International Journal of Mathematics And its Applications
Volume 4, Issue 4 (2016), 373–379. (Special Issue)
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Direct Method of Fuzzy Transportation Problem Using Hexagonal Fuzzy Number with Alpha Cut

Research Article*

A.Arokia Mary[1] and R.Sivasankari[1]

1 Department of Mathematics, St. Joseph's College of Arts & Science, Manjakuppam, Cuddalore (Tamil Nadu), India.

Abstract: The purpose of this paper is to find an optimal solution of fuzzy transportation problem, where transportation cost, supply and demand are fuzzy quantities. Using the method of robust ranking, we find the rank [a b c d e f] of hexagonal fuzzy number by arithmetic operation. We develop a new algorithm to find a fuzzy optimal solution with alpha-level cut for computation. The parameter such as supply, demand and fuzzy coefficients are represented by fuzzy numbers. We have derived it with numerical illustration.

Keywords: Fuzzy set, Hexagonal fuzzy number, Robust ranking, Fuzzy transportation problem, Optimal solution.

1. Introduction

In this paper, we shall study fuzzy transportation problem, by using a robust ranking method which is very useful tool and is applied for ranking of fuzzy coefficients, supply and demand. The objective of fuzzy transportation model is to find the optimal solution for the products transported from origin to each destination to maintain the product availability and demand requirement at the minimum transportation cost. A fuzzy transportation problem is more appropriate to solve real life problem. It is basically a transportation problem where the fuzzy cost, supply, demand are fuzzy quantities. Since the objective function is also considered as a fuzzy number and in such a manner the total transportation cost of fuzzy transportation is minimized. This typical problem is the technique, which is applied only when we convert source demand fuzzy coefficient into crisp value. By the method of ranking, convert all parameters into fuzzy numbers. We use arithmetic operations for solving hexagonal fuzzy numbers in order to maximizing profit or minimizing cost such that deliver the quantity of supply to satisfy the demand.

The fuzzy linear programming model is a special type where a single product can be distributed to various destinations. Srinivasan and Geetharamani [3] develop an effective algorithm, for solving fuzzy transportation problems using robust ranking method with fixed fuzzy numbers. Basically transportation problem was developed by Hitchock. In Zimmerman [15] we bring out fuzzy optimization methods in which total fuzzy supply is equal to total fuzzy demand, and fuzzy maximization or minimization linear programming for computation.

Finally, feasibility of original conventional transportation problem is checked with numerical example.

* *Proceedings : National Conference on `Recent Trends in Applied Mathematics` held on 22 & 23.07.2016, organized by Department of Mathematics, St. Joseph's College of Arts & Science, Manjakuppam, Cuddalore (Tamil Nadu), India.*

2. Preliminaries

Definition 2.1 (Fuzzy numbers). *A fuzzy number is a generalisation of a regular real number and which doesn't refer to a single value, but rather refer to a connected set of possible values, where each possible value has its weight between 0 and 1.*

Definition 2.2 (Strict and Strong Convexity). *A fuzzy set $\tilde{A}$ is strictly convex if the sets Γ_α, $0 < \alpha \leq 1$ are strictly convex i.e. if the midpoint of any 2 distinct points in Γ_α lies in the interior of Γ_α. A fuzzy set $\tilde{A}$ is strongly convex if for any 2 distinct points x_1 and x_2 and any in the open interval (0, 1). (i.e.) $f_{\tilde{A}}[\lambda x_1 + (1-\lambda)x_2] > \min[f_{\tilde{A}}(x_1), f_{\tilde{A}}(x_2)]$.*

