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Ranking of Generalized Hexagonal fuzzy numbers based on Rank, Mode, Divergence and Spread

Research Article*

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Abstract: Ranking of fuzzy number used mainly in risk analysis,decision making, data analysis etc. In this paper, a new approach for ranking of generalized hexagonal fuzzy number. The proposed approach based on rank, mode, divergenge and spread.The proposed approach is compared with different existing approaches.

Keywords: Ranking function, Generalised hexagonal fuzzy number, centroid points.

1. Introduction

Fuzzy set theory is most powerful method to deal the real life situations.The method for ranking was first proposed by Jain, 1976. Kaufmann and Gupta, 1988 proposed an approach is presented for the ranking of fuzy numbers. Liou and Wang, 1992 developed a ranking method based on integrand value index. A method for ranking fuzzy numbers was proposed by Chu and Tsao 2002 with the area between the centroid point and original point. In this paper a new approach is proposed for the ranking of generalized Hexagonal fuzzy numbers. The results of the proposed approach are compared with different approaches. This paper is organized as follows. In section 2, some basic definition are given. In section 3, some important results are proved. In section 4, a new approach is proposed for the ranking of hexagonal fuzzy numbers. In section 5, the ranking results of proposed approach are compared with different approaches. The conclusion is discussed in section 6.

2. Pre-Requisites

Definition 2.1 (Fuzzy number)**.** *A fuzzy set $\tilde{A}$, defined on the universal set of real number R, is said to be a fuzzy number of its membership function has the following characteristics.*

- *$\tilde{A}$ is convex. i.e., $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1),\ \mu_{\tilde{A}}(x_2))$ for all x_1, $x_2 \in R$, $\lambda \in [0,1]$.*
- *$\tilde{A}$ is normal is there exists $x_0 \in R$ that $\mu_{\tilde{A}}(x_0) = 1$.*
- *$\mu_{\tilde{A}}$ is piecewise continuous.*

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Definition 2.2 (Triangular fuzzy number). *It is a fuzzy number represented with three points as follows:* $\tilde{A} = (a_1, a_2, a_3)$. *This representation is interpreted as membership functions and holds the following conditions,*

1. a_1 *to* a_2 *is increasing function.*
2. a_2 *to* a_3 *is decreasing function.*
3. $a_1 \leq a_2 \leq a_3$.

Definition 2.3 (Trapezoidal fuzzy number). *A fuzzy number* $\tilde{A}$ *is a trapezoidal fuzzy number denoted by* (a_1, a_2, a_3, a_4) *where* a_1,a_2,a_3 *and* a_4 *are real numbers and its membership function* $\mu_{\tilde{A}}(x)$ *is given below:*

$$\mu_{\tilde{A}} = \begin{cases} 0, & x \leq a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 0, & x \geq a_4 \end{cases}$$

Definition 2.4 (Pentagonal fuzzy number). *A pentagonal fuzzy number of a fuzzy set* $\tilde{P}$ *is defined as* $\tilde{P} = (a_1, a_2, a_3, a_4, a_5; k, \omega)$, *and its membership is given by,*

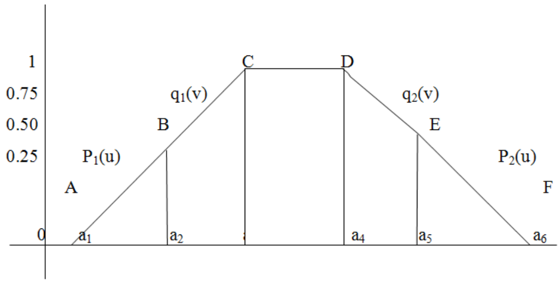
$$\mu_{\tilde{P}} = \begin{cases} 0, & \text{for } x \leq a_1 \\ \left(\frac{x-a_1}{a_2-a_1}\right), & \text{for } a_1 \leq x \leq a_2 \\ \left(\frac{x-a_2}{a_3-a_2}\right), & \text{for } a_2 \leq x \leq a_3 \\ 1, & \text{for } x = a_3 \\ \left(\frac{a_4-x}{a_4-a_3}\right), & \text{for } a_3 \leq x \leq a_4 \\ \left(\frac{a_5-x}{a_5-a_4}\right), & \text{for } a_4 \leq x \leq a_5 \\ 0, & \text{for } x \geq a_5 \end{cases}$$

Definition 2.5 (Hexagonal fuzzy number). *A fuzzy number* $\tilde{A}_H$ *is a hexagonal fuzzy number denoted by,* $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ *where* a_1, a_2, a_3, a_4, a_5, a_6 *are real numbers and its membership function* $\mu_{\tilde{A}}(X)$ *is given below.*

