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Solving Fuzzy Assignment Problem for Hexagonal Fuzzy Number Using Revised Ones Assignment Method

Research Article*

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Abstract: In this paper, revised ones assignment method is used to solve the fuzzy assignment problem to find maximum and minimum objective function using hexagonal fuzzy number. Finally, examples are given to explain this method. We find the minimum total cost and maximum total cost.

Keywords: Fuzzy assignment problem, revised ones assignment algorithm and hexagonal fuzzy number.

1. Introduction

Fuzzy sets, introduced by zadeh in 1965. A new mathematical tool to deal with uncertainly of information we used basic concepts of fuzzy sets, fuzzy number and fuzzy linear programming. The fuzzy assignment problem is a special type of linear programming problems. Revised ones assignment method is used to solve fuzzy assignment problem to find maximum and minimum objective function. Then such as fuzzy number, hexagonal have been introduced in this method. Six persons are to be performed by six jobs depending in on their jobs. Examples are given to explain this method and find minimum and maximum total cost.

1.1. Preliminaries

Definition 1.1 (Fuzzy set). *A Let X be a non empty set. A fuzzy set $\tilde{A}$ of X is defined as $\tilde{A} = \{x, \mu_A(x)/x \in X\}$ where $\mu_{\tilde{A}}(x)$ is called the membership function where $\mu_{\tilde{A}}(x) : R \to [0,1]$.*

Definition 1.2 (Fuzzy numbers). *A fuzzy number f in the real line R is a fuzzy set $f : R \to [0,1]$ that satisfies the following properties,*

- *f is piecewise continuous*
- *there exist an $x \in R$ such that $f(x) = 1$*
- *f is convex. ie, if x_1, $x_2 \in R$ and $a \in [0,1]$.*

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Definition 1.3 (Hexagonal fuzzy number)**.** *A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by* $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ *where* $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6$ *are real number and its membership function is given below*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-a^1}{a^2-a^1}\right), & for\ a^1 \le x \le a^2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a^2}{a^3-a^2}\right), & for\ a^2 \le x \le a^3 \\ 1, & for\ a^3 \le x \le a^4 \\ 1 - \frac{1}{2}\left(\frac{x-a^4}{a^5-a^4}\right), & for\ a^4 \le x \le a^5 \\ \frac{1}{2}\left(\frac{a^6-x}{a^6-a^5}\right), & for\ a^5 \le x \le a^6 \\ 0, & otherwise \end{cases}$$

Definition 1.4 (α-cut)**.** *The α-cut of a fuzzy number $A(\alpha)$ is defined as* $A(\alpha) = \{x/\mu(x) \ge \alpha,\ \alpha \in [0,1]\}$.

2. Revised Ones Assignment Using Hexagonal Fuzzy Number

2.1. Mathematical Formulation of Assignment Problem's

The mathematical formulation of the assignment problem is, associated to each assignment problem there is a matrix called cost or effectiveness matrix $[c_{ij}]$ where c_{ij} is the cost of assigning i^{th} products to j^{th} salesmen. In this paper we call it assignment matrix and represent it as follows:

$$\begin{array}{c} \\ 1 \\ \vdots \\ n \end{array} \begin{array}{c} \begin{array}{ccc} 1 & \dots & n \end{array} \\ \begin{pmatrix} c_{11} & \dots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} & \dots & c_{nn} \end{pmatrix} \end{array}$$

The mathematical formulation of the assignment problem is,

$$\begin{aligned} \text{Minimize} \quad & Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}X_{ij} \\ \text{subject to} \quad & \sum_{i=1}^{n} X_{ij} = 1,\ i = 1,\dots,n \\ & \sum_{j=1}^{n} X_{ij} = 1,\ j = 1,\dots,n; \qquad X_{ij} = 0 \ \ or \ \ 1 \end{aligned}$$

2.2. Algorithm

Revised Ones Assignment Algorithm for using Hexagonal Fuzzy Number

Step 1: In a minimization or maximization case, find the minimum or maximum element of each row in the assignment matrix and write it on the right hand side of the matrix. Then divide each element of 6 row of the matrix by a_1 to a_6. These operations create at least one ones in each rows. In term of ones for each row and column. Otherwise go to Step 2.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{pmatrix} \end{array} \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{array}$$