Definition 2.3 (Hexagonal fuzzy number). *A fuzzy number $\tilde{A}_{\alpha H}$ is a Hexagonal fuzzy number denoted by $\tilde{A}_{\alpha H}(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6)$, where $\tilde{a}_1$, $\tilde{a}_2$, $\tilde{a}_3$, $\tilde{a}_4$, $\tilde{a}_5$, $\tilde{a}_6$ are real numbers*

$$\delta_{\tilde{a}H}(z) = \begin{cases} 0, & for\ z < \tilde{a}_1 \\ \frac{1}{2}\left(\frac{z-\tilde{a}_1}{\tilde{a}_2-\tilde{a}_1}\right), & for\ \tilde{a}_1 \leq z \leq \tilde{a}_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{z-\tilde{a}_2}{\tilde{a}_3-\tilde{a}_2}\right), & for\ \tilde{a}_2 \leq z \leq \tilde{a}_3 \\ 1, & for\ \tilde{a}_3 \leq z \leq \tilde{a}_4 \\ 1-\frac{1}{2}\left(\frac{z-\tilde{a}_4}{\tilde{a}_5-\tilde{a}_4}\right), & for\ \tilde{a}_4 \leq z \leq \tilde{a}_5 \\ \frac{1}{2}\left(\frac{\tilde{a}_6-z}{\tilde{a}_6-\tilde{a}_5}\right), & for\ \tilde{a}_5 \leq z \leq \tilde{a}_6 \\ 0, & for\ z > a_6 \end{cases}$$

Definition 2.4 (Alpha cut). *The classical set $\tilde{A}_\alpha$ called alpha cut set is the set of elements whose degree of membership is the set of elements whose degree of membership in $\tilde{A}_H = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6)$ is no less than α, it defines $\tilde{A}_\alpha = \{x \in X/\mu_{\tilde{A}H}(x) \geq \alpha\}$.*

Definition 2.5 (Fuzzy matrix). *A matrix $\tilde{A} = (\tilde{a}_{ij})$, if each element of A is a fuzzy number is called the fuzzy matrix. If each element of $\tilde{A}$ will be positive (negative) and denoted by $\tilde{A} > 0$ ($\tilde{A} < 0$), if each element of $\tilde{A}$ be positive (negative). $\tilde{A}$ will be non positive (non negative) and denoted by $\tilde{A} < 0$ ($\tilde{A} > 0$), if each element of $\tilde{A}$ be non positive (non negative). We represent $n \times m$ fuzzy matrices $\tilde{A} = (\tilde{a}_{ij})_{nxm}$, where, $\tilde{a}_{ij} = (\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij})$.*

Definition 2.6 (Fuzzy zero). *A Hexagonal fuzzy number $\tilde{A} = (a, b, c, d, e, f)$ is said to be fuzzy zero if $R(\tilde{A}) = 0$.*

Definition 2.7 (Arithmetic operation on Hexagonal fuzzy number). *Suppose $\tilde{A}_H = (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6)$ and $\tilde{B}_H = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4, \tilde{b}_5, \tilde{b}_6)$ are two Hexagonal fuzzy numbers.*

Addition:

$$\begin{aligned} \tilde{A}_H + \tilde{A}_H &= (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6) + (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4, \tilde{b}_5, \tilde{b}_6) \\ &= (\tilde{a}_1 + \tilde{b}_1, \tilde{a}_2 + \tilde{b}_2, \tilde{a}_3 + \tilde{b}_3, \tilde{a}_4 + \tilde{b}_4, \tilde{a}_5 + \tilde{b}_5, \tilde{a}_6 + \tilde{b}_6) \end{aligned}$$

Subtraction:

$$\begin{aligned} \tilde{A}_H - \tilde{B}_H &= (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6) - (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4, \tilde{b}_5, \tilde{b}_6) \\ &= (\tilde{a}_1 - \tilde{b}_6, \tilde{a}_2 - \tilde{b}_5, \tilde{a}_3 - \tilde{b}_4, \tilde{a}_4 - \tilde{b}_3, \tilde{a}_5 - \tilde{b}_2, \tilde{a}_6 - \tilde{b}_1) \end{aligned}$$

