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Fuzzy Critical Path Analysis In A Project Network Using Fuzzy Topsis Method

Research Article*

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Abstract: In this Paper, we present the project scheduling problem using fuzzy TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method. The ratings of each activity and the weight of each criterion are given in terms of linguistic Variables which are expressed in terms of pentagonal fuzzy number. Then, by the relative closeness coefficient, we compute the ranking order of each alternative and the alternative with the highest ranking score is chosen to be the fuzzy critical path of the project network. An illustration is given to explain the method.

Keywords: Pentagonal fuzzy number, Fuzzy TOPSIS method, linguistic Variables and fuzzy project network.

1. Introduction

Network optimization is a very popular and frequently applied field among the well studied areas of operation research. Fuzzy TOPSIS is a well known Multi Criteria Decision Making method (MCDM) such that its solution is identified from a finite set of alternatives. The basic principle of the TOPSIS method is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. Classical project scheduling methods such as CPM, PERT, etc, considers only the time criterion to determine the criticality of the path, but in fuzzy TOPSIS method, we could consider various criteria to compute the fuzzy Critical path. So, in this paper we have considered each path in the project network as the alternative and moreover, we have considered three criteria such as time, cost and quality in order to find the Critical path of the fuzzy project network.

Definition 1.1 (Fuzzy Set)**.** *Let X be a non empty set. A fuzzy set $\tilde{A}$ of X is defined as $\tilde{A} = \{(x,\ \mu_{\tilde{A}}((x))\ /\ x \in X\}$ where, $\mu_{\tilde{A}}(x)$ is called the membership function which maps each element of X to a value between 0 and 1.*

Definition 1.2 (Fuzzy Number)**.** *A fuzzy set $\tilde{A}$ defined on the set of real numbers R, is said to be a fuzzy number if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ has the following characteristics,*

1. *$\tilde{A}$ is fuzzy convex, (i.e) $\mu_{\tilde{A}}[\lambda x_1 + (1-\lambda)x_2] \geq \mu_{\tilde{A}}(x_1) \bigwedge \mu_{\tilde{A}}(x_2), x_1, x_2 \in R, \forall\ \lambda \in [0,1]$.*
2. *$\tilde{A}$ is normal (i.e) there exists an element x_0 such that $\mu_{\tilde{A}}(x_0) = 1$.*
3. *$\mu_{\tilde{A}}$ is piecewise continuous.*

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Definition 1.3 (Pentagonal Fuzzy Number). *A pentagonal Fuzzy Number (PFN) of a fuzzy set* $\tilde{A}$ *is defined as* $\widetilde{A}_p = \{a_1, a_2, a_3, a_4, a_5\}$ *where* a_1, a_2, a_3, a_4, a_5 *are real numbers and its membership function is given below.*

$$\mu_{\widetilde{A}_p}(x) = \begin{cases} 0, & x < a_1 \\ \left[\frac{x-a_1}{a_2-a_1}\right], & a_1 \le x \le a_2 \\ \left[\frac{x-a_2}{a_3-a_2}\right], & a_2 \le x \le a_3 \\ 1, & x = a_3 \\ \left[\frac{a_4-x}{a_4-a_3}\right], & a_3 \le x \le a_4 \\ \left[\frac{a_5-x}{a_5-a_4}\right], & a_4 \le x \le a_5 \\ 0, & x > a_5. \end{cases}$$

Definition 1.4 (Distance between pentagonal fuzzy numbers). *Let* $\widetilde{a} = (a_1, a_2, a_3, a_4, a_5)$ *and* $\widetilde{b} = (b_1, b_2, b_3, b_4, b_5)$ *be two pentagonal fuzzy numbers. The distance between them is given by,*

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{5}((a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2 + (a_4 - b_4)^2 + (a_5 - b_5)^2)}$$

