathcal{R}(\tilde{B})}, \frac{a_2^L}{\mathcal{R}(\tilde{B})}, \frac{a_1^L}{\mathcal{R}(\tilde{B})}\right), \left(\frac{a_4^U}{\mathcal{R}(\tilde{B})}, \frac{a_3^U}{\mathcal{R}(\tilde{B})}, \frac{a_2^U}{\mathcal{R}(\tilde{B})}, \frac{a_1^U}{\mathcal{R}(\tilde{B})}\right)\right], \quad \text{if } \mathcal{R}(\tilde{B}) < 0$$

5. Scalar Multiplication for IVFNS

   If $k \geq 0$ and $k \in R$, then $k\tilde{A} = [(ka_1^L,\ ka_2^L,\ ka_3^L,\ ka_4^L), (ka_1^U,\ ka_2^U,\ ka_3^U,\ ka_4^U)]$

   If $k < 0$ and $k \in R$ then $k\tilde{A} = [(ka_4^L,\ ka_3^L,\ ka_2^L,\ ka_1^L), (ka_4^U,\ ka_3^U,\ ka_2^U,\ ka_1^U)]$.

3.2. L_R Representation of IVFNS [17]

Let $\tilde{A}_{LR} = \left[\left(a_1^L, a_2^L, \alpha_1^L, \alpha_2^L\right), \left(a_1^U, a_2^U, \alpha_1^U, \alpha_2^U\right)\right]$ and $\tilde{B}_{LR} = \left[\left(b_1^L, b_2^L, \beta_1^L, \beta_2^L\right), \left(b_1^U, b_2^U, \beta_1^U, \beta_2^U\right)\right]$

1. Addition for L_R IVFNS

$$\tilde{A}_{LR} \oplus \tilde{B}_{LR} = \left[\ \left(a_1^L + b_1^L, a_2^L + b_2^L, \alpha_1^L + \beta_1^L, \alpha_2^L + \beta_2^L\right), \left(a_1^U + b_1^U, a_2^U + b_2^U, \alpha_1^U + \beta_1^U, \alpha_2^U + \beta_2^U\right)\ \right]$$

2. Subtraction for L_R IVFNS

$$\tilde{A}_{LR} \ominus \tilde{B}_{LR} = \left[\ \left(a_1^L - b_2^L, a_2^L - b_1^L, \alpha_1^L + \beta_2^L, \alpha_2^L + \beta_1^L\right), \left(a_1^U - b_2^U, a_2^U - b_1^U, \alpha_1^U + \beta_2^U, \alpha_2^U + \beta_1^U\right)\ \right]$$

3. Multiplication for L_R IVFNS

$$\tilde{A}_{LR} \otimes \tilde{B}_{LR} = \begin{bmatrix} \left(a_1^L.\mathcal{R}\left(\tilde{B}_{LR}\right),\ a_2^L.\mathcal{R}\left(\tilde{B}_{LR}\right), \alpha_1^L.\mathcal{R}\left(\tilde{B}_{LR}\right),\ \alpha_2^L.\mathcal{R}\left(\tilde{B}_{LR}\right)\right), \\ \left(a_1^U.\mathcal{R}\left(\tilde{B}_{LR}\right),\ a_2^U.\mathcal{R}\left(\tilde{B}_{LR}\right), \alpha_1^U.\mathcal{R}\left(\tilde{B}_{LR}\right),\ \alpha_2^U.\mathcal{R}\left(\tilde{B}_{LR}\right)\right) \end{bmatrix}$$

If $\mathcal{R}(\tilde{B}_{LR}) = \left(\frac{2\left(b_1^L+b_2^L+b_1^U+b_2^L\right)+\left(\beta_2^L+\beta_2^U\right)-\left(\beta_1^L+\beta_1^U\right)}{8}\right)$ when $\mathcal{R}(\tilde{B}_{LR}) \geq 0$

$$\tilde{A}_{LR} \otimes \tilde{B}_{LR} = \begin{bmatrix} \left(a_2^L.\mathcal{R}\left(\tilde{B}_{LR}\right),\ a_1^L.\mathcal{R}\left(\tilde{B}_{LR}\right), \alpha_2^L.\mathcal{R}\left(\tilde{B}_{LR}\right),\ \alpha_1^L.\mathcal{R}\left(\tilde{B}_{LR}\right)\right), \\ \left(a_2^U.\mathcal{R}\left(\tilde{B}_{LR}\right),\ a_1^U.\mathcal{R}\left(\tilde{B}_{LR}\right), \alpha_2^U.\mathcal{R}\left(\tilde{B}_{LR}\right),\ \alpha_1^U.\mathcal{R}\left(\tilde{B}_{LR}\right)\right) \end{bmatrix}$$

