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Dusty Couple Stress Fluid Heated Below in Hydromagnetic Field Through Porous Medium

Research Article*

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Abstract: The Combined effect of Solute gradient and Magnetic field on Dusty Couple stress fluid through Porous medium is considered. The analysis is carried out within the framework of linear stability theory and normal mode technique. For the case of stationary convection, solute gradient has stabilizing effect whereas Dust particles are found to have a destabilizing effect on the system. Couple Stress and Medium permeability have dual character to its stabilizing effect in the absence of magnetic field and rotation. Magnetic field has succeed in stabilizing effect in the absence of rotation. The oscillatory modes are introduced due to the presence of magnetic field in the system. Graphs have been plotted by giving numerical values to the parameters to depict the stability characteristics.

Keywords: Dusty Couple Stress Fluid, Hydromagnetic Field, Porous Medium.

1. Introduction

The theoretical and experimental results on the onset of thermal instability(B'enard convection) in a fluid layer under varying assumption of hydrodynamics, has been discussed in detail by Chandrasekhar (1981). With the growing of non-Newtonian fluids in technology and industries, the investigations of such fluids are desirable. The theory of couple-stress fluids is proposed by Stokes (1966). Couple-stress appear in noticeable magnitude in fluids with very large molecules. Applications of couple-stress fluid occur in the attention the study of the mechanism of lubrication of synovial joints, at which currently attract the attention of researchers. A human joint is a dynamically loaded bearing that has an articular cartilage as the bearing and synovial fluid as the lubricant. Normal synovial fluid is clear or yellowish and is a non-Newtonian, viscous fluid. Walicki and Walicka (1999) Modeled synovial fluid as couple-stress fluid in human joints because of the long chain of lauronic acid molecules found as additives in synovial fluid. The problem of a couple-stress fluid heated from below in a porous medium is considered by Sharma and Sharma (2001) and Sharma and Thankur (2000).

Recent spacecraft observations have confirmed that the dust particles play a significant role in the dynamics of the atmosphere as well as in the diural and surface variations in the temperature of the martain whether. Further, environmental pollution is the main cause of the dust to enter the human body. The metal dust which filters into the blood stream of those working near furnace causes extensive damage to the chromosomes and genetic mutation so observed are likely to breed censer as malformations in the coming progeny. It is essential, therefore to study the presence of dust particles in astrophysical

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situations and fluid flow. Sunil et al.(2004) have studied the effect of suspended particles on couple-stress fluid heated and soluted from below in a porous medium and found that suspended particles have destabilizing effect on the system. Sharma and Sharma (2004) have studied the effect of suspended particles on couple-stress fluid heated from below in the presence of rotation and magnetic field and found that rotation has a stabilizing effect while dust particles have destabilizing effect on the system. Kumar et al. (2004) have studied the thermal stability of Walters 'B' visco-elastic fluid permeated with suspended particles in hydromagnetics in a porous medium and found that magnetic fields stabilize the system. The problem on a Rivlin-Ericksen fluid in a porous medium in the presence of uniform vertical magnetic field and rotation is also considered by Sharma et al (2001). They have found that rotation has a stabilizing effect on the system. Keeping in mind the importance of non-Newtonian fluids, convection in a fluid layer heated from below, dust particles and magnetic field, we propose to study the Stokes (1966) incompressible couple-stress fluid in the presence of dust particles and magnetic field in the present paper.

2. Formulation of the Problem

Consider a static in which an incompressible, Stokes couple-stress fluid layer of thickness d, is arranged, confined between two infinite horizontal planes situated at $z = 0$ and $z = d$, which is acted upon by a vertical magnetic field $\mathbf{H}(0,0,H)$, where H is a constant, uniform rotation $\Omega(0,0,\Omega)$, and variable gravity field $\mathbf{g}(0,0,-g)$. The particles are assumed to be non-conducting. The fluid layer is heated from below leading to an adverse temperature gradient $\beta = \frac{T_0 - T_1}{d}$, where T_0 and T_1 are the constant temperatures of the lower and upper boundaries with $T_0 > T_1$ and $\beta' = \frac{C_0 - C_1}{d}$, Where C_0 and C_1 are the constant concentrations of lower and upper boundaries with $C_0 > C_1$. This fluid particles layer is assumed to be flowing through an isotroic and homogeneous porous medium of porosity ε and medium permeablity k_1.

