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# On Determinant of Hexagonal Fuzzy Number Matrices

**Research Article***

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**Abstract:** The fuzzy set theory has been applied in many fields such as management, engineering, theory of matrices and so on. In this paper, some elementary operations on hexagonal fuzzy numbers (HFNs) have been studied. We also studied some special operations on hexagonal fuzzy matrices (HFMs). The notion of Determinant on HFM is also introduced and discussed. Some of their relevant properties have also been verified.

**Keywords:** Fuzzy Arithmetic, Fuzzy number, Hexagonal fuzzy number, Hexagonal fuzzy matrices.


## 1. Introduction

Fuzzy sets have been introduced by Lotfi.A.Zadeh [14]. Fuzzy set theory permits the gradual assessment of the membership of elements in a set which is described in the interval [0, 1]. It can be used in a wide range of domains where information is incomplete and imprecise. Interval arithmetic was first suggested by Dwyer [2] in 1951.by means of Zadeh's extension principle [15,16], the usual Arithmetic operations on real numbers can be extended to the ones defined on Fuzzy numbers. Dubois and Prade [1] has defined any of the fuzzy numbers as a fuzzy subset of the real line [2,3,4,7]. A fuzzy number is a quantity whose values are imprecise, rather than exact as is the case with single-valued numbers. Among the various shapes of fuzzy numbers, Triangular fuzzy number and Trapezoidal fuzzy number are the most commonly used membership function [1,16].

In this paper, a new fuzzy number called 'Hexagonal fuzzy numbers' is utilized in developing the notion of "Determinant". Though Rajarajesvari et.al [8] proposed the above said fuzzy number without any restrictions of parameter, Recently Stephen Dinagar and Rajesh Kanna [11] introduced the "modified definition" of the Hexagonal Fuzzy number by including conditions for the convexity of the number and few more results have been verified in this work. Thomason [13] introduced fuzzy matrices and discussed about the convergence of powers of fuzzy matrix. Fuzzy matrices play an important role in scientific development. Kim [5] presented some important results on determinant of a square fuzzy matrices. Ragab and Emam [9] presented some properties of the min-max composition of fuzzy matrices. Shyamal and pal [10] introduced triangular fuzzy matrices. Recently Stephen Dingar and Latha [12] discussed the Determinant of Type 2 Triangular fuzzy number matrices.

The paper is organized as follows, Firstly in Section 2 of this paper, we recall the definition of Hexagonal fuzzy number and some operations on Hexagonal fuzzy numbers (HFNs). In Section 3 we have reviewed the definition of Hexagonal fuzzy matrix (HFM) and some operations on Hexagonal fuzzy matrices (HFMs). In Section 4 we defined determinant of HFMs. In Section 5, we have presented some properties of determinant of HFMs. Finally in section6, conclusion is included.

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## 2. Hexagonal Fuzzy Numbers

**Definition 2.1** (Fuzzy set)**.** *A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0,1]. A fuzzy set A in a universe of discourse X is defined as the following set of pairs*

$$A = \{(x, \mu_A(x)\,; x \in X)\}$$

*Here* $\mu_A : X \to [0,1]$ *is mapping called the degree of membership function of the fuzzy set A and* $\mu_A(x)$ *is called the membership value of* $x \in X$ *in the fuzzy set A. These membership grades are often represented by real ranging from [0,1].*

**Definition 2.2** (Convex fuzzy set)**.** *A fuzzy set* $A = \{(x, \mu_A(x))\} \subseteq X$ *is called convex fuzzy set if all* $A_\alpha$ *are convex set (i.e) for every element* $x_1 \in A_\alpha$ *and* $x_2 \in A_\alpha$ *for every* $\alpha \in [0,1]$. $\lambda x_1 + (1-\lambda)x_2 \in A_\alpha$ *for all* $\lambda \in [0,1]$. *Otherwise the fuzzy set is called non-convex fuzzy set.*

**Definition 2.3** (Fuzzy number)**.** *A fuzzy set* $\tilde{A}$*, defined on the set of real number R is said to be fuzzy number if its membership function has the following characteristics*

- $\tilde{A}$ *is normal*
- $\tilde{A}$ *is convex set*
- *The support of* $\tilde{A}$ *is closed and bounded then* $\tilde{A}$ *is called fuzzy number.*