Multiplication:

$$\begin{aligned} \tilde{A}_H \cdot \tilde{A}_H &= (\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6) \cdot (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4, \tilde{b}_5, \tilde{b}_6) \\ &= (\tilde{a}_1 \cdot \tilde{b}_1, \tilde{a}_2 \cdot \tilde{b}_2, \tilde{a}_3 \cdot \tilde{b}_3, \tilde{a}_4 \cdot \tilde{b}_4, \tilde{a}_5 \cdot \tilde{b}_5, \tilde{a}_6 \cdot \tilde{b}_6) \end{aligned}$$

3. Mathematical Formulation of a Transportation Problem

$$\text{Minimize} \qquad \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$$
$$\text{subject to constraints} \qquad \sum_{j=1}^{n} x_{ij} = a_i, \quad j = 1, \ldots, n,$$
$$\sum_{i=1}^{m} x_{ij} = b_j, \quad i = 1, \ldots, m.$$

Where $x_{ij} \geq 0$, $i = 1, \ldots, m$; $j = 1, \ldots, n$, c_{ij} is the cost of transportation of a unit from i^{th} supply to the j^{th} demand, and the quantity x_{ij} is to be some positive integer or zero, which is to be transported from the i^{th} origin to j^{th} destination. An obvious necessary and sufficient condition for the linear programming problem is to have a solution is of the form, $\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} x_{ij} = \sum\limits_{i=1}^{m} a_i = \sum\limits_{j=1}^{n} b_j$ (i.e.) the total availability is equal to the total requirement, so, the solution is feasible.

4. Mathematical Formulation of Fuzzy Transportation Problem

$$\text{Minimize} \qquad \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$$
$$\text{subject to constraints} \qquad \sum_{j=1}^{n} x_{ij} = a_i, \quad j = 1, \ldots, n,$$
$$\sum_{i=1}^{m} x_{ij} = b_j, \quad i = 1, \ldots, m.$$

for all $\tilde{x}_{ij}$ represents fuzzy variables. The fuzzy transportation cost c_{ij}, and fuzzy supply a_i of product at i^{th} source and fuzzy demand b_j of product at j^{th} destination are quantities or constants. A necessary and sufficient condition for fuzzy linear programming problem is to have a solution is $\sum\limits_{i=1}^{m} \tilde{a}_i = \sum\limits_{j=1}^{n} \tilde{b}_j$, $i = 1, \ldots, m$; $j = 1, \ldots, n$, otherwise, it is unbalanced.

Fuzzy Quantities	1	2	$\cdots$	$\cdots$	n	Fuzzy Supply
1	x_{11} $\tilde{c}_{11}$	x_{12} $\tilde{c}_{12}$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	x_{1n} $\tilde{c}_{1n}$	$\tilde{S}_1$
2	x_{21} $\tilde{c}_{21}$	x_{22} $\tilde{c}_{22}$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	x_{2n} $\tilde{c}_{2n}$	$\tilde{S}_2$
3	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\tilde{S}_3$
. .	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	. .
. m	x_{m1} $\tilde{c}_{m1}$	x_{m2} $\tilde{c}_{m2}$	$\cdots$ $\cdots$	$\cdots$ $\cdots$	x_{mn} $\tilde{c}_{mn}$	. $\tilde{S}_m$
Fuzzy Demand	$\tilde{D}_1$	$\tilde{D}_2$	$\cdots$	$\cdots$	$\tilde{D}_n$	$\sum\limits_{i=1}^{m} \tilde{S}_i = \sum\limits_{j=1}^{n} \tilde{D}_j$