Let p, ρ, T, C, α, v, μ', k_T and $\mathbf{q}(u,v,w)$ denote respectively pressure, density, temperature, concentration, thermal coefficient of expansion an analogous coefficient of expansion, kinematic viscosity, couple-stress viscosity, thermal diffusivity, solute diffusivity velocity of the fluid. $\mathbf{q}_d(\overline{x},t)$ and $N(\overline{x},t)$ denote the velocity and number density of particles. $K = 6\pi\mu\eta$, where η is the particle radius, is a constant and $\overline{x} = (x,y,z)$. Then equation of motion, continuity and heat conduction of couple-stress rotating dusty fluid in hydromagnetics through porous medium are

$$\frac{1}{\varepsilon}\left[\frac{\partial q}{\partial t} + \frac{1}{\varepsilon}(q.\nabla)q\right] = -\frac{1}{\rho_0}\nabla p + g\left(1 + \frac{\delta\rho}{\rho_0}\right) - \frac{1}{k_1}\left(v - \frac{\mu^1}{\rho_0}\nabla^2\right)q + \frac{KN}{\varepsilon\rho_0}(q_d - q) + \frac{2}{\varepsilon}(q \times \Omega) + \frac{\mu_e}{4\pi\rho_0}[(\nabla \times H) \times H] \quad (1)$$

$$\nabla \cdot q = 0 \quad (2)$$

$$\varepsilon\frac{\partial H}{\partial t} = (H \cdot \nabla)q + \varepsilon\eta\nabla^2 H \text{ and} \quad (3)$$

$$\nabla \cdot H = 0 \quad (4)$$

The equation of state is

$$P = \rho_0[1 - \alpha(T - T_0) + \alpha'(C - C_0)] \quad (5)$$

Where the suffix zero refers to value at the reference level $z = 0$. The presence of particles add an extra force term, proportional to the velocity difference between particles and fluid and appears in equation of motion (1). Since the force exerted by the fluid on the particles is equal and opposite to the exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equation of motion for the particles. The buoyancy force on the particles is neglected. Interparticle reactions are not considered for we assume that the distance between particles is quite large as compared with their diameter. The equations of motion continuity for the particle, under the above approximation, are

$$mN\left[\frac{\partial q_d}{\partial t}+\frac{1}{\varepsilon}(q_d\cdot\nabla)q_d\right]=KN(q-q_d) \tag{6}$$

and

$$\varepsilon\frac{\partial N}{\partial t}+\nabla(N\cdot q_d)=0 \tag{7}$$

Here mN is represent the mass of the particles per unit volume. Let c_v, c_{pt}, ρ_s, c_s denote the heat capacity of the fluid at constant volume and the heat capacity of the particles, density of the solid (porous) material and heat capacity of the solid (porous) material. Assuming that the particles and fluids are in thermal equilibrium, the equation of heat conduction give

$$\rho_0 c_v\varepsilon+\rho_s c_s(1-\varepsilon)\frac{\partial T}{\partial t}+\rho_0 c_v(q\cdot\nabla)T+mNc_{pt}\left(\varepsilon\frac{\partial}{\partial t}+q_d\cdot\nabla\right)T=q\nabla^2 T \text{ or}$$

$$E\frac{\partial T}{\partial t}+(q\cdot\nabla)T+\frac{mNc_{pt}}{\rho_0 C_v}\left(\varepsilon\frac{\partial}{\partial t}+q_d\cdot\nabla\right)T=K_T\nabla^2 T \tag{8}$$

$$E'\frac{\partial C}{\partial t}+(q\cdot\nabla)C+\frac{mNc_{pt}}{\rho_0 C_v}\left(\varepsilon\frac{\partial}{\partial t}+q_d\cdot\nabla\right)C=K_s\nabla^2 C \tag{9}$$

Where $E=\varepsilon+(1\varepsilon)\frac{\rho_s c_s}{\rho_0 C_v}$ is a constant and E' is a constant analogous to E. The kinematic viscosity v, couple-stress viscosity μ', thermal diffusivity k_T, solute diffusivity k_S, coefficient of thermal expansion α and an analogous coefficient of expansion α' are assumed to be constants.