**Definition 2.4** (Hexagonal fuzzy number)**.** *A fuzzy number on* $\tilde{A}_h$ *is hexagonal fuzzy number denoted by* $\tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6)$*, where* $(a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6)$ *are real number satisfying* $a_2 - a_1 \le a_3 - a_2$ *and* $a_5 - a_4 \ge a_6 - a_5$ *and its membership function* $\mu_{\tilde{A}_h}(x)$ *is given by*

$$\mu_{\tilde{A}_h}(x) = \begin{cases} 0, & x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right), & a_2 \le x \le a_3 \\ 1, & a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right), & a_5 \le x \le a_6 \\ 0, & x > a_6 \end{cases}$$

**Remark 2.5.** *The hexagonal fuzzy number* $\tilde{A}_h$ *becomes trapezoidal fuzzy number if* $a_2 - a_1 = a_3 - a_2$ *and* $a_5 - a_4 = a_6 - a_5$. *The hexagonal fuzzy number* $\tilde{A}_h$ *becomes non-convex fuzzy number if* $a_2 - a_1 > a_3 - a_2$ *and* $a_5 - a_4 < a_6 - a_5$.

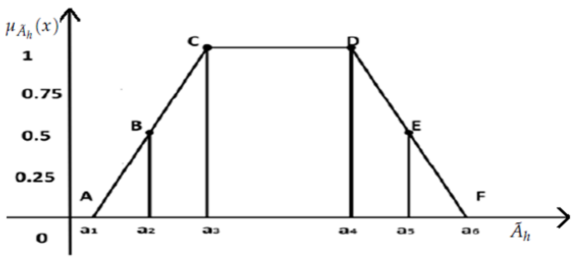


**Figure 1.** **Hexagonal Fuzzy Number** $\tilde{A}_h$

The arithmetic operations between hexagonal fuzzy number (HFNs) are proposed given below.

**Definition 2.6** (Arithmetic Operations On Hexagonal Fuzzy Numbers (HFNs)). *Let us consider* $\tilde{A}_h = (a_1, a_2, a_3, a_4, a_5, a_6)$ *and* $\tilde{B}_h = (b_1, b_2, b_3, b_4, b_5, b_6)$ *be two hexagonal fuzzy numbers then,*

*(1).* ***Addition:*** $\tilde{A}_h\,(+)\,\tilde{B}_h = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6)$.

*(2).* ***Subtraction:*** $\tilde{A}_h\,(-)\,\tilde{B}_h = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1)$.

*(3).* ***Multiplication:*** $\tilde{A}_h\,(\times)\,\tilde{B}_h = \left(\frac{a_1}{6}\sigma_b, \frac{a_2}{6}\sigma_b, \frac{a_3}{6}\sigma_b, \frac{a_4}{6}\sigma_b, \frac{a_5}{6}\sigma_b, \frac{a_6}{6}\sigma_b\right)$, *where* $\sigma_b = b_1 + b_2 + b_3 + b_4 + b_5 + b_6$.

*(4).* ***Division:*** $\tilde{A}_h\,(\div)\,\tilde{B}_h = \left(\frac{6a_1}{\sigma_b}, \frac{6a_2}{\sigma_b}, \frac{6a_3}{\sigma_b}, \frac{6a_4}{\sigma_b}, \frac{6a_5}{\sigma_b}, \frac{6a_6}{\sigma_b}\right)$, *where* $\sigma_b = b_1 + b_2 + b_3 + b_4 + b_5 + b_6$.

*(5).* ***Scalar Multiplication:*** *If* $k \neq 0$ *is scalar* $k\tilde{A}_h$ *is defined as* $k\tilde{A}_h = \begin{cases} (ka_1, ka_2, ka_3, ka_4, ka_5, ka_6) \; if \; k \geq 0 \\ (ka_6, ka_5, ka_4, ka_3, ka_2, ka_1) \;\; if \; k \leq 0 \end{cases}$

**Definition 2.7** (Ranking Function). *We define a ranking function* $\check{R} : F(R) \to R$ *which maps each fuzzy numbers to real line* $F(R)$ *represent the set of all hexagonal fuzzy numbers. If* $R$ *be any linear ranking functions.*

$$\check{R}(\tilde{A}_h) = \left(\frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6}\right).$$