2. Procedures of Fuzzy TOPSIS Method

This fuzzy TOPSIS method can deal with the ratings of both quantitative as well as qualitative criteria and can effectively select a suitable alternative. We apply the proposed fuzzy TOPSIS method to find out the best alternative. This method is given as the following steps. The decision group has k members, the k^{th} members, the k^{th} decision makers ratings and weights are the i^{th} alternative on j^{th} criterion. Let $\{A_1, A_2, \ldots, A_m\}$ be the set of m alternatives which are to be evaluated against n criteria, $\{C_1, C_2, \ldots, C_n\}$, D_K be the committee of decision makers and the fuzzy rating and the important weight of criteria are represented as $x_{ij}^k = (a_{ij}^k, b_{ij}^k, c_{ij}^k, d_{ij}^k, e_{ij}^k)$ and $\tilde{w}_{ij}^k = (w_{j1}^k, w_{j2}^k, w_{j3}^k, w_{j4}^k, w_{j5}^k)$ respectively. Where $i = 1, 2, \ldots, m$; $j = 1, 2, 3, \ldots, n$. Then, compute the aggregated fuzzy rating $\tilde{x}_{ij}$ for each activity (i) with respect to each criterian (j) by using $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}, e_{ij})$. Such that $a_{ij} = \min_k\{a_{ij}^k\}$, $b_{ij} = \frac{1}{k}\sum_{k=1}^{k} b_{ij}^k$, $c_{ij} = \frac{1}{k}\sum_{k=1}^{k} c_{ij}^k$, $d_{ij} = \frac{1}{k}\sum_{k=1}^{k} d_{ij}^k$, $e_{ij} = \max_k\{e_{ij}^k\}$. The aggregated fuzzy weights $\tilde{w}_{ij}$ of each criterion are calculated as

$$\tilde{w}_j^k = (w_{j1}^k, w_{j2}^k, w_{j3}^k, w_{j4}^k, w_{j5}^k)$$

Where

$$w_{j1} = \min_k\{w_{jk1}\},$$
$$w_{j2} = \frac{1}{k}\sum_{k=1}^{k} w_{jk2},$$
$$w_{j3} = \frac{1}{k}\sum_{k=1}^{k} w_{jk3},$$
$$w_{j4} = \frac{1}{k}\sum_{k=1}^{k} w_{jk4},$$
$$w_{j5} = \max_k\{w_{jk5}\},$$

Construct the normalized fuzzy decision matrix: The set of criteria can be classified into benefit criteria and cost criteria. And thus, the normalized fuzzy decision matrix can be represented as

$$\tilde{R} = [r_{ij}]_{m \times n}$$

Where

$$\tilde{r}_{ij} = \left[\frac{a_{ij}}{e_j{}^+}, \frac{b_{ij}}{e_j{}^+}, \frac{c_{ij}}{e_j{}^+}, \frac{d_{ij}}{e_j{}^+}, \frac{e_{ij}}{e_j{}^+}\right], \ j \in B$$

$$\tilde{r}_{ij} = \left[\frac{a_j^-}{e_{ij}}, \frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right], \qquad j \in C$$

Where B and C are the set of benefit and cost criteria, respectively

$$e_j{}^+ = \max_i(e_{ij})$$

$$a_j^- = \min_i(a_{ij})$$

Construct weighted normalized fuzzy-decision matrix: The weighted normalized fuzzy decision matrix is constructed as $\tilde{v} = [\tilde{v}_{ij}]_{m\times n}$; $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. Where $\tilde{v}_{ij} = w_j(.)r_{ij}$. Determine FPIS and FNIS: The fuzzy positive ideal solution, FPIS (A^+) and fuzzy negative ideal Solution, FNIS (A^-) can be defined as

$$A^+ = (\tilde{v}_1^+, \tilde{v}_2^+, \ldots, \tilde{v}_n^+)$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-)$$

$$\tilde{v}_j^- = \min_i(v_{ij1}),$$

$$\tilde{v}_j^+ = \max_i(v_{ij5}),$$

$i = 1, 2, 3, \ldots, m$ and $j = 1, 2, \ldots, n$. The index v_{ij1} and v_{ij5}, 1 and 5 determine the first and fifth elements in a pentagonal fuzzy number respectively. Calculate the distance of each alternative from FPIS and FNIS: The distance of each alternative from A^+ and A^- can be calculated by using the following equations,

$$d_i^+ = \sum_{j=1}^{n} d_v(\tilde{v}_{ij}, \tilde{v}_j^+); \ \ i = 1, 2, \ldots, m$$

$$d_i^- = \sum_{j=1}^{n} d_v(\tilde{v}_{ij}, \tilde{v}_j^-); \ \ i = 1, 2, \ldots, m$$