3. Basic State

The basic state is described by $q=(0,0,0)$, $q_d=(0,0,0)$, $\Omega(0,0,\Omega)$, $H=(0,0,H)$, $T=T_0-\beta z$, and

$$C=C_0-\beta' z,\;\; N=N_0 \tag{10}$$

Where β and β' may be either positive or negative and

$$\rho=\rho(z),\;\; p=p(z)\;\;\text{ and }\;\;\rho=\rho_0[1+\alpha\beta z-\alpha'\beta' z] \tag{11}$$

4. Perturbation Equations and Normal Mode Analysis

Let $q(u,v,w)$, $q_d(l,r,s)$, $h(h_x,h_y,h_z)$, θ, γ, $\delta\rho$, δp denote respectively the perturbations in fluid velocity $q=(0,0,0)$, dust particles velocity $q_d=(0,0,0)$, magnetic field H, temperature T, Concentration C density ρ and pressure p. After linearizing the perturbation and analyzing the perturbation into normal modes, we assume that the perturbation quantities are of the form

$$[w,\theta,\gamma,h_z,\zeta,\xi]=[W(z),\Theta(z),K(z),\Gamma(z),Z(z),X(z)]\cdot\exp\{ik_x x+ik_y y+nt\}. \tag{12}$$

Where k_x and k_y are the wave number in x and y directions respectively and $k=\sqrt{K_x^2+K_y^2}$ is the resultant wave number of propagation and n is the frequency of any arbitrary disturbance which is, in general, a complex constant, $\zeta=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial t}$ and $\xi=\frac{\partial hy}{\partial x}-\frac{\delta hx}{\delta x}$ are the z-components of the vorticity and current density respectively. The perturbations in equation (12), equations (1) to (9), after eliminating the physical quantities using the non-demensional parameters $a=kd$, $\sigma=\frac{nd^2}{v}$, $\tau=\frac{m}{K}$, $p_1=\frac{v}{k_T}$, $p_2=\frac{v}{\eta}$, $q=\frac{v}{k_s}$, $F=\frac{\mu^1}{\rho_0 d^2 v}$, $\tau_1=\frac{\tau v}{d^2}$, $M=\frac{mN_0}{\rho_0}$, $B=1+b$, $B'=1+b'$, $p_e=\frac{k_1}{d^2}$, $E_1=E+b\varepsilon$, $E_1'=E'+b'\varepsilon$

and $D^* = dD$ and dropping $(*)$ for convenience, give

$$\begin{aligned}&\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M_1}{+}\sigma t_1\right)+\frac{1}{p_e}\{1-F(D^2-a^2)^2\}\right][D^2-a^2-\sigma E_1p_1][D^2-a^2-\sigma E_1q][D^2-a^2-\sigma p_2][D^2-a^2]W\\&-Ra^2\left[\frac{B+\sigma\tau_1}{1+\sigma\tau_1}\right][D^2-a^2-\sigma E_1'q][D^2-a^2-\sigma p_2]W+Sa^2\left[\frac{B'+\sigma\tau_1}{1+\sigma\tau_1}\right][D^2-a^2-\sigma E_1p_1][D^2-a^2-\sigma p_2]W\\&+\frac{Q}{\varepsilon}[D^2-a^2-\sigma E_1p_1][D^2-a^2]D^2W+\frac{T_A[D^2-a^2-\sigma E_1'q][D^2-a^2-\sigma p_2]^2[D^2-a^2-\sigma E_1p_1]D^2W}{\varepsilon^2\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\sigma\tau_1}\right)+\frac{1}{p_e}\{1-F(D^2-a^2)\}[D^2-a^2-\sigma p_2]+\frac{Q}{\varepsilon}D^2\right]}=0\end{aligned}\tag{13}$$

Here $R = \frac{g\alpha\beta d^4}{vk_T}$ is the thermal Rayleigh number, $T_A = \left(\frac{2Wd^2}{v}\right)^2$ is the Taylor number and $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0 v\eta}$ is the Chandrasekhar number. The perturbation in the temperature is zero at the boundaries because both the boundaries are maintained at constant temperatures. The appropriate boundary conditions for the equation (13) are

$$W=0,\ D^2W=0,\ D^4W=0,\ \Theta=0,\ \Gamma=0,\ DZ=0,\ DK=0\ \text{ at }\ z=0\ \text{ and }\ z=1 \tag{14}$$