*Also we defined orders on* $F(R)$ *by*
$\check{R}(\tilde{A}_h) \geq \check{R}(\tilde{B}_h)$ *if and only if* $\tilde{A}_h \underset{\check{R}}{\geq} \tilde{B}_h$,
$\check{R}(\tilde{A}_h) \leq \check{R}(\tilde{B}_h)$ *if and only if* $\tilde{A}_h \underset{\check{R}}{\leq} \tilde{B}_h$ *and*
$\check{R}(\tilde{A}_h) = \check{R}(\tilde{B}_h)$ *if and only if* $\tilde{A}_h \underset{\check{R}}{=} \tilde{B}_h$.

**Definition 2.8** (Zero Hexagonal fuzzy number). *If* $\tilde{A}_h = (0, 0, 0, 0, 0, 0)$ *then* $\tilde{A}_h$ *is said to be zero hexagonal fuzzy number. It is denoted by 0.*

**Definition 2.9** (Zero-Equivalent Hexagonal fuzzy number). *A hexagonal fuzzy number* $\tilde{A}_h$ *is said to be a zero-equivalent hexagonal fuzzy number if* $\check{R}(\tilde{A}_h) = 0$. *It is denoted by* $\tilde{0}$.

**Definition 2.10** (Unit Hexagonal fuzzy number). *If* $\tilde{A}_h = (1, 1, 1, 1, 1, 1)$ *then* $\tilde{A}_h$ *is said to be unit hexagonal fuzzy number. It is denoted by 1.*

**Definition 2.11** (Unit-Equivalent Hexagonal fuzzy number). *A hexagonal fuzzy number* $\tilde{A}_h$ *is said to be a unit-equivalent hexagonal fuzzy number if* $\check{R}(\tilde{A}_h) = 1$. *It is denoted by* $\tilde{1}$.

**Definition 2.12** (Inverse Hexagonal fuzzy number). *If* $\tilde{a}_h$ *is hexagonal fuzzy number and* $\tilde{a}_h \neq \tilde{0}$ *then we define* $\tilde{a}_h^{-1} = \frac{\tilde{1}}{\tilde{a}_h}$.

## 3. Hexagonal Fuzzy Matrices (HFMs)

**Definition 3.1** (Hexagonal fuzzy matrix). *A hexagonal fuzzy matrix of order* $m \times n$ *is defined as* $\hat{A} = (\tilde{a}_{hij})_{m\times n}$ *where* $\tilde{a}_{hij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}, a_{ij5}, a_{ij6})$ *is* $ij^{th}$ *element of* $\hat{A}$.

**Definition 3.2** (Operations on Hexagonal Fuzzy Matrices (HFMs)). *Let* $\hat{A} = (\tilde{a}_{hij})$ *and* $\hat{B} = (\tilde{b}_{hij})$ *be two HFMs of same order. Then we have the following*

*(1).* $\hat{A} + \hat{B} = (\tilde{a}_{hij} + \tilde{b}_{hij})$

*(2).* $\hat{A} - \hat{B} = (\tilde{a}_{hij} - \tilde{b}_{hij})$

*(3). For* $\hat{A} = (\tilde{a}_{hij})_{m\times n}$ *and* $\hat{B} = (\tilde{b}_{hij})_{n\times k}$ *then* $\hat{A}\hat{B} = (\tilde{c}_{hij})_{m\times k}$ *where* $\tilde{c}_{hij} = \sum_{p=1}^{n} \tilde{a}_{hip}\ \tilde{b}_{hpj}$*,* $i = 1, 2, \ldots\ldots m$ *and* $j = 1, 2, \ldots\ldots k$*.*

*(4).* $\hat{A}^T$ *or* $\hat{A}' = (\tilde{a}_{hji})$*.*

*(5).* $k\hat{A} = (k\tilde{a}_{hij})$*, where k is scalar.*

# 4. Determinant of HFM

**Definition 4.1** (Determinant of HFM)**.** *The determinant of* $n \times n$ *HFM* $\hat{A} = (\tilde{a}_{hij})$ *is denoted by* $|\hat{A}|$ *or det(*$\hat{A}$*) and is defined as follows*

$$|\hat{A}| = \sum_{q\in s_n} sgn\ q \prod_{i=1}^{n} \tilde{a}_{hiq(i)} = \sum_{q\in s_n} sgn\ q\ \tilde{a}_{h1q(1)}.\tilde{a}_{h2q(2)} \ldots \tilde{a}_{hnq(n)}$$