When $\mathcal{R}(\tilde{B}_{LR}) < 0$

4. Division for L_R IVFNS

$$\begin{aligned} \frac{\tilde{A}_{LR}}{\tilde{B}_{LR}} &= \left[\left(\frac{a_1^L}{\mathcal{R}(\tilde{B}_{LR})}, \frac{a_2^L}{\mathcal{R}(\tilde{B}_{LR})}, \frac{\alpha_1^L}{\mathcal{R}(\tilde{B}_{LR})}, \frac{\alpha_2^L}{\mathcal{R}(\tilde{B}_{LR})}\right), \left(\frac{a_1^U}{\mathcal{R}(\tilde{B}_{LR})}, \frac{a_2^U}{\mathcal{R}(\tilde{B}_{LR})}, \frac{\alpha_1^U}{\mathcal{R}(\tilde{B}_{LR})}, \frac{\alpha_2^U}{\mathcal{R}(\tilde{B}_{LR})}\right)\right], \quad \text{if } \mathcal{R}(\tilde{B}_{LR}) > 0 \\ &= \left[\left(\frac{a_2^L}{\mathcal{R}(\tilde{B}_{LR})}, \frac{a_1^L}{\mathcal{R}(\tilde{B}_{LR})}, \frac{\alpha_2^L}{\mathcal{R}(\tilde{B}_{LR})}, \frac{\alpha_1^L}{\mathcal{R}(\tilde{B}_{LR})}\right), \left(\frac{a_2^U}{\mathcal{R}(\tilde{B}_{LR})}, \frac{a_1^U}{\mathcal{R}(\tilde{B}_{LR})}, \frac{\alpha_2^U}{\mathcal{R}(\tilde{B}_{LR})}, \frac{\alpha_1^U}{\mathcal{R}(\tilde{B}_{LR})}\right)\right], \quad \text{if } \mathcal{R}(\tilde{B}_{LR}) < 0 \end{aligned}$$

5. Scalar Multiplication for L_R IVFNS

If $k \geq 0$ and $k \in R$, then $k\tilde{A}_{LR} = [(ka_1^L,\ ka_2^L,\ k\alpha_1^L,\ k\alpha_2^L),\ (ka_1^U,\ ka_2^U,\ k\alpha_1^U,\ k\alpha_2^U)]$

If $k < 0$ and $k \in R$ then $k\tilde{A}_{LR} = [(ka_2^L,\ ka_1^L,\ k\alpha_2^L,\ k\alpha_1^L),\ (ka_2^U,\ ka_1^U,\ k\alpha_2^U,\ k\alpha_1^U)]$.

4. Algorithm for Finding the Critical Path Method [16]

In this section two distinct method are used to find the critical path be discussed as follows.

4.1. Ranking Value of fuzzy number Approach

Let $\tilde{A}_i = [(a_i^L,\ b_i^L,\ c_i^L,\ d_i^L),\ (a_i^U,\ b_i^U,\ c_i^U,\ d_i^U)]$, $i = 1, 2, \ldots, n$, be fuzzy numbers with membership functions $\mu_{\tilde{A}_i}$ respectively. Let $x_1 = \inf\ D$, $x_2 = \sup\ D$, and $D_i = \{x|\mu_{\tilde{A}_i}(x) \geq 0\}$, $i = 1, 2, 3, \ldots, n$, $\mu_{\tilde{A}}^L(x) > 0$, $\mu_{\tilde{A}}^U(x) > 0$. Then ranking value of fuzzy number $\tilde{A}_i$, $R(\tilde{A}_i)$, is defined as

$$\begin{aligned} R(\tilde{A}_i) = \frac{\beta}{2}&\left[(d_i^L + d_i^U) - x_1/x_2 - x_1 - (c_i^L + c_i^U) + (d_i^L + d_i^U)\right] \\ &+ \left(\frac{1-\beta}{2}\right)\left[2 - [x_2 - (a_i^L + a_i^U)]/(x_2 - x_1) + (b_i^L + b_i^U) - (a_i^L + a_i^U)\right] \end{aligned} \tag{1}$$