From equation (12), it is clear that all the even order derivatives of W vanish on the boundaries. Therefore, the proper solution of equation (11) characterizing the lowest mode is

$$W = W_0 \sin \pi z \tag{15}$$

Here W_0 is constant. Using equation (15), equation (13) gives

$$\begin{aligned}R_1=&\frac{1+x}{x}\left[\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{1+i\sigma_1\pi^2\tau_1}\right)+\frac{1}{p}\{1+F_1(1+x)\}\right]\left(\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right)[1+x+i\sigma_1E_1p_1]\\&+S_1\left(\frac{B'+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right)\left[\frac{1+x+i\sigma_1E_1p_1}{1+x+i\sigma_1E_1'q}\right]+\frac{Q_1}{\varepsilon}\frac{(1+x)}{x}\left(\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right)\left[\frac{1+x+i\sigma_1E_1p_1}{1+x+i\sigma_1p_2}\right]\\&+\frac{T_{A_1}\left(\frac{1+i\sigma_1\pi^2\tau_1}{B+i\sigma_1\pi^2\tau_1}\right)[1+x+i\sigma_1E_1p_1][1+x+i\sigma_1p_2]}{\varepsilon^2x\left[\left\{\frac{i\sigma_1}{\varepsilon}\left(1+\frac{M}{1+i\sigma_1\pi^2\tau_1}\right)+\frac{1}{p}\{1+F_1(1+x)\}\right\}\right][1+x+i\sigma_1p_2+\frac{Q_1}{\varepsilon}]}\end{aligned}\tag{16}$$

where, $R_1 = \frac{R}{\pi^4}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$, $F_1 = \pi^2 F$, $T_{A_1} = \frac{T_A}{\pi^4}$, $Q_1 = \frac{Q}{\pi^2}$, $P = \pi^2 p$ and $x = \frac{a^2}{\pi^2}$. Equation (16) gives the required dispersion relation including the effect of solute gradient, medium permeability, magnetic field, couple-stress, rotation, dust particles and kinematic viscoelasticity in the present problem.

5. Analytical Discussion

5.1. Stationary Convection

At stationary convection,when the instability sets, the marginal state will be characterized by $\sigma = 0$. Thus, putting $\sigma = 0$ in the equation (16), we get

$$R_1=\frac{1+x}{xB}\left[\frac{(1+x)}{p}\{F_1(1+x)+1\}+\frac{Q_1}{\varepsilon}+\frac{T_{A_1}(1+x)}{\varepsilon^2\left[\frac{(1+x)}{p}\{F_1(1+x)+1\}+\frac{Q_1}{\varepsilon}\right]}\right]+S1\frac{B'}{B}\tag{17}$$

Which express the Rayleigh number R_1 as a function of the parameters S_1, P, B, F_1, T_{A_1}, Q_1 and dimensionless wave number x. To study the effect of dust particles, couple-stress and magnetic fields, we study the behavior of $\frac{dR_1}{dS_1}$, $\frac{dR_1}{dP}$, $\frac{dR_1}{dB}$, $\frac{dR_1}{dF_1}$ and $\frac{dR_1}{dQ_1}$ analytically. From equation (17), we have

$$\frac{dR_1}{dS_1} = \frac{B'}{B} \tag{18}$$

Which confirm that solute gradient has a stabilizing effect on a couple - stress rotating dusty fluid on the thermal convection in porous medium. From equation (17), we have

$$\frac{dR_1}{dP} = \frac{\{F_1(1+x)+1\}(1+x)^2}{xBP^2}\left[\frac{T_{A_1}(1+x)}{\varepsilon^2\left[\frac{(1+x)}{p}\{F_1(1+x)+1\}+\frac{Q_1}{\varepsilon}\right]^2} - 1\right] \tag{19}$$

Which shows that medium permeability has a stabilizing or destabilizing effect on the thermal convection of couple-stress rotating fluid in porous medium under the conditions

$$T_{A_1}(1+x) > \quad \text{or} \quad < \varepsilon^2\left[\frac{(1+x)}{p}\{F_1(1+x)+1\}+\frac{Q_1}{\varepsilon}\right]^2$$

But, for the permissible values of various parameters, the said effect is stabilizing only if

$$T_{A_1}(1+x) < \varepsilon^2\left[\frac{(1+x)}{p}\{F_1(1+x)+1\}+\frac{Q_1}{\varepsilon}\right]^2$$