Step 2: Find the minimum or maximum element of each column in assignment matrix and write it below 6 columns. Then divide each element of 6 column of the matrix by b_1 to b_6, these operations create at least one ones in each columns. Make assignment in terms of ones. If no feasible assignment can be achieved from Step 1 and 2 then go to Step 3.

$$\begin{array}{c} \\ 1\\2\\3\\4\\5\\6 \\ \\ \end{array}\begin{array}{c} \quad 1 \qquad\quad 2 \qquad\quad 3 \qquad\quad 4 \qquad\quad 5 \qquad\quad 6 \\ \begin{pmatrix} c_{11}/a_1 & c_{12}/a_1 & c_{13}/a_1 & c_{14}/a_1 & c_{15}/a_1 & c_{16}/a_1 \\ c_{21}/a_2 & c_{22}/a_2 & c_{23}/a_2 & c_{24}/a_2 & c_{25}/a_2 & c_{26}/a_2 \\ c_{31}/a_3 & c_{32}/a_3 & c_{33}/a_3 & c_{34}/a_3 & c_{35}/a_3 & c_{36}/a_3 \\ c_{41}/a_4 & c_{42}/a_4 & c_{43}/a_4 & c_{44}/a_4 & c_{45}/a_4 & c_{46}/a_4 \\ c_{51}/a_5 & c_{52}/a_5 & c_{53}/a_5 & c_{54}/a_5 & c_{55}/a_5 & c_{56}/a_5 \\ c_{61}/a_6 & c_{62}/a_6 & c_{63}/a_6 & c_{64}/a_6 & c_{65}/a_6 & c_{66}/a_6 \end{pmatrix} \\ \quad b_1 \qquad\quad b_2 \qquad\quad b_3 \qquad\quad b_4 \qquad\quad b_5 \qquad\quad b_6 \end{array}\begin{array}{c} \\ a_1\\a_2\\a_3\\a_4\\a_5\\a_6 \\ \\ \end{array}$$

Step 3: Draw the minimum number of lines to cover all the ones of the matrix. If the number of drawn lines less than 6, then the complete assignment is not possible, while if the number of lines is exactly equal to 6, then the complete assignment is obtained.

Step 4: If a complete assignment program is not possible in Step 3, then select the smallest or largest element (say d_{ij}) out of those which do not lie on any of the lines in the above matrix. Then divide by d_{ij} each element of the uncovered rows or columns, which d_{ij} lies on it. This operation creates some new ones to this row or column.

If still a complete optimal assignment is not achieved in this new matrix, then use Step 4 and 3 iteratively. By repeating the same procedure the optimal assignment will be obtained. (To assign one we have add Step 5 which is mentioned below.)

Step 5:

i). For minimization problem select maximum number from calculated matrix and write it on right hand side as well as bottom side.

- To assign one, start from minimum number of columns (bottom side) and select ones.
- If there are more than one ones in any column then ignore temporarily, and give last priority to that column.
- If still there are identical ones in column then give the priority to maximum number of rows (right hand side).

(or)

ii). For maximization problem select minimum number from calculated matrix and write it on right hand side as well as bottom side.

- To assign one, start from maximum number of columns (bottom side) and select ones.
- If there are more than one ones in any column then ignore temporarily, and give last priority to that column.
- If still there are identical ones in column then give the priority to minimum number of rows (right hand side).

Example 2.1. *Consider a fuzzy assignment problem with row representing 6 persons and 6 jobs so as to find minimize the* *total cost.*

	1	2	3	4	5	6
1	10	5	4	14	9	7
2	7	1	10	6	4	3
3	12	7	9	7	6	4
4	1	7	5	3	2	3
5	9	7	5	6	3	1
6	1	3	16	8	9	6

Solution:

Step 1: In a minimization case, find the minimum element of each row in the assignment matrix and write it on the right hand side of the matrix. Then divide each element of 6 row of the matrix by a_1 to a_6. These operations create at least one ones in each rows.In term of ones for each row and column. Otherwise go to Step 2.

	1	2	3	4	5	6	*min*
1	10	5	4	14	9	7	4
2	7	1	10	6	4	3	1
3	12	7	9	7	6	4	4
4	1	7	5	3	2	3	1
5	9	7	5	6	3	1	1
6	1	3	16	8	9	6	1

	1	2	3	4	5	6
1	2.5	1.3	1	3.5	2.3	1.8
2	7	1	10	6	4	3
3	3	1.8	2.3	1.8	1.5	1
4	1	7	5	3	2	3
5	9	7	5	6	3	1
6	1	3	16	8	9	6

Step 2: Find the minimum element of each column in assignment matrix and write it below 6 columns. Then divide each element of 6 column of the matrix by b_1 to b_6. These operations create at least one ones in each columns. Make assignment in terms of ones. If no feasible assignment can be achieved from Step 1 and 2 then go to Step 3.