Table 1. **Tabular Representation of Fuzzy Transportation Problem**

Where,

$$\tilde{C}_{ij} = [C_{ij}^{(1)}, C_{ij}^{(2)}, C_{ij}^{(3)}, C_{ij}^{(4)}, \ldots, C_{ij}^{(6)}]$$
$$\tilde{X}_{ij} = [X_{ij}^{(1)}, X_{ij}^{(2)}, X_{ij}^{(3)}, X_{ij}^{(4)}, \ldots, X_{ij}^{(6)}]$$

$$\tilde{S}_i = [S_i^{(1)}, S_i^{(2)}, S_i^{(3)}, S_i^{(4)}, \ldots, S_i^{(6)}] \ \ (\tilde{S}_i \geq 0)$$
$$\tilde{D}_j = [D_j^{(1)}, D_j^{(2)}, D_j^{(3)}, D_j^{(4)}, \ldots, D_j^{(6)}] \ \ (\tilde{D}_j \geq 0)$$

Definition 4.1 (Index Sets). *m – Sources; n – destinations; i Index for sources for all i = 1, 2, 3,. . . , m; j Index for destination for all j = 1, 2, 3,. . . , n; g Index for objective for all g = 1, 2,. . . , k.*

Definition 4.2 (Decision Variables). $\tilde{x}_{ij}$ *Fuzzy Quantity transported from source i to destination j (units).*

Definition 4.3 (Objective functions). $\tilde{Z}$ *Transportation costs (Rs.)*

Definition 4.4 (Parameters). $\tilde{c}_{ij}$ *Cost of fuzzy transportation per unit delivered from source i to destination j (Rs / unit).* $\tilde{S}_i$ *Total fuzzy availability of product at each source i (units).* $\tilde{D}_j$ *Total forecast fuzzy requirement of product n at each destination j (units).*

5. Robust Ranking Technique

If $\tilde{a}$ is a fuzzy number (a, b, c, d, e, f; 1), then the Robust ranking is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^l a_\alpha^U) d\alpha,$$

where $(a_\alpha^L a_\alpha^U)$ is the α-level cut of the fuzzy number. Such that

$$(a_\alpha^L a_\alpha^U) = \{(b-a)\alpha + a, d - (d-c)\alpha\} + \{(d-c)\alpha + c, f - (f-e)\alpha\}.$$

Here we use robust ranking function which satisfy additive, linearity, and compensation properties provides results, which are consistent with human intuition.

6. Methodology

Step 1: Construct the table from given fuzzy transportation problem.

Step 2: Subtract each row entries of the transportation table from the respective row minimum and then from the resulting transportation table, we subtract each column entries from corresponding column minimum.

Step 3: Now, we identify that there must be at least one zero in each row and in each column in the reduced cost matrix. Select the first zero which occurring in the cost matrix (row-wise). Suppose $(i, j)^{th}$ zero is selected, count the total number of zero's (except the selected one) in the i^{th} row and j^{th} column. Now, we select the next zero and count the total number of zero's in the same manner, continue it for all zero's in that matrix.

Step 4: Now, we choose a zero for which the minimum number of zero's is counted in Step 3 and allocate maximum positive amount to that cell. If tie occurs for some zero's in step 3, then alternatively choose a $(k, l)^{th}$ zero to break a tie, such that total sum of all the elements in the k^{th} row and l^{th} column is maximum. Allocate as much as possible amount to that cell.

Step 5: After performing step 4, delete the selected row or column in cost matrix for further proceedings where the supply from a given source is exhausted or the demand from a given destination is satisfied.

Step 6: Check whether the resultant reduced cost matrix possesses at least one zero in each row and in each column. If not repeat Step 2, otherwise go to Step 7.

Step 7: Repeat Step 3 to Step 6 until and unless all the requirements are fulfilled and all the supplies are depleted.

Step 8: Finally, we calculate fuzzy optimal solution.