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0, & \text{for } x \leq a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & \text{for } a_2 \leq x \leq a_3 \\ 1, & \text{for } a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & \text{for } a_5 \leq x \leq a_6 \\ 0, & \text{for } x \geq a_6 \end{cases}$$

Definition 2.6 (Graphical representation of hexagonal fuzzy number).
An hexagonal fuzzy number denoted $\tilde{A}_H$ *is defined as* $\tilde{A}_W = P_1(u), Q_1(v), Q_2(v), P_2(u)$ *for* $u \in [0.5, w]$ *where*

- $P_1(u)$ *is a bounded left continuous non-decreasing function over [0, 0.5]*
- $Q_1(v)$ *is a bounded left continuous non-decreasing function over [0.5, W]*
- $Q_2(v)$ *is a bounded continuous non-increasing function over [w, 0.5]*



- *$P_2(u)$ is a bounded continuous non-increasing function over [0.5, 0]*

Definition 2.7 (Operations on hexagonal fuzzy numbers)**.** *Following are the three operations that can be performed on hexagonal fuzzy numbers.*

Suppose $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two hexagonal fuzzy numbers then

Addition: $\tilde{A}_H + \tilde{B}_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$

For example, let $\tilde{A}_H = (1, 2, 3, 4, 5, 6)$ and $\tilde{B}_H = (2, 4, 5, 6, 7, 7)$ be two fuzzy numbers then $\tilde{A}_H + \tilde{B}_H = (3, 6, 7, 9, 11, 12)$.

Subtraction: $\tilde{A}_H - \tilde{B}_H = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6)$

For example, let $\tilde{A}_H = (3, 4, 5, 6, 7, 8)$ and $\tilde{B}_H = (6, 8, 10, 12, 14, 16)$ be two fuzzy numbers then $\tilde{A}_H + \tilde{B}_H = (-3, -4, -5, -6, -7, -8)$.

Multiplication: $\tilde{A}_H * \tilde{B}_H = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6)$

For example, let $\tilde{A}_H = (1, 2, 3, 4, 5, 6)$ and $\tilde{B}_H = (1, 2, 3, 4, 5, 6)$ be two fuzzy numbers then $\tilde{A}_H * \tilde{B}_H = (1, 4, 9, 16, 25, 36)$.

Definition 2.8 (Ordering of Hexagonal fuzzy numbers)**.** *Let $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$ be set of all real numbers.*

- *$\tilde{A}_H = \tilde{B}_H$ if and only if $a_i = b_i$, $i = 1, 2, 3, 4, 5, 6$.*
- *$\tilde{A}_H \leq \tilde{B}_H$ if and only if $a_i \leq b_i$, $i = 1, 2, 3, 4, 5, 6$.*
- *$\tilde{A}_H \geq \tilde{B}_H$ if and only if $a_i \geq b_i$, $i = 1, 2, 3, 4, 5, 6$.*

Definition 2.9 (Ranking of Hexagonal fuzzy numbers)**.** *An efficient approach for comparing the fuzzy numbers is by the use of a ranking function $R : F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into a real number, where a natural order exists.*

- *$\tilde{A}_H \approx B_H \Leftrightarrow R(\tilde{A}_H) = R(\tilde{B}_H)$.*
- *$\tilde{A}_H \leq \tilde{B}_H \Leftrightarrow R(\tilde{A}_H) \leq R(\tilde{B}_H)$.*
- *$\tilde{A}_H \geq \tilde{B}_H \Leftrightarrow R(\tilde{A}_H) \geq R(\tilde{A}_H)$.*

3. Proposed Ranking Method

The centroid of a hexagonal fuzzy number is considered to be the balancing point of the hexagon. The circumcenter of the centroids of these three plane figures is taken as the point of reference to define the ranking of generalized hexagonal fuzzy numbers. Let centroid of the three plane figures G_1, G_2, G_3. The centroid of the three plane is $G_1 = \left(\frac{a_1+a_2+a_3}{3}\right)$; $G_2 = \left(\frac{a_2+2a_3+2a_4+a_5}{6}, \frac{w}{2}\right)$; $G_3 = \left(\frac{a_4+a_5+a_6}{3}, \frac{w}{6}\right)$ respectively. Equation of the line G_1G_2 is $y = \frac{w}{6}$ and G_2 does not lie on the line G_1G_3. Therefore G_1G_2 and G_3 are non collinear and they form a triangle. We define the centroid $G_{\tilde{A}}(\bar{x_0}, \bar{y_0})$ of the triangle with vertices G_1, G_2 and G_3 are of the generalized hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ as

$$G_{\tilde{A}}(\bar{x_0}, \bar{y_0}) = \left(\frac{2a_1 + 3a_2 + 4a_3 + 4a_4 + 3a_5 + 2a_6}{18}; \frac{5w}{18}\right) \tag{1}$$