	1	2	3	4	5	6
1	2.5	1.3	1	3.5	2.3	1.8
2	7	1	10	6	4	3
3	3	1.8	2.3	1.8	1.5	1
4	1	7	5	3	2	3
5	9	7	5	6	3	1
6	1	3	16	8	9	6
min	1	1	1	1.8	1.5	1

	1	2	3	4	5	6
1	2.5	1.3	1	1.9	1.5	1.8
2	7	1	10	3.3	2.7	3
3	3	1.8	2.3	1	1	1
4	1	7	5	1.6	1.3	3
5	9	7	5	2.2	3.5	1
6	1	3	16	1.6	1.3	6

Step 3: Draw the minimum number of lines to cover all the ones of the matrix. if the number of lines is exactly equal to 6, then the complete assignment is obtained while If the number of draw lines less than 6 go to Step 4.

	1	2	3	4	5	6
1	2.5	1.3	(1)	1.9	1.5	1.8
2	7	(1)	10	3.3	2.7	3
3	3	1.8	2.3	(1)	1	1
4	(1)	7	5	1.6	1.3	3
5	9	7	5	2.2	3.5	(1)
6	1	3	16	1.6	1.3	6

Step 4: If a complete assignment program is not possible in Step 3, then select the smallest or largest element (say d_{ij}) out of those which do not lie on any of the lines in the above matrix. Then divide by d_{ij} each element of the uncovered rows or columns, which d_{ij} lies on it. This operation creates some new ones to this row or column.

	1	2	3	4	5	6
1	2.5	1.3	(1)	1.9	1.5	1.8
2	7	(1)	10	3.3	2.7	3
3	3	1.8	2.3	(1)	1	1
4	(1)	5.3	3.8	1.2	1	2.3
5	9	7	5	2.2	3.5	(1)
6	1	2.3	12.3	1.2	(1)	4.6

Step 5:

(1). We select maximum number from matrix for minimization problem and write it to right as well as bottom side.

(2). In column 1, 5, 6 contain more than one ones, so we will give it last priority.

(3). To assign ones from matrix select minimum number from columns (bottom side). So 3.3 is minimum number from all other and assign ones.

(4). After giving next priority to column 2 and next column 3 in identical ones. Then more than one ones in column 6 and column1 but the minimum value is column 6.similarly at this stage give maximum priority of minimization problem.

	1	2	3	4	5	6	max
1	2.5	1.3	(1)	1.9	1.5	1.8	2.5
2	7	(1)	10	3.3	2.7	3	7
3	3	1.8	2.3	(1)	1	1	3
4	(1)	5.3	3.8	1.2	1	2.3	5.3
5	9	7	5	2.2	3.5	(1)	9
6	1	2.3	12.3	1.2	(1)	4.6	12.3
max	9	7	12.3	3.3	3.5	4.6	
	↓	↓	↓	↓	↓	↓	
	(5)	(2)	(3)	(1)	(6)	(4)	

And we can assign the ones and the solution is (1,3), (2,2), (3,4), (4,1), (5,6), (6,5). Minimum total cost = 15

Example 2.2. *Consider a fuzzy assignment problem with row representing 6 persons and 6 jobs so as to find maximum the total cost.*

	1	2	3	4	5	6
1	1	4	6	3	2	1
2	5	1	7	9	8	2
3	8	6	7	1	5	3
4	2	4	6	8	4	2
5	4	3	2	1	7	8
6	10	6	5	12	3	7

Solution:

Step 1: In a maximization case, find the maximum element of each row in the assignment matrix and write it on the right hand side of the matrix. Then divide each element of 6 row of the matrix by a_1 to a_6. These operations create at least one ones in each rows.In term of ones for each row and column. Otherwise go to Step 2.