In the absence of rotation and magnetic field, equation (19) becomes

$$\frac{dR_1}{dP} = -\frac{\{F1(1+x)+1\}(1+x)^2}{xBP^2} \tag{20}$$

Which confirm that permeability has a destabilising effect on the system in the absence of rotation and magnetic field derived by Sunil, Sharma and Chandel. From equation (20), we have

$$\frac{dR_1}{dB} = -\frac{1}{B^2}\left[\frac{1+x}{x}\left\{\frac{1+x}{P}\{F_1(1+x)+1\}+\frac{Q_1}{\varepsilon}+\frac{T_{A_1}(1+x)}{\varepsilon^2\left[\frac{(1+x)}{p}\{F_1(1+x)+1\}+\frac{Q_1}{\varepsilon}\right]}\right\}+S_1B'\right] \tag{21}$$

Which clearly show that dust particles have a destabilizing effect on the thermal convection of couple-stress rotating fluid in porous medium. From equation (17), we have

$$\frac{dR_1}{dF_1} = \frac{(1+x)^3}{xBP}\left[1 - \frac{T_{A_1}(1+x)}{\varepsilon^2[\frac{(1+x)}{p}\{F_1(1+x)+1\}+\frac{Q_1}{\varepsilon}]^2}\right] \tag{22}$$

Which shows that couple-stress has a stabilizing or destabilizing effect on the thermal convection of couple-stress rotating fluid in porous medium under the conditions

$$T_{A_1}(1+x) < \quad \text{or} \quad > \varepsilon^2\left[\frac{(1+x)}{p}\{F_1(1+x)+1\}\frac{Q_1}{\varepsilon}\right]^2$$

In absence of rotation and magnetic field, equation (22) becomes

$$\frac{dR_1}{dF_1} = \frac{(1+x)^3}{xBP} \tag{23}$$

Which confirm that couple -stress has a stablising effect on a couple-stress rotating dusty fluid on thermal convection in porous medium in the absence of rotation magnetic field as Sunil, Sharma and Chandel. From equation (17), we have

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{xB\varepsilon}\left[1 - \frac{T_{A_1}(1+x)}{\varepsilon^2[\frac{(1+x)}{p}\{F_1(1+x)+1\}+\frac{Q_1}{\varepsilon}]^2}\right] \tag{24}$$

Which shows that magnetic field has a stabilizing/destabilizing effect on a couple-stress dusty fluid on thermal convection in porous medium under the condition

$$T_{A_1}(1+x) < \quad \text{or} \ \varepsilon^2 \left[\frac{(1+x)}{p}\{F_1(1+x)+1\}+\frac{Q_1}{\varepsilon}\right]^2$$

But for the permissible values of various parameters, magnetic field has a stabilizing effect to its destabilizing influence if

$$T_{A_1}(1+x) < \varepsilon^2 \left[\frac{(1+x)}{p}\{F_1(1+x)+1\}+\frac{Q_1}{\varepsilon}\right]^2$$

In absence of rotation ($T_{A_1}=0$), equation (24) becomes

$$\frac{dR_1}{dQ_1}=\frac{(1+x)}{xB\varepsilon} \tag{25}$$

Which shows that in the absence of rotation, magnetic field has a stabilizing effect on a couple-stress rotating dusty fluid on thermal convection in porous medium.

5.2. Stability of the system and Oscillatory Modes

Using equations (1) to (9) with the boundary condition (14), we get

$$\begin{aligned}&\left[\frac{\sigma}{\varepsilon}\left(1+\frac{M}{1+\sigma\tau_1}\right)+\frac{1}{p_e}\right]I_1+\frac{F}{P_e}I_2-\frac{g\alpha K_T a^2}{v\beta}\left(\frac{1+\sigma^*\tau_1}{B+\sigma^*\tau_1}\right)[I_3+\sigma^* E_1 p_1 I_4]+\frac{\mu_e\eta}{4\pi\rho_0 v}[I_5+\sigma^* p_2 I_6]\\&+d^2\left[\left\{\frac{\sigma^*}{\varepsilon}\left(1+\frac{M}{1+\sigma^*\tau_1}\right)+\frac{1}{p_e}\right\}I_7+\frac{F}{P_e}I_8\right]+\frac{\mu_e\eta d^2}{4\pi\rho_0 v}[I_9+\sigma p_2 I_{10}]+\frac{g\alpha' K_s a^2}{v\beta'}\left(\frac{1+\sigma^*\tau_1}{B'+\sigma^*\tau_1}\right)[I_{11}+E_1' q\sigma^* I_{12}]=0\end{aligned} \tag{26}$$