The value of β can be referred to as decision makers risk index. If $\beta < 0.5$, it implies that the decision maker is a risk averter. If $\beta = 0.5$, it implies that the risk attitude of decision maker is natural. If $\beta > 0.5$, it implies that the decision maker is risk lover. For a fuzzy critical path (FCP) analysis problem, using the IVFNS such as $F\tilde{E}T_{ij} = [(a_{ij}^L, b_{ij}^L, c_{ij}^L, d_{ij}^L), (a_{ij}^U, b_{ij}^U, c_{ij}^U, d_{ij}^U)]$ to denote the fuzzy activity time of activity $\tilde{A}_{ij}$, the decision maker's risk attitude index β can be obtained by

$$\beta = \sum_i \sum_j \frac{(b_{ij}^L + b_{ij}^U) - (a_{ij}^L + a_{ij}^U)}{\left[[(b_{ij}^L + b_{ij}^U) - (a_{ij}^L + a_{ij}^U)] + \left[(d_{ij}^L + d_{ij}^U) - (c_{ij}^L + c_{ij}^U)\right]\right]} \Big/ t, \quad \tilde{A}_{ij} \in \quad ACT \tag{2}$$

where ACT and t denote the set of all activities and the number of activities in a project network, respectively. We can write x_1 as $x_1 = \min\{(a_1^L + a_1^U), (a_2^L + a_2^U), \ldots, (a_n^L + a_n^U)\}$ and x_2 as $x_2 = \max\{(d_1^L + d_1^U), (d_2^L + d_2^U), \ldots, (d_n^L + d_n^U)\}$. Taking β value calculated by the equation (2) and by using equation (1), we can easily calculate the ranking values of the n IVFNS. Then based on the ranking rules described above, the ranking of the IVFNS can be effectively determined.

Notations 4.1 ([13,14]). *The notations that will be used in the presented methods are as follows.*

N	The set of all nodes in a project network.
A_{ij}	The activity between nodes i and j
$F\tilde{E}T_{ij}$	The fuzzy activity time of A_{ij}.
$F\tilde{E}S_j$	The earliest fuzzy time of node j.
$F\tilde{L}F_j$	The latest fuzzy time of node j.
$F\tilde{T}S_{ij}$	The total slack fuzzy time of A_{ij}.
$S(j)$	The set of all successor activities of node j.
$NS(j)$	The set of all nodes connected to all successor activities of node j. i.e., $NS(j) = \{k \mid A_{jk} \in S(j),\ k \in N\}$.
$F(j)$	The set of all predecessor activities of node j.
$NP(j)$	The set of all nodes connected to all predecessor activities of node j. i.e., $NP(j) = \{i/A_{ij} \in N\}$.
P_i	The i^{th} path.
P	The set of all paths in a project network.
$F\tilde{C}PM(P_K)$	The fuzzy completion time of path P_k in a project network.

4.2. Some Important Properties

In this section, some properties that will be used in this method for analyzing the FCP are presented. Set the initial node to be zero for starting. i.e., $F\tilde{E}S_1 = [(0,0,0,0),(0,0,0,0)]$. Then the following properties are true :

Property 4.2. $F\tilde{E}S_j = \max\{F\tilde{E}S_i \oplus F\tilde{E}T_{ij} \mid i \in NP(j),\ j \neq 1,\ j \in N\}$.

Property 4.3. $F\tilde{L}F_j = \min\{F\tilde{L}F_k \ominus F\tilde{E}T_{jk} \mid k \in NS(j),\ j \neq n,\ j \in N\}$.

Property 4.4. $F\tilde{T}S_{ij} = F\tilde{L}F_j \ominus (F\tilde{E}S_j \oplus F\tilde{E}T_{ij}),\ 1 \leq i < j \leq n;\ i, j \in N$.