Calculate the closeness coefficient of each alternative: The coefficient of closeness represents the distances of each alternative from the fuzzy positive ideal solution A^+ and the fuzzy negative ideal solution A^-. The closeness coefficient cc_i of each alternative is calculated as $cc_i = \frac{d_i^-}{d_i^- + d_i^+}$. According to the closeness coefficient, we determine the ranking order of all alternatives and the alternative with the highest closeness coefficient is chosen to be the best alternative.

2.1. Method for the Critical Path Selection

A massive project is divided into a series of activities. We determine the activity duration and precedence relationship among these activities. A path is one of the routes from starting node to the ending node. Identify all the paths in the fuzzy project network which start with starting event and end with the ending event. Choose the suitable linguistic variables for each criterion. Then, all linguistic evaluations are converted into suitable pentagonal fuzzy numbers. The length of a path is the sum of durations of activities on the path, so add up pentagonal fuzzy number to establish the final assessment value of each criterion for the paths. Then by following the steps of fuzzy TOPSIS method, we compute the relative closeness coefficient of each path. By comparing the relative closeness coefficient among the paths, we would get the required fuzzy critical path of the fuzzy project network. The length of the longest path of the entire project network is the project duration.

2.2. Algorithm of Fuzzy TOPSIS Method

Step 1: construct the fuzzy project network.

Step 2: Assign the linguistic variable in the form of pentagonal fuzzy number. Choose the appropriate linguistic Variable for the importance weight of criteria and assign the linguistic variables for each activity.

Step 3: Compute the aggregated fuzzy weight with respect to each criteria and aggregated fuzzy rating for each activity.

Step 4: Construct the fuzzy decision matrix and normalized fuzzy decision matrix.

Step 5: Construct the weighted normalized fuzzy decision matrix.

Step 6: From the weighted normalized fuzzy decision matrix, compute FPIS and FNIS.

Step 7: Calculate the distance of each path from FPIS and FNIS by using the distance formula of pentagonal fuzzy number.

Step 8: Calculate the closeness coefficient of each path.

Step 9: According to the closeness coefficient, the ranking order of all the alternatives could be determined.

Example 2.1. *Consider the following fuzzy project network, whose activities are represented as pentagonal fuzzy number. In the fuzzy project network we have eight nodes and five paths. This example presents the scheduling of construction project by means of project Network. Our aim is to find the best path that is the required Critical path for the construction project. Here we have five alternatives (path) and three criteria that is, time, cost and quality respectively.*

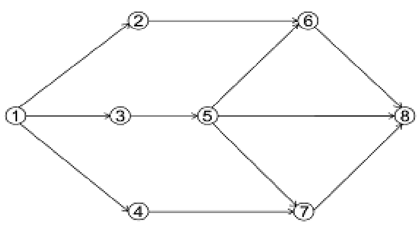

Figure 1: Fuzzy project network

The following table describes the activities and their description related to construction project

Activity	Activity Description
1-2	Lay the foundation
1-3	Put up the rough wall
1-4	Put up the roof
2-6	Install the exterior plumbing
3-5	Install the interior plumbing
4-7	Put up the exterior siding
5-6	Exterior painting
5-7	Electrical work
5-8	Exterior painting
6-8	Install the flooring
7-8	Interior painting

Table 1: Activity list for the reliable construction project

Linguistic variables	Fuzzy number
Very Low	(0,0,0,0,1)
Low	(0,0,1,2,3)
Medium Low	(1, 2,3,4,5)
Medium	(3,4,,5,6,7)
Medium High	(5,6,7,8,9)
High	(7,8,9,10,10)
Very High	(9,910,10,10)