*Where* $\tilde{a}_{hiq(i)} = (a_{iq(i)1}, a_{iq(i)2}, a_{iq(i)3}, a_{iq(i)4}, a_{iq(i)5}, a_{iq(i)6})$ *are hexagonal fuzzy number (HFNs) and* $s_n$ *denotes the symmetric group of all permutations of the indices {1,2,....n} and* $sgn\ q = 1$ *or* $-1$ *according as the permutation* $q = \begin{pmatrix} 1 & 2 & \ldots & n \\ q(1) & q(2) & \ldots & q(n) \end{pmatrix}$ *is even or odd respectively.*

**Definition 4.2** (Minor of HFM)**.** *Let* $\hat{A} = (\tilde{a}_{hij})$ *be a square HFM of order n. The minor of an element* $\tilde{a}_{hij}$ *in* $\hat{A}$ *is a determinant of order* $(n-1) \times (n-1)$ *which is obtained by deleting the* $i^{th}$ *row and* $j^{th}$ *column from* $\hat{A}$ *and is denoted by* $\tilde{M}_{hij}$*.*

**Definition 4.3** (Cofactor of HFM)**.** *Let* $\hat{A} = (\tilde{a}_{hij})$ *be a square HFM of order n. The cofactor of an element* $\tilde{a}_{hij}$ *in* $\hat{A}$ *is denoted by* $\tilde{A}_{hij}$ *and is defined as* $\tilde{A}_{hij} = (-1)^{i+j}\tilde{M}_{hij}$*.*

**Definition 4.4** (Aliter Definition of Determinant)**.** *Alternatively, the determinant of square HFM* $\hat{A} = (\tilde{a}_{hij})$ *of order n may be expanded in the form*

$$|\hat{A}| = \sum_{j=1}^{n} \tilde{a}_{hij}\ \tilde{A}_{hij},\ \ i \in \{1, 2, \ldots, n\}$$

*where* $\tilde{A}_{hij}$ *is the cofactor of* $\tilde{a}_{hij}$*. Thus the determinant is the sum of the products of the elements of any row(or column) and the cofactors of the corresponding elements of the same row(or column).*

**Definition 4.5** (Adjoint of HFM)**.** *Let* $\hat{A} = (\tilde{a}_{hij})$ *be a square HFM of order n. Find the cofactor* $\tilde{A}_{hij}$ *of every element* $\tilde{a}_{hij}$ *in* $\hat{A}$ *and replace every* $\tilde{a}_{hij}$ *by its cofactor* $\tilde{A}_{hij}$ *in* $\hat{A}$ *and let it be* $\hat{B}$*. i.e.,* $\hat{B} = (\tilde{A}_{hij})$*. Then the transpose of* $\hat{B}$ *is called the adjoint or adjugate of* $\hat{A}$ *and is denoted by adj* $\hat{A}$*. I.e.,* $\hat{B}' = \tilde{A}_{hji} = adj\ \hat{A}$*.*

# 5. Properties of Determinants of HFM

**Property 5.1.** *Let* $\hat{A} = (\tilde{a}_{hij})$ *be a square HFM of order n. if all the elements of a row(or column) of* $\hat{A}$ *are 0 then* $|\hat{A}|$ *is also 0.*

*Proof.* Let $\hat{A} = (\tilde{a}_{hij})$ be a square HFM of order n and let all the elements of $r^{th}$ row, where $1 \le r \le n$ be 0. ie, $\tilde{a}_{hrj} = (0, 0, 0, 0, 0, 0)$, $j = 1, 2, \ldots, n$. Then $|\hat{A}| = \sum_{j=1}^{n} \tilde{a}_{hij}\ \tilde{A}_{hij}\ i \in \{1, 2, \ldots, n\}$ where $\tilde{A}_{hij}$ is the cofactor of $\tilde{a}_{hij}$. Particularly if we expand through $r^{th}$ row then we have $|\hat{A}| = \sum_{j=1}^{n} \tilde{a}_{hrj}\ \tilde{A}_{hrj}$, which is the sum of the products of the elements of the $r^{th}$ row and the cofactor of the corresponding elements of the $r^{th}$ row. Since all $\tilde{a}_{hrj}$ are 0, each term in this summation is 0 and hence $|\hat{A}|$ is also 0. □