Property 4.5. $F\tilde{C}PM(P_k) = \sum_{\substack{1 \leq i < j \leq n, \\ i,j \in P_k}} F\tilde{T}S_{ij},\ P_k \in p$.

Definition 4.6. *If there exists a path P_c in a project network such that $F\tilde{C}PM(P_c) = \min\{F\tilde{C}PM(P_i) \mid P_i \in P\}$ then the path P_c is a FCP.*

4.3. Algorithm for Finding the Fuzzy Critical Path

The FCP of a Project network can be obtained by using the following steps :

1. Identify activities in a project.
2. Establish precedence relationship of all activities.
3. Estimate the fuzzy activity time with respect to each activity.
4. Construct the project network.
5. Let $F\tilde{E}S_1 = [(0,0,0,0),(0,0,0,0)]$ and calculate $F\tilde{E}S_j$, $j = 2, 3, \ldots, n$ by using Property 4.2
6. Let $F\tilde{L}F_n = F\tilde{E}S_n$ and calculate $F\tilde{L}F_j$, $j = n-1, n-2, \ldots, 2, 1$, by using Property 4.3
7. Calculate $F\tilde{T}S_{ij}$ with respect to each activity in a project network by using Property 4.4
8. Find all the possible paths and calculate $F\tilde{C}PM(P_k)$ by using Property 4.5

9. Find the FCP by Using Definition 4.6

10. Find the grade of membership that the project can be completed at scheduled time.

4.4. Sanguansat and Chen Ranking Method [10]

Let $F\tilde{E}S_i$ and $F\tilde{L}S_i$ be the earliest fuzzy event time, and the latest Fuzzy event time for event i, respectively Functions that define the earliest starting times, latest starting times and floats in terms of fuzzy activity durations are convex, normal whose membership functions are piecewise continues, hence the quantities such as earliest fuzzy event time $F\tilde{E}S_i$, the latest fuzzy event time $F\tilde{L}S_i$ and the floats $\tilde{T}_i$ are also IVFNS for an event i respectively.

Step 1: Identify fuzzy activities in a fuzzy project

Step 2: Establish precedence relationship of all fuzzy activities, by applying fuzzy ranking function

Step 3: Construct the fuzzy project network with IVFNS as fuzzy activity times.

Step 4: Let $F\tilde{E}S_1$ be the earliest Fuzzy event time and $F\tilde{L}S_1$ be the latest fuzzy event time for the initial event $\tilde{V}_1$ of the project network and assume that $F\tilde{E}S_1 = F\tilde{L}S_1 = \tilde{0}$ Compute the earliest fuzzy event time $F\tilde{E}S_j$ of the event $\tilde{V}_j$ by using the formula

$$F\tilde{E}S_j = \max_{i\in N: i\to j}\{F\tilde{E}S_i + \tilde{A}_{ij}\} \tag{3}$$

Step 5: Let $F\tilde{E}S_n$ be the earliest fuzzy event time and $F\tilde{L}S_n$ be the latest fuzzy event time for the terminal event $\tilde{V}_n$ of the fuzzy project network and assume that $F\tilde{E}S_n = F\tilde{L}S_n$. Compute the latest fuzzy event time $F\tilde{L}S_i$ by using the following equation

$$F\tilde{L}S_i = \min_{i\in N}\{F\tilde{L}S_j - \tilde{A}_{ij}\} \tag{4}$$

Step 6: Compute the total float $\tilde{T}_{ij}$ of each fuzzy activity $\tilde{a}_{ij}$ by using the following equation

$$\tilde{T}_{ij} = \{F\tilde{L}S_j - F\tilde{E}S_i\} \tag{5}$$

Step 7: Find all the possible paths and finding the ranking value by using the Definition 2.3 which is denoted as k

Step 8: Divide all the possible paths P_i by the value of k which is denoted by P_i^* and find the mean value of P_i^* which is also denoted by $\overline{P}_i$

Step 9: Find the value of $\sigma_{P_i^*} = \sqrt{\frac{\sum_{j=1}^{4}(P_{ij}^{*L} - \overline{P}_i)^2 + \sum_{j=1}^{4}(P_{ij}^{*U} - \overline{P}_i)^2}{8-1}}$, where $1 \le i \le n$