Where

$$\begin{aligned}
I_1&=\int(|DW|^2+a^2|W|^2)dz, & I_2&=\int(|D^2W|^2+2a^2|DW|^2+a^4|W|^2)dz,\\
I_3&=\int(|D\Theta|^2+a^2|\Theta|^2)dz, & I_4&=\int(|\Theta|^2)dz,\\
I_5&=\int(|D^2K|^2+2a^2|DK|^2+a^4|K|^2)dz, & I_6&=\int(|DK|^2+a^2|K|^2)dz,\\
I_7&=\int(|Z|^2)dz, & I_8&=\int(|DZ|^2+a^2|Z|^2)dz,\\
I_9&=\int(|DX|^2+a^2|X|^2)dz, & I_{10}&=\int(|X|^2)dz,\\
I_{11}&=\int(|D\Gamma|^2+a^2|\Gamma|^2)dz, & I_{12}&=\int(|\Gamma|^2)dz
\end{aligned}$$

Where $\sigma*$ is the complex conjugate of σ. All the integrals I_1 to I_{12} are positive definite, putting $\sigma=i\sigma$, in equation (27) and equating the imaginary parts, we obtain

$$\begin{aligned}&\sigma_i\left[\frac{1}{\varepsilon}\left(1+\frac{M}{1+\sigma_i^2\tau_1^2}\right)I_1+\frac{g\alpha k_T a^2}{v\beta}\left\{\frac{\tau_1(B-1)}{B^2+\sigma_i^2\tau_1^2}I_3+\frac{B+\sigma_i^2\tau_1^2}{B^2+\sigma_i^2\tau_1^2}E_1p_1I_4\right\}-\frac{\mu_e\varepsilon\eta}{4\pi\rho_0 v}p_2I_6-\frac{d^2}{\varepsilon}\left\{I_7+\frac{MI_7}{1+\sigma_i^2\tau_1^2}\right\}+\frac{\mu_e d^2P_2\eta I_{10}}{4\pi\rho_0 v}\right]\\&-\frac{g\alpha' K_s a^2}{v\beta'}\left\{\frac{\tau_1(B'-1)I_{11}}{{B'}^2+\sigma_i^2\tau_1^2}+\frac{\left(B'+\sigma_i^2\tau_1^2\right)E_{1q}'I_{12}}{{B'}^2+\sigma_i^2\tau_1^2}\right\}=0\end{aligned} \tag{27}$$

In the absence of stable solute gradient, magnetic field and rotation, equation (27) becomes

$$\sigma_i\left[\frac{1}{\varepsilon}\left(1+\frac{M}{1+\sigma_i^2\tau_1^2}\right)I_1+\frac{g\alpha k_T a^2}{v\beta}\left\{\frac{\tau_1(B-1)}{B^2+\sigma_i^2\tau_1^2}I_3+\frac{B+\sigma_i^2\tau_1^2}{B^2+\sigma_i^2\tau_1^2}E_1p_1I_4\right\}\right]+\frac{g\alpha' K_s a^2}{v\beta'}\left\{\frac{\tau_1(B'-1)I_{11}}{{B'}^2+\sigma_i^2\tau_1^2}+\frac{\left(B'+\sigma_i^2\tau_1^2\right)E_{1q}'I_{12}}{{B'}^2+\sigma_i^2\tau_1^2}\right\}=0 \tag{28}$$

From equation (28), it is obvious that all terms in the bracket are positive definite. Thus $\sigma_i = 0$, which means that oscillatory modes are not allowed in the system and Principle of Exchange of Stabilities (PES) is satisfied in the absence of magnetic field and rotation. This is true for porous as well as non-porous medium. It is evident form equation (27) that presence of stable solute gradient and magnetic field brings oscillatory modes (as, σ_i may not be zero) which were non-existent in their absence.

6. Numerical Computations

Now, the critical thermal Rayleigh number for the onset of instability is determined numerically using Newton-Raphson method by the condition $\frac{dR_1}{dx} = 0$. The numerical values of critical thermal Rayleigh number R_1 and critical wave number x determined for various values of solute gradient S_1, dust particles B,magnetic field Q_1, and couple-stress F_1. Graphs have been potted between critical Rayleigh number R_1 and Parameters S_1, B, Q_1 and F_1 by giving some numerical values to them.