7. Numerical Illustration

A company has three production S_1, S_2 and S_3 and four requirements D_1, D_2, D_3 and D_4 the fuzzy transportation cost for unit quantity of the product form i^{th} source to j^{th} destinations is C_{ij}, where

$$[c_{ij}]_{m\times n} = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{bmatrix} (1,2,3,4,5,6) & (1,6,3,4,7,8) & (9,8,6,7,4,5) & (4,5,2,6,3,2) \\ (4,5,2,3,6,3) & (7,6,5,2,3,5) & (5,6,1,2,7,4) & (5,6,5,7,4,3) \\ (6,2,7,6,5,1) & (7,8,6,6,5,1) & (4,5,6,7,9,6) & (1,8,10,7,5,6) \end{bmatrix} \end{array}$$

Fuzzy Supply: (2, 3, 5, 6, 2, 1) (5, 10, 12, 17, 11, 10) (8, 0, 12, 12, 6, 4)

Fuzzy Demand: (5, 8, 8, 8, 5, 1) (5, 6, 6, 8, 5, 1) (2, 3, 6, 6, 5, 3) (3, 6, 9, 13, 4, 10)

Solution:

$$R(1,2,3,4,5,6) = \int_0^1 0.5\{14\}d\alpha = [14\alpha]_0^1(0.5) = 7$$
$$R(1,6,3,4,7,8) = \int_0^1 0.5\{16+4\alpha\}d\alpha = [8\alpha - 4\alpha^2/2]_0^1(0.5) = 9$$
$$R(9,8,6,7,4,5) = \int_0^1 0.5\{27-2\alpha\}d\alpha = [27\alpha - 2\alpha^2/2]_0^1(0.5) = 13$$
$$R(4,5,2,6,3,2) = \int_0^1 0.5\{14+2\alpha\}d\alpha = [14\alpha + 2\alpha^2/2]_0^1(0.5) = 7.5$$
$$R(4,5,2,3,6,3) = \int_0^1 0.5\{12+4\alpha\}d\alpha = [12\alpha + 4\alpha^2/2]_0^1(0.5) = 7$$
$$R(7,6,5,2,3,5) = \int_0^1 0.5\{19-3\alpha\}d\alpha = [19\alpha - 3\alpha^2/2]_0^1(0.5) = 8.75$$
$$R(5,6,1,2,7,4) = 0.5\int_0^1 \{12+4\alpha\}d\alpha = [12\alpha + 4\alpha^2/2]_0^1(0.5) = 7$$
$$R(5,6,5,7,4,3) = 0.5\int_0^1 \{20+2\alpha\}d\alpha = [20\alpha + 2\alpha^2/2]_0^1(0.5) = 10.5$$
$$R(6,2,6,5,3,1) = \int_0^1 0.5\{18-2\alpha\}d\alpha = [18\alpha - 2\alpha^2/2]_0^1(0.5) = 8.5$$
$$R(7,8,6,6,5,1) = \int_0^1 0.5\{20+5\alpha\}d\alpha = [20\alpha + 5\alpha^2/2]_0^1(0.5) = 11.25$$
$$R(4,5,6,7,9,6) = 0.5\int_0^1 \{23+4\alpha\}d\alpha = [23\alpha + 4\alpha^2/2]_0^1(0.5) = 12.5$$
$$R(1,8,0,6,5,6) = 0.5\int_0^1 \{13+6\alpha\}d\alpha = [13\alpha + 6\alpha^2/2]_0^1(0.5) = 8$$

Rank of Fuzzy supply: R(2, 3, 5, 6, 2, 1) = 7.5; R(5, 10, 12, 17, 11, 10) = 23.5; R(8, 10, 12, 12, 6, 4) = 19

Rank of Fuzzy Demand: R(5, 8, 8, 5, 5, 1) = 12.75; R(5, 6, 6, 8, 5, 1) = 9.25; R(3, 6, 9, 13, 4, 10) = 16.75

Step 1: Construct the fuzzy transportation matrix, after ranking we get

$$\begin{array}{cccccc} & D_1 & D_2 & D_3 & D_4 & Fuzzy\ Supply \\ S_1 & 14 & 9 & 13 & 7.5 & 7.5 \\ S_2 & 7 & 8.75 & 7 & 10.5 & 23.5 \\ S_3 & 10 & 11.25 & 12.5 & 8.5 & 14 \\ Fuzzy\ Demand & 12.75 & 11.25 & 9.25 & 16.75 & \end{array}$$