	1	2	3	4	5	6	max
1	1	4	6	3	2	1	6
2	5	1	7	9	8	2	9
3	8	6	7	1	5	3	8
4	2	4	6	8	4	2	8
5	4	3	2	1	7	8	8
6	10	6	5	12	3	7	12

	1	2	3	4	5	6
1	0.16	0.6	1	0.5	0.3	0.16
2	0.5	0.11	0.7	1	0.8	0.2
3	1	0.75	0.7	0.12	0.62	0.37
4	0.25	0.5	0.75	1	0.5	0.25
5	0.5	0.37	0.25	0.12	0.87	1
6	0.83	0.5	0.41	1	0.25	0.58

Step 2: Find the maximum element of each column in assignment matrix and write it below 6 columns. Then divide each element of 6 column of the matrix by b_1 to b_6. These operations create at least one ones in each columns. Make assignment in terms of ones. If no feasible assignment can be achieved from Step 1 and 2 then go to Step 3.

	1	2	3	4	5	6
1	0.16	0.6	1	0.5	0.3	0.16
2	0.5	0.11	0.7	1	0.8	0.2
3	1	0.75	0.7	0.12	0.62	0.37
4	0.25	0.5	0.75	1	0.5	0.25
5	0.5	0.37	0.25	0.12	0.87	1
6	0.83	0.5	0.41	1	0.25	0.58
max	1	0.75	1	1	0.87	1

	1	2	3	4	5	6
1	0.16	0.8	1	0.5	0.34	0.16
2	0.5	0.14	0.7	1	0.91	0.2
3	1	1	0.7	0.12	0.71	0.37
4	0.25	0.66	0.75	1	0.57	0.25
5	0.5	0.49	0.25	0.12	1	1
6	0.83	0.66	0.41	1	0.28	0.58

Step 3: Draw the minimum number of lines to cover all the ones of the matrix. if the number of lines is exactly equal to 6, then the complete assignment is obtained while If the number of draw lines less than 6 go to Step 4

	1	2	3	4	5	6
1	~~0.16~~	~~0.8~~	~~(1)~~	~~0.5~~	~~0.34~~	~~0.16~~
2	0.5	0.14	0.7	1	0.91	0.2
3	~~(1)~~	~~1~~	~~0.7~~	~~0.12~~	~~0.71~~	~~0.37~~
4	0.25	0.66	0.75	1	0.57	0.25
5	~~0.5~~	~~0.49~~	~~0.25~~	~~0.12~~	~~(1)~~	~~1~~
6	0.83	0.66	0.41	(1)	0.28	0.58

Step 4: If a complete assignment program is not possible in Step 3, then select the smallest element (say d_{ij}) out of those which do not lie on any of the lines in the above matrix. Then divide by d_{ij} each element of the uncovered rows, which d_{ij} lies on it. This operation creates some new ones to this row.

	1	2	3	4	5	6
1	0.16	0.8	(1)	0.5	0.34	0.16
2	0.54	0.15	0.76	1	(1)	0.21
3	1	(1)	0.7	0.12	0.71	0.37
4	0.3	0.88	1	(1)	0.76	0.33
5	0.5	0.49	0.25	0.12	1	(1)
6	(1)	0.79	0.49	1	0.33	0.69

Step 5:

(1). We select minimum number from matrix for maximization problem and write it to right as well as bottom side.

(2). In column 1, 3, 4, 5 contain more than one ones, so we will give it last priority.

(3). To assign ones from matrix select maximum number from columns (bottom side). So 0.16 is maximum number from all other and assign ones.

(4). After giving next priority to column 2 in identical ones. Then more than one ones in column 3 and column 1, column 5 but the maximum value is column.

(5). Similarly at this stage give min priority of maximization problem.

	1	2	3	4	5	6	*max*
1	0.16	0.8	(1)	0.5	0.34	0.16	0.16
2	0.54	0.15	0.76	1	(1)	0.21	0.15
3	1	(1)	0.7	0.12	0.71	0.37	0.12
4	0.3	0.88	1	(1)	0.76	0.33	0.3
5	0.5	0.49	0.25	0.12	1	(1)	0.12
6	(1)	0.79	0.49	1	0.33	0.69	0.33
min	0.16	0.15	0.25	0.12	0.33	0.16	
	↓	↓	↓	↓	↓	↓	
	(5)	(2)	(4)	(6)	(3)	(1)	

We can assign the ones and the solution is (1,3), (2,5), (3,2), (4,4), (5,6), (6,1). Maximum total cost = 46

3. Conclusion

In this paper Revised ones assignment method solved the fuzzy assignment problem using hexagonal fuzzy number. This method can be used to find the maximum and minimum total cost .In future we find revised ones assignment algorithm to using fuzzy transportation problems.

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