The ranking function of the generalized hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$, which maps the set of all fuzzy numbers to a set of real numbers is defined as:

$$R(\tilde{A}_H) = (\bar{x_0})(\bar{y_0}) = \left(\frac{2a_1 + 3a_2 + 4a_3 + 4a_4 + 3a_5 + 2a_6}{18}\right)\left(\frac{5w}{18}\right) \tag{2}$$

This is the area between the centroid of the centroids $G_{\tilde{A}}(\bar{x_0}, \bar{y_0})$ as defined in (1) and (2) the original point. The mode of the generalized hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ is defined as:

$$Mode = \frac{1}{2}\int_0^w (a_3 + a_4)dx \tag{3}$$

The divergence of the generalized hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ is defined as:

$$Divergence = \frac{1}{2}\int_0^w (a_6 - a_1)dx \tag{4}$$

The left spread of the generalized hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ is defined as:

$$\begin{aligned} Left\ \ spread &= \int_0^{w/2} (a_2 - a_1)dx + \frac{1}{2}\int_{w/2}^w (a_3 - a_2)dx \\ Left\ \ spread &= \int_0^w (a_3 - a_2)dx \end{aligned} \tag{5}$$

The right spread of the generalized hexagonal fuzzy number $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ is defined as:

$$\begin{aligned} Right\ \ spread &= \int_0^{w/2} (a_6 - a_5)dx + \frac{1}{2}\int_{w/2}^w (a_5 - a_4)dx \\ Right\ \ spread &= \int_0^w (a_6 - a_4)dx \end{aligned} \tag{6}$$

Proposition 3.1. *If $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; w)$ and $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6; w)$ are two generalized Hexagonal fuzzy numbers then*

(1). $R(\tilde{A}_H) = R(\tilde{B}_H)$

(2). $Mode(\tilde{A}_H) = Mode(\tilde{B}_H)$

(3). $Divergence(\tilde{A}_H) = Divergence(\tilde{B}_H)$, then

(a). $Left\ \ spread(\tilde{A}_H) > Left\ \ spread(\tilde{B}_H)$, if $w_1a_3 > w_2b_3$.

(b). $Left\ \ spread(\tilde{A}_H) < Left\ \ spread(\tilde{B}_H)$, if $w_1a_3 < w_2b_3$.

(c). $Left\ \ spread(\tilde{A}_H) = Left\ \ spread(\tilde{B}_H)$, if $w_1a_3 = w_2b_3$.

Proof. From the assumption

(1). $R(\tilde{A}_H) = R(\tilde{B}_H)$. i.e.,

$$w_1\left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}\right)\left(\frac{5}{18}\right) = w_2\left(\frac{2a_1+3a_2+4a_3+4a_4+3a_5+2a_6}{18}\right)\left(\frac{5}{18}\right)$$

$$w_1(2a_1+3a_2+4a_3+4a_4+3a_5+2a_6) = w_2(2b_1+3b_2+4b_3+4b_4+3b_5+2b_6) \tag{7}$$

(2). $Mode(\tilde{A}_H) = Mode(\tilde{B}_H)$. i.e.,

$$w_1(a_3+a_4) = w_2(b_3+b_4) \tag{8}$$

(3). $Divergence(\tilde{A}_H) = Divergence(\tilde{B}_H)$. i.e.,

$$w_1(a_6-a_1) = w_2(b_6-b_1) \tag{9}$$

Solving (7), (8) and (9)

$$w_1a_1 = w_2b_1$$

$$w_1a_6 = w_2b_6$$

$$w_1(a_3+a_4) = w_2(b_3+b_4)$$

$$w_1(a_2+a_5) = w_2(b_2+b_5)$$

(a). $Left\ \ spread(\tilde{A}_H) > Left\ \ spread(\tilde{B}_H)$

$$w_1(a_3-a_1) > w_2(b_3-b_1)$$

$$w_1a_3 > w_2b_3 \quad (\text{since } w_1a_1 = w_2b_1)$$

Hence $Left\ \ spread(\tilde{A}_H) > Left\ \ spread(\tilde{B}_H)$ iff $w_1a_3 > w_2b_3$.