Step 2: After a row minimum and column minimum, we get the resulting matrix form is

$$\begin{array}{ccc} & \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} & \textit{Fuzzy Supply} \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array} & \begin{pmatrix} 6.5 & 0 & 5.5 & 0 \\ 0 & 0.25 & 0 & 3.5 \\ 1.5 & 1.25 & 4 & 0 \end{pmatrix} & \begin{array}{c} 7.5 \\ 23.5 \\ 19 \end{array} \\ \textit{Fuzzy Demand} & \begin{array}{cccc} 12.75 & 11.25 & 9.25 & 16.75 \end{array} & \end{array}$$

Step 3: Check, whether each row and each column contains at least one zero.

$$\begin{array}{ccc} & \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} & \textit{Fuzzy Supply} \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array} & \begin{pmatrix} 6.5 & 0(7.5) & 5.5 & 0 \\ 0 & 0.25 & 0 & 3.5 \\ 1.5 & 1.25 & 4 & 0 \end{pmatrix} & \begin{array}{c} 0 \\ 23.5 \\ 19 \end{array} \\ \textit{Fuzzy Demand} & \begin{array}{cccc} 12.75 & 3.75 & 9.25 & 16.75 \end{array} & \end{array}$$

Step 4: Then, choose a zero (row wise), such that the total number of zero's in that row is minimum (except the selected one). Allocate a minimum supply to that cell.

$$\begin{array}{ccc} & \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} & \textit{Fuzzy Supply} \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array} & \begin{pmatrix} 6.5 & 0(7.5) & 5.5 & 0 \\ 0 & 0(3.75) & 0(9.25) & 3.5 \\ 1.5 & 1 & 4 & 0 \end{pmatrix} & \begin{array}{c} 0 \\ 10.5 \\ 19 \end{array} \\ \textit{Fuzzy Demand} & \begin{array}{cccc} 12.75 & 0 & 0 & 16.75 \end{array} & \end{array}$$

$$\begin{array}{ccc} & \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} & \textit{Fuzzy Supply} \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array} & \begin{pmatrix} 6.5 & 0(7.5) & 5.5 & 0 \\ 0(10.5) & 0(3.75) & 0(9.25) & 3.5 \\ 1.5 & 1 & 4 & 0 \end{pmatrix} & \begin{array}{c} 0 \\ 0 \\ 19 \end{array} \\ \textit{Fuzzy Demand} & \begin{array}{cccc} 2.25 & 0 & 0 & 16.75 \end{array} & \end{array}$$

Step 5: Delete the corresponding row or column from where the allotment is made. Again check whether each row and each column contains at least one zero, otherwise either row minima or column minima must be needed.

$$\begin{array}{ccc} & \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} & \textit{Fuzzy Supply} \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array} & \begin{pmatrix} 6.5 & 0(7.5) & 5.5 & 0 \\ 0(10.5) & 0(3.75) & 0(9.25) & 3.5 \\ 1.5 & 1 & 4 & 0(16.75) \end{pmatrix} & \begin{array}{c} 0 \\ 0 \\ 2.25 \end{array} \\ \textit{Fuzzy Demand} & \begin{array}{cccc} 2.25 & 0 & 0 & 0 \end{array} & \end{array}$$

Step 6: Repeat the procedure until the requirements are satisfied. At last all the allocations are mentioned on the original fuzzy transportation problem.