**Property 5.2.** *Let $\hat{A} = (\tilde{a}_{hij})$ be a square HFM of order n. If all the elements of a row(or column) of $\hat{A}$ are $\tilde{0}$ then $|\hat{A}|$ is either 0 or $\tilde{0}$.*

*Proof.* Let $\hat{A} = (\tilde{a}_{hij})$ be a square HFM of order n and let all the elements of $r^{th}$ row be $\tilde{0}$. Since for any hexagonal fuzzy number $\tilde{a}_h \neq 0$, $\tilde{a}_h * \tilde{0} = 0$ and $\tilde{0} * \tilde{a}_h = \tilde{0}$, if we expand through $r^{th}$ row then in this case $|\hat{A}| = \tilde{0}$ and we expand through other then we have $|\hat{A}| = 0$. □

**Property 5.3.** *Let $\hat{A} = (\tilde{a}_{hij})$ be a square HFM of order n. where $\tilde{a}_{hij} = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}, a_{ij5}, a_{ij6})$. If a row(or column) is multiplied by a scalar k, then $|\hat{A}|$ is multiplied by k.*

*Proof.*

**Case (1):** $k = 0$. If $k = 0$, then the result is obvious since $|\hat{A}| = 0$ when $\hat{A}$ has a 0 row.

**Case (2):** $k \neq 0$. Let $\hat{B} = (\tilde{b}_{hij})_{n\times n}$ where $\tilde{b}_{hij} = (b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4}, b_{ij5}, b_{ij6})$ obtained from $\hat{A} = (\tilde{a}_{hij})_{n\times n}$ by multiplying $r^{th}$ row by a scalar $k \neq 0$. Obviously $(b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4}, b_{ij5}, b_{ij6}) = (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}, a_{ij5}, a_{ij6})$ for all $i \neq r$ and when $k > 0$

$$(b_{rj1}, b_{rj2}, b_{rj3}, b_{rj4}, b_{rj5}, b_{rj6}) = (ka_{rj1}, ka_{rj2}, ka_{rj3}, ka_{rj4}, ka_{rj5}, ka_{rj6})$$

and when $k < 0$

$$(b_{rj1}, b_{rj2}, b_{rj3}, b_{rj4}, b_{rj5}, b_{rj6}) = (ka_{rj6}, ka_{rj5}, ka_{rj4}, ka_{rj3}, ka_{rj2}, ka_{rj1})$$

Then by definition,

$$\begin{aligned}
|\hat{B}| &= \sum_{q\in s_n} sgn\ q \prod_{i=1}^{n} \tilde{b}_{hiq(i)} \\
&= \sum_{q\in s_n} sng\ q \begin{bmatrix} \left(b_{1q(1)1}, b_{1q(1)2}, b_{1q(1)3}, b_{1q(1)4}, b_{1q(1)5}, b_{1q(1)6}\right) \dots \\ \left(b_{rq(r)1}, b_{rq(r)2}, b_{rq(r)3}, b_{rq(r)4}, b_{rq(r)5}, b_{rq(r)6}\right) \dots \\ \left(b_{nq(n)1}, b_{nq(n)2}, b_{nq(n)3}, b_{nq(n)4}, b_{nq(n)5}, b_{nq(n)6}\right) \end{bmatrix}
\end{aligned}$$

when $k > 0$

$$\begin{aligned}
|\hat{B}| &= \sum_{q\in s_n} sng\ q \begin{bmatrix} \left(a_{1q(1)1}, a_{1q(1)2}, a_{1q(1)3}, a_{1q(1)4}, a_{1q(1)5}, a_{1q(1)6}\right) \dots \\ \left(ka_{rq(r)1}, ka_{rq(r)2}, ka_{rq(r)3}, ka_{rq(r)4}, ka_{rq(r)5}, ka_{rq(r)6}\right) \dots \\ \left(a_{nq(n)1}, a_{nq(n)2}, a_{nq(n)3}, a_{nq(n)4}, a_{nq(n)5}, a_{nq(n)6}\right) \end{bmatrix} \\
&= k \sum_{q\in s_n} sng\ q \begin{bmatrix} \left(a_{1q(1)1}, a_{1q(1)2}, a_{1q(1)3}, a_{1q(1)4}, a_{1q(1)5}, a_{1q(1)6}\right) \dots \\ \left(a_{rq(r)1}, a_{rq(r)2}, a_{rq(r)3}, a_{rq(r)4}, a_{rq(r)5}, a_{rq(r)6}\right) \dots \\ \left(a_{nq(n)1}, a_{nq(n)2}, a_{nq(n)3}, a_{nq(n)4}, a_{nq(n)5}, a_{nq(n)6}\right) \end{bmatrix} \\
&= k \sum_{q\in s_n} sgn\ q \prod_{i=1}^{n} \tilde{a}_{hiq(i)} \\
|\hat{B}| &= k|\hat{A}|.
\end{aligned}$$