In Figure 1, critical Rayleigh number R_1 is plotted against stable solute gradient parameter S_1 for fixed value of $F_1 = 10$, $B = 20$, $B' = 10$, $\varepsilon = 0.5$, $P = 0.005$, $T_{A_1} = 100$ and $Q_1 = 100, 200, 300$. The critical Rayleigh number R_1 increases with increase in stable solute gradient parameter S_1 which shows that stable solute gradient has stabilizing effect on the system.

In Figure 2, critical Rayleigh number R_1 is plotted against medium permeability P for fixed value of $F_1 = 10$, $S_1 = 10$, $B = 20$, $B' = 10$, $\varepsilon = 0.5$, $T_{A_1} = 500$ and $Q_1 = 100, 300, 500$. The critical Rayleigh number R_1 decreases with increase in medium permeability P which shows that medium permeability P has destabilizing effect on the system.

In Figure 3, critical Rayleigh number R_1 is plotted against medium permeability P for fixed value of $F_1 = 10$, $S_1 = 10$, $B = 20$, $B' = 10$,$\varepsilon = 0.5$, $Q_1 = 100$ and $T_{A_1} = 8000, 9000, 10000$. The critical Rayleigh number R_1 increases /decreases with increase in medium permeability P which shows that medium permeability P has both stabilizing and destabilizing effect on the system.

In Figure 4, critical Rayleigh number R_1 is plotted against medium permeability P for fixed value of $F_1 = 10$, $S_1 = 10$, $B = 20$, $B' = 10$, $\varepsilon = 0.5$, $T_{A_1} = 8000$ and $Q_1 = 80, 100, 120$. The critical Rayleigh number R_1 increases with increase in medium permeability P which shows that medium permeability P has stabilizing effect on the system.

In Figure 5, critical Rayleigh number R_1 is plotted against magnetic field Q_1 for fixed value of $F_1 = 10$, $S_1 = 10$, $B = 20$, $B' = 10$, $\varepsilon = 0.5$, $T_{A_1} = 0$ and $P = 0.001, 0.003, 0.005$. The critical Rayleigh number R_1 increases with increase in magnetic field Q_1 which shows that medium permeability P has stabilizing effect on the system.

7. Conclusion

In this present paper, the combined effect of solute gradient on a couple-stress dusty fluid heated from below in hydromagnetics in porous medium is considered. Dispersion relation governing the effects of solute gradient, dust particles, couple-stress and magnetic field is derived. The main results from the analysis are summarized as follows.:

(i) For the case of stationary convection, stable solute gradient has a stabilizing effect on the system as can be seen from equation (18), and graphically from Figure 1.

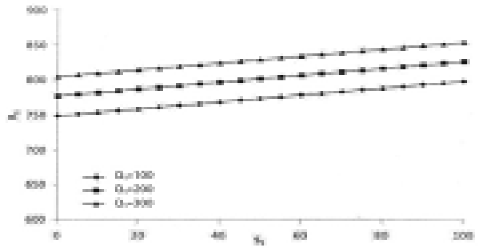

Fig. 1 Variation of critical Rayleigh number R_1 with S_1 for fixed values of $F_1 = 10, B = 20, B' = 10, T_A = 100, \varepsilon = 0.5,\ P = 0.005$ and $Q_1 = 100, 200, 300$.

(ii) For the case of stationary convection, medium permeability has a stabilizing / destabilizing effect on the system as can be seen from equation (19), and graphically from Figure 2, 3, 4.

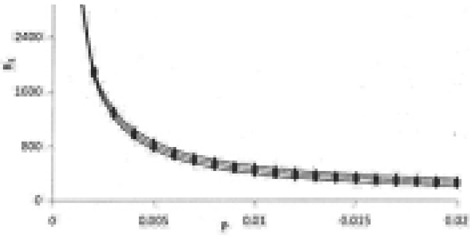

Fig. 2 Variation of critical Rayleigh number R_1 with P for fixed values of $F_1 = 10, S_1 = 10, B = 20, B' = 10, T_A = 500, \varepsilon = 0.5$ and $Q_1 = 100, 300, 500$.