(b). $Left\ \ spread(\tilde{A}_H) < Left\ \ spread(\tilde{B}_H)$

$$w_1(a_3-a_1) < w_2(b_3-b_1)$$

$$w_1a_3 < w_2b_3 \quad (\text{since } w_1a_1 = w_2b_1)$$

Hence $Left\ \ spread(\tilde{A}_H) < Left\ \ spread(\tilde{B}_H)$ iff $w_1a_3 < w_2b_3$.

(c). $Left\ \ spread(\tilde{A}_H) = Left\ \ spread(\tilde{B}_H)$

$$w_1(a_3-a_1) = w_2(b_3-b_1)$$

$$w_1a_3 = w_2b_3 \quad (\text{since } w_1a_3 > w_2b_3)$$

Hence $Left\ \ spread(\tilde{A}_H) = Left\ \ spread(\tilde{B}_H)$ iff $w_1a_3 = w_2b_3$.

□

4. Proposed Approach for Ranking Generalized Hexagonal Fuzzy Number

If $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ are two generalized hexagonal fuzzy number then

Step 1: Find $R(\tilde{A}_H)$ and $R(\tilde{B}_H)$

Case (a) If $R(\tilde{A}_H) > R(\tilde{B}_H)$ then $\tilde{A}_H > \tilde{B}_H$.

Case (b) If $R(\tilde{A}_H) < R(\tilde{B}_H)$ then $\tilde{A}_H < \tilde{B}_H$.

Case (c) $R(\tilde{A}_H) = R(\tilde{B}_H)$ then $\tilde{A}_H = \tilde{B}_H$ then go to Step 2.

Step 2: Find mode($\tilde{A}_H$) and mode ($\tilde{B}_H$)

Case (a) If $mode(\tilde{A}_H) > mode(\tilde{B}_H)$ then $\tilde{A}_H > \tilde{B}_H$.

Case (b) If $mode(\tilde{A}_H) < mode(\tilde{B}_H)$ then $\tilde{A}_H < \tilde{B}_H$.

Case (c) If $mode(\tilde{A}_H) = mode(\tilde{B}_H)$ then $\tilde{A}_H = \tilde{B}_H$ then go to Step 3.

Step 3: Find divergence($\tilde{A}_H$) and divergence($\tilde{B}_H$)

Case (a) If $divergence(\tilde{A}_H) > divergence(\tilde{B}_H)$ then $\tilde{A}_H > \tilde{B}_H$.

Case (b) If $divergence(\tilde{A}_H) < divergence(\tilde{B}_H)$ then $\tilde{A}_H < \tilde{B}_H$.

Case (c) If $divergence(\tilde{A}_H) = divergence(\tilde{B}_H)$ then $\tilde{A}_H = \tilde{B}_H$ then go to Step 4.

Step 4: Find Left spread($\tilde{A}_H$) and Left spread($\tilde{B}_H$)

Case (a) If $Left\ \ spread(\tilde{A}_H) > Left\ \ spread(\tilde{B}_H)$ then i.e., $w_1a_3 > w_2b_3$ then $\tilde{A}_H > \tilde{B}_H$.

Case (b) If $Left\ \ spread(\tilde{A}_H) < Left\ \ spread(\tilde{B}_H)$ then i.e., $w_1a_3 < w_2b_3$ then $\tilde{A}_H < \tilde{B}_H$.

Case (c) If $Left\ \ spread(\tilde{A}_H) = Left\ \ spread(\tilde{B}_H)$ then i.e., $w_1a_3 = w_2b_3$ then $\tilde{A}_H = \tilde{B}_H$.

Step 5: Find w_1 and w_2

Case (a) If $w_1 > w_2$ then $\tilde{A}_H > \tilde{B}_H$.

Case (b) If $w_1 > w_2$ then $\tilde{A}_H < \tilde{B}_H$.

Case (c) If $w_1 = w_2$ then $\tilde{A}_H = \tilde{B}_H$.

5. Results and Discussions

In this section seven sets of fuzzy numbers are compared using the proposed approach and existing approaches. The results are shown in Table 1.

Set 1: Let $\tilde{A}_H = (0.2, 0.4, 0.6, 0.8, 0.9, 0.9; 0.35)$ and $\tilde{B}_H = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6; 0.7)$.

Step 1: $R(\tilde{A}_H) = 4.41$, $R(\tilde{B}_H) = 4.41$. Since $R(\tilde{A}_H) = R(\tilde{B}_H)$ so go to Step 2.

Step 2: $mode(\tilde{A}_H) = 0.49$ and $mode(\tilde{A}_H) = 0.49$. Since $mode(\tilde{A}_H) = mode(\tilde{B}_H)$, So go to Step 3.