Similarly when $k < 0$,

$$\left|\hat{B}\right| = \sum_{q\in s_n} sng\ q \begin{bmatrix} \left(a_{1q(1)1}, a_{1q(1)2}, a_{1q(1)3}, a_{1q(1)4}, a_{1q(1)5}, a_{1q(1)6}\right) \dots \\ \left(ka_{rq(r)6}, ka_{rq(r)5}, ka_{rq(r)4}, ka_{rq(r)3}, ka_{rq(r)2}, ka_{rq(r)1}\right) \dots \\ \left(a_{nq(n)1}, a_{nq(n)2}, a_{nq(n)3}, a_{nq(n)4}, a_{nq(n)5}, a_{nq(n)6}\right) \end{bmatrix}$$

$$
\begin{aligned}
&= k \sum_{q \in s_n} sng\ q \begin{bmatrix} (a_{1q(1)1}, a_{1q(1)2}, a_{1q(1)3}, a_{1q(1)4}, a_{1q(1)5}, a_{1q(1)6}) \dots \\ (a_{rq(r)1}, a_{rq(r)2}, a_{rq(r)3}, a_{rq(r)4}, a_{rq(r)5}, a_{rq(r)6}) \dots \\ (a_{nq(n)1}, a_{nq(n)2}, a_{nq(n)3}, a_{nq(n)4}, a_{nq(n)5}, a_{nq(n)6}) \end{bmatrix} \\
&= k \sum_{q \in s_n} sgn\ q \prod_{i=1}^{n} \tilde{a}_{hiq(i)} \\
|\hat{B}| &= k|\hat{A}|.
\end{aligned}
$$

Hence the result ◻

**Property 5.4.** *The Determinant of triangular HFM is given by the product of diagonal elements.*

*Proof.* Let $\hat{A} = (\tilde{a}_{hij})$ be a square triangular HFM without loss of generality, let us assume that A is a lower triangular HFM. ie, $\tilde{a}_{hij} = 0$ for $i < j$. Take a term t of $|\hat{A}|.t = \tilde{a}_{h1q(1)}.\tilde{a}_{h2q(2)} \dots \tilde{a}_{hnq(n)}$. Let $q(1) \neq 1$. So that $1 < q(1)$ and so $\tilde{a}_{h1q(1)} = 0$ and thus $t = 0$. This means that each term is 0 if $q(1) \neq 1$. Now, let $q(1) = 1$ but $q(2) \neq 2$. Then $2 < q(2)$ and so $\tilde{a}_{h2q(2)} = 0$ and thus $t = 0$. This means that each term is 0 if $q(2) \neq 2$. However in a similar manner we can see that each term must be 0 if $q(1) \neq 1$, or $q(2) \neq 2$ or $\dots$, $q(n) \neq n$. Consequently,

$$
\begin{aligned}
|\hat{A}| &= \tilde{a}_{h11}.\tilde{a}_{h22} \dots .\tilde{a}_{hnn} \\
&= \prod_{i=1}^{n} \tilde{a}_{hii} \\
&= \text{product of its diagonal elements}
\end{aligned}
$$

Similarly when a is an upper triangular HFM, the result follows. ◻

## 6. Conclusion

In this article Determinant of Hexagonal fuzzy matrices are defined and some relevant properties of determinant of HFMs have also been proved. Using this results of HFMs, some important properties of HFMs, involving the notion like adjoint of matrix and inverse of matrix can be studied in future. Also the theories of the discussed HFMs may be utilized in further works.

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