Step 7: Finally, calculate the Total fuzzy optimal cost to the problem.

$$\begin{array}{ccc} & \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} & \textit{Fuzzy Supply} \\ \begin{array}{c} S_1 \\ S_2 \\ S_3 \end{array} & \begin{pmatrix} 14 & 9(7.5) & 13 & 7.5 \\ 7(10.5) & 8.75(3.75) & 7(9.25) & 10.5 \\ 10(2.25) & 11.25 & 12.5 & 8(16.75) \end{pmatrix} & \begin{array}{c} 7.5 \\ 23.5 \\ 14 \end{array} \\ \textit{Fuzzy Demand} & \begin{array}{cccc} 12.75 & 11.25 & 9.25 & 16.75 \end{array} & \end{array}$$

$$\begin{aligned}\text{Total Fuzzy optimal solution is} &= C_{12}x_{12} + C_{21}x_{21} + C_{22}x_{22} + C_{23}x_{23} + C_{31}x_{31} + C_{34}x_{34} \\ &= 9 \times 7.5 + 7 \times 10.5 + 8.75 \times 3.75 + 7 \times 9.25 + 8.5 \times 2.25 + 8.5 \times 16.75 \\ &= 400\end{aligned}$$

8. Conclusion

In this paper, direct method is used for solving fuzzy transportation problem involving hexagonal fuzzy number to get an optimal solution directly, which is more realistic in nature. The solution obtained by using ranking in less iteration is optimal. So this method is easy to understand and apply.

References

[1] A.Thamaraiselvi and R.Santhi, *Optimal Solution of Fuzzy Transportation Problem using Hexagonal Fuzzy Numbers*, International Journal of Scientific & Engineering Research, 6(3)(2015), 40-45.

[2] P.Rajeswari and A.Sahaya Sudha, *A new Operation on Hexagonal Fuzzy Number*, International Journal of Fuzzy Logic Systems, 3(3)(2013), 15-26.

[3] Srinivasan and G.Geetharamani, *Direct Method for finding an optimal solution for fuzzy Transportation Problem*, International Journal of Computational Engineering Research, 3(11)(2013), 2250-3005.

[4] S.Narayanamoorthy, S.Saranya and S.Maheswari, *Method of solving Fuzzy Transportation problem using fuzzy Russel's method*, I.J. Intelligent Systems and Applications, 2(2013), 71-75.

[5] C.Sharmila Devi and G.Geetharamani, *An Innovative Method for solving Fuzzy Transportation Problem*, Indian Journal Of Applied Research, 4(5)(2014), 399-402.

[6] S.U.Malini and Felbin C.Kennedy, *An approach for Solving Fuzzy Transportation Problem Using Octagonal fuzzy numbers*, Applied Mathematical Sciences, 7(54)(2013), 2661-2673.

[7] S.Nareshkumar and S.Kumara Ghuru, *Solving Fuzzy Transportation Problem using Symmetric Triangular Fuzzy number*, International Journal of Advanced Research in Mathematics and Applications, 1(1)(2014), 74-83.

[8] Abdul Quddoos, Shakeel Javaid and M.M.Khalid, *New Method for finding an Optimal Solution for Transportation problem*, International Journal on Computer Science and Engineering, 4(7)(2012), 1271-1274.

[9] B.Srinivas and G.Ganeshan, *Optimal solution for Fuzzy Transportation problem using Stepping stone Method*, International Journal of Innovative Trends in Engineering, 3(3)(2015), 185-198.

[10] P.Jayaraman and R.Jahirhussian, *Fuzzy Optimal Transportation Problems by improved zero suffix method via Robust Rank Techniques*, International Journal of Fuzzy Mathematics and Systems, 3(4)(2013), 303-311.

[11] L.A.Zadeh, *Fuzzy set*, Information and Control, 8(3)(1965), 338-353.

[12] J.K.Sharma, *operation Research*, 4^{th} edition, (2010).

[13] Premkumar Gupta and D.S.Hira, *Problems in Operations Research*, Revised edition, (2008).

[14] Kanti swarup and P.K.Gupta Manmohan, *Operation Research -I*, 12^{th} edition, (2004).

[15] H.J.Zimmerman, *Fuzzy set theory*, wire's computational statistics, (2010).

[16] Sundhir, K.Pundir and Rimple Pundir, *Fuzzy sets and their applications*, 1^{st} edition.