Step 3: $Divergence(\tilde{A}_H) = 0.245$ and $Divergence(\tilde{A}_H) = 0.35$. Since $Divergence(\tilde{A}_H) < Divergence(\tilde{B}_H)$ then $\tilde{A}_H < \tilde{B}_H$.

Set 2: Let $\tilde{A}_H = (0.1, 0.2, 0.4, 0.5, 0.6, 0.7, 1)$ and $\tilde{B}_H = (0.1, 0.1, 0.5, 0.5, 0.6, 0.7, 1)$, $R(\tilde{A}_H) = 7.5$, $R(\tilde{B}_H) = 7.5$. Since $R(\tilde{A}_H) = R(\tilde{B}_H)$ so go to Step 2.

$mode(\tilde{A}_H) = 0.9$ and $mode(\tilde{B}_H) = 1$. Since $mode(\tilde{A}_H)) < mode(\tilde{B}_H)$ then $\tilde{A}_H < \tilde{B}_H$.

Set 3: Let $\tilde{A}_H = (0.1, 0.2, 0.4, 0.5, 0.6, 0.7, 1)$ and $\tilde{B}_H = (1, 1, 1, 1, 1, 1, 1)$, $R(\tilde{A}_H) = 7.5$, $R(\tilde{B}_H) = 18$. Since $R(\tilde{A}_H) < R(\tilde{B}_H)$ then $\tilde{A}_H < \tilde{B}_H$.

Set 4: Let $\tilde{A}_H = (0.1, 0.3, 0.3, 0.4, 0.5, 0.6, 1)$ and $\tilde{B}_H = (0.1, 0.3, 0.3, 0.6, 0.7, 0.8, 1)$, $R(\tilde{A}_H) = 6.6$, $R(\tilde{B}_H) = 8.4$. Since $R(\tilde{A}_H) < R(\tilde{B}_H)$ then $\tilde{A}_H < \tilde{B}_H$.

Set 5: Let $\tilde{A}_H = (0.3, 0.5, 0.5, 0.6, 0.6, 0.7, 1)$ and $\tilde{B}_H = (0.1, 0.2, 0.2, 0.3, 0.4, 0.5, 1)$, $R(\tilde{A}_H) = 9.7$, $R(\tilde{B}_H) = 6.5$. Since $R(\tilde{A}_H) > R(\tilde{B}_H)$ then $\tilde{A}_H > \tilde{B}_H$.

Set 6: Let $\tilde{A}_H = (0, 0.4, 0.6, 0.6, 0.7, 0.7, 1)$, $\tilde{B}_H = (0.2, 0.5, 0.5, 0.7, 0.7, 0.8, 1)$ and $\tilde{C}_H = (0.1, 0.6, 0.7, 0.7, 0.8, 0.9, 1)$, $R(\tilde{A}_H) = 9.5$, $R(\tilde{B}_H) = 10.4$, $R(\tilde{C}_H) = 11.8$. Since $R(\tilde{A}_H) < R(\tilde{B}_H) < R(\tilde{C}_H)$ then $\tilde{A}_H < \tilde{B}_H < R(\tilde{C}_H)$.

Set 7: Let $\tilde{A}_H = (0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 1)$ and $\tilde{B}_H = (0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 1)$, $R(\tilde{A}_H) = 8.1$, $R(\tilde{B}_H) = 6.3$. Since $R(\tilde{A}_H) > R(\tilde{B}_H)$ then $\tilde{A}_H > \tilde{B}_H$.

The main advantage of the proposed approach is provides the correct ordering of generalized hexagonal fuzzy numbers and also proposed approach is very easy to apply real life situation

Approaches	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7
Cheng	$A < B$	$A \sim B$	Not Comparable	$A \sim B$	$A > B$	$A > B > C$	Not Comparable
Chu and Tsao	$A < B$	$A \sim B$	Not Comparable	$A < B$	$A > B$	$A < B < C$	Not Comparable
Chen and Chen	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < C < B$	$A > B$
Abbasbandy and Hajjari	Not Comparable	$A \sim B$	$A < B$	$A \sim B$	$A < B$	$A < B < C$	$A > B$
Chen and Chen	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Kumar et al.	$A > B$	$A \sim B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Proposed approach	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$

6. Conclusion

In this paper a simple approach is proposed for the ranking of generalized hexagonal fuzzy number. The main advantage of the proposed approach is provides the correct ordering of generalized hexagonal fuzzy numbers and it is very simple to apply in the real life problems. The result of the proposed approach are compared with different approaches.

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