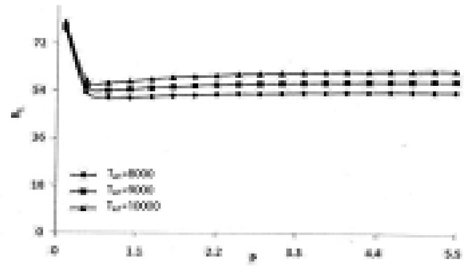

Fig. 3 Variation of critical Rayleigh number R_1 with P for fixed values of $F_1 = 10, S_1 = 10, B = 20, B' = 10, Q_1 = 100, \varepsilon = 0.5$ and $T_A = 8000, 9000, 10000$.

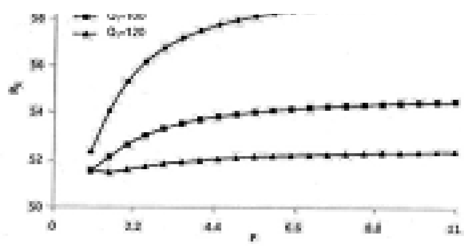

Fig. 4 Variation of critical Rayleigh number R_1 with P for fixed values of $F_1 = 10, S_1 = 10, B = 20, B' = 10, \varepsilon = 0.5, T_A = 8000$ and $Q_1 = 80, 100, 120$.

(iii) For the case of stationary convection, dust particles have a destabilizing effect on the system as can be seen from equation (21).

(iv) Couple-stress has stabilizing /destabilizing effects on the system for the permissible values of various parameters as can be seen from equation (22). In the absence of rotation, couple-stress clearly has a stabilizing effect on the system as can be seen from equation (23) as derived by Sharma and Sharma (2004).

(v) Magnetic field has stabilizing/destabilizing effect on the system for the permissible values of various parameters as can be seen from equation (24). And graphically from, in the absence of rotation, magnetic field has a stabilizing effect on the system as can seen from equation (25) as derived by Sharma and Sharma (2004).

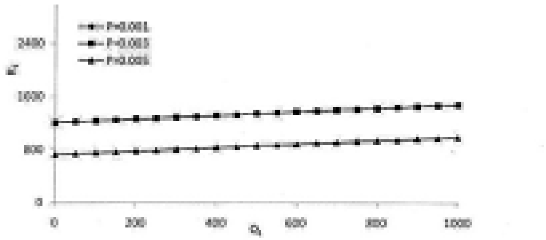

Fig.5 Variation of critical Rayleigh number R_1 with Q_1 for fixed values of $F_1 = 10, S_1 = 10, \delta = 20, B' = 10, \epsilon = 0.5, T_A = 0$ and $P = 0.001, 0.003, 0.005$.

(iv) The Principle of Exchange of Stabilities(PES) is found to hold true in the absence of magnetic field and rotation. It is evident from equation (28) that presence of magnetic field brings oscillatory modes (as σ_i may not be zero) which were non-existent in their absence.

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Nomenclature

A	Dimensionless wave number, [-]
F	Couple-stress parameter,
g	Acceleration due to gravity, $[m/s^2]$
k	Wave number, [1/m]
k_x, k_y	Horizontal wave number, [1/m]
n	Growth rule, [1/s]
P	Fluid pressure, [pa]
Q	Chandrasekhar number, [-]
T_A	Taylor number, [-]
S	Solute Rayleigh number, [-]
R	Rayleigh number, [-]
T	Temperature, [K]
t	Time, [s]
$\Omega(0,0,\Omega)$	Rotation vector having components $(0,0,\Omega)$
$H(h_x,h_y,h_z)$	Magnetic field having components (h_x,h_y,h_z)
$q(u,v,w)$	Component of velocity after perturbation,
$q_d(l,r,s)$	Component of particles velocity after perturbation,
α	Coefficient of thermal expansion, [1/K]
β	Uniform temperature gradient, [K/m]
β'	Uniform solute gradient, [K/m]
Θ	Perturbation in temperature, [K]
K_r	Thermal diffusivity, $[m^2/s]$
K_s	Solute diffusivity, $[m^2/s]$
v	Kinematic Viscosity, $[m^2/s]$
v'	Kinematic viscoelaticity, $[m^2/s]$
ρ	Density, $[Kg/m^3]$
∇, ∂, D	Del operator, curly operator and Derivative with respect to $z(= d/dz)$