Step 10: Find the values of

$$Area_{iL}^- = \frac{(p_{i1}^{*L}+1) + (p_{i2}^{*L}+1) + (p_{i1}^{*U}+1) + (p_{i2}^{*U}+1)}{4}, \quad Area_{iR}^- = \frac{(p_{i3}^{*L}+1) + (p_{i4}^{*L}+1) + (p_{i3}^{*U}+1) + (p_{i4}^{*U}+1)}{4}$$

$$Area_{iL}^+ = \frac{(1-p_{i1}^{*L}) + (1-p_{i2}^{*L}) + (1-p_{i1}^{*U}) + (1-p_{i2}^{*U})}{4}, \quad Area_{iR}^+ = \frac{(1-p_{i3}^{*L}) + (1-p_{i4}^{*L}) + (1-p_{i3}^{*U}) + (1-p_{i4}^{*U})}{4}$$

Step 11: Find $Score\,(P_i^*) = \frac{XI_{P_i^*} - XD_{P_i^*}}{XI_{P_i^*} + XD_{P_i^*} + k}$ by using $XI_{P_i^*} = Area_{iL}^- + Area_{iR}^-$ and $XD_{P_i^*} = Area_{iL}^+ + Area_{iR}^+$. Minimum value of $Score\,(P_i^*)$ gives the required critical path.

5. Numerical Illustration

Suppose there is a project network, as shown in Fig 5.1 with the set of node $N = \{1, 2, 3, 4\}$, the fuzzy activity time for each activity as shown in Table 1. All of the duration are in hours. Find the FCP for the given network.

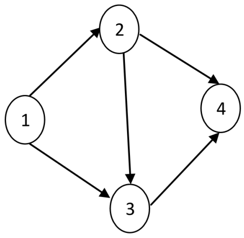

Figure 3. **Project network**

Activity A_{ij}	Fuzzy activity times (Hours) $F\tilde{E}T_{ij}$
A_{12}	Approximately 5 Hours [(4,5,5,6), (3,5,5,7)]
A_{13}	Approximately 10 Hours [(8,10,10,15), (5,10,10,15)]
A_{23}	Approximately between 3 and 4 Hours [(2,3.3,3.7,4), (1,3,4,5)]
A_{24}	Approximately between 4 and 5 Hours [(3,4.4,4.6,5), (2,4,5,6)]
A_{34}	Approximately between 8 and 10 Hours[(7,8.2,9.8,10), (6,8,10,11)]

Table 1. **Fuzzy activity time for each activity.**

By using the algorithm 4.1, By using the algorithm 4.2,

POSSIBLE PATHS	$F\tilde{C}PM(P_i)$	$R(F\tilde{C}PM(P_i))$
$P_1 = 1-2-4$	[(0,8.3,13.7,24),(-14,7,15,37)]	0.50962
$P_2 = 1-2-3-4$	[(-17,-2.2,8.2,25),(-39,-4,10,47)]	0.46723
$P_3 = 1-3-4$	[(-14,-3.2,3.2,14),(-30,-4,4,30)]	0.44126

Table 2.

POSSIBLE PATHS	$XI_{P_i^*}$	$XD_{P_i^*}$	$Score\,(P_i^*)$
$P_1 = 1-2-4$	3.8665075	0.1385	0.09169
$P_2 = 1-2-3-4$	3.9999	0.00006	0.098223
$P_3 = 1-3-4$	3.752768	0.24713	0.086107

Table 3.

The Minimum Value of $Score\,(P_i^*)$ is 0.086107. $P_3 = 1-3-4$ is the required critical path. From the Table 2 and 3, it is noted that Sanguansat and chen method is better to find the critical path in a fuzzy project network.

6. Conclusion

In this paper we have computed total fuzzy slack time for each path in the fuzzy project network to find the critical path for a given fuzzy project network. A new ranking value of fuzzy number approach of IVFN is utilized to solve the problem A numerical example has provided to explain the ranking method by using interval valued fuzzy numbers. A comparison have been made between sanguansat and chen method and the proposed method and the results have been given in the Table 2 and 3. The Same notion can be extended using L_R type representation of IVFN also.

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