Table 2: Linguistic variables

Criteria	D_1	D_2	D_3
Time	ML	M	VH
Cost	M	MH	M
Quality	H	ML	M

Table 3: Importance weight of criteria

Activity	Time			Cost			Quality		
	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
1-2	ML	M	M	M	ML	MH	M	M	MH
1-3	M	MH	VH	L	M	ML	H	ML	H
1-4	M	M	H	H	M	VH	MH	H	H
2-6	H	MH	VH	ML	M	MH	L	ML	M
3-5	ML	M	MH	MH	M	M	ML	VH	H
4-7	MH	L	M	M	L	MH	M	MH	VH
5-6	L	H	M	ML	MH	H	L	M	M
5-7	M	ML	VH	ML	M	VH	MH	M	ML
5-8	MH	H	VH	ML	MH	M	MH	M	H
6-8	MH	MH	VH	M	MH	H	ML	L	M
7-8	ML	L	M	L	M	M	M	H	H

Table 4: Rating of the activity by decision makers under various criteria

Activity	Time	Cost	Quality
1-2	(1,3.3,4.3,5.3,5)	(1,4,5,6,9)	(3,4.6,5.6,6.6,9)
1-3	(3,6.3,7.3,8,10)	(0,2,3,4,7)	(1,6,7,8,10)
1-4	(3,5.3,6.3,7.3,10)	(3,7,8,8.6,10)	(5,7.3,8.3,9.3,10)
2-6	(5,7.6,8.6,9.3,10)	(1,4,5,6,9)	(0,2,3,4,7)
3-5	(1,4,5,6,9)	(3,4.6,5.6,6.6,9)	(1,6.3,7.3,8,10)
4-7	(0,3.3,4.3,5.3,9)	(0,3.3,4.3,5.3,9)	(3,6.3,7.3,8,10)
5-6	(0, 4,5,6,10)	(1,5.3,6.3,7.3,10)	(0,2.6,3.6,4.6,7)
5-7	(1,5,6,6.6,10)	(1,5,6,6.6,10)	(1,4,5,6,9)
5-8	(5,7.6,8.6,9.3,10)	(1,4,5,6,9)	(3,6,7,8,10)
6-8	(5,7,8,8.6,10)	(3,6,7,8,10)	(0,2,3,4,7)
7-8	(0,2,3,4,5)	(0,2.6,3.6,4.6,7)	(3,6.6,7.6,8.6,10)

Table 5: Converted linguistic evaluation into pentagonal fuzzy number

Paths	Time (1,5,6,6.7,10)	Cost(3,4.7,5.7,6.7,9)	Quality(1,4.7,5.7,6.7,10)
1-2-6-8	(11,17.9,20.9,23.2,25)	(5,16,17,20,28)	(3,8.6,11.6,14.6,23)
1-3-5-6-8	(9,21.3,25.3,28.6,39)	(7,17.9,21.9,25.9,36)	(2,16.9,20.9,24.6,34)
1-3-5-8	(9,17.9,20.9,23.3,29)	(4,10.6,13.6,16.6,25)	(5,18.3,21.3,24,30)
1-3-5-7-8	(5,17.3,21.3,24.6,34)	(4,14.2,18.2,21.8,33)	(6,22.9,26.9,30.6,39)
1-4-7-8	(3,10.6,13.6,16.6,24)	(3,12.9,15.9,18.5,26)	(11,20.2,23.2,25.9,30)

Table 6: Fuzzy-decision matrix, Fuzzy weight of criteria

Paths	Time (1,5,6,6.7,10)	Cost (3,4.7,5.7,6.7,9)	Quality(1,4.7,5.7,6.7,10)
1-2-6-8	(0.12,0.13,0.14,0.17,0.27)	(0.11,0.15,0.18,0.19,0.6)	(0.08,0.14,0.17,0.2,0.67)
1-3-5-6-8	(0.08,0.1,0.12,0.14,0.3)	(0.08,0.12,0.14,0.17,0.4)	(0.06,0.08,0.09,0.1,1)
1-3-5-8	(0.1,0.13,0.14,0.17,0.3)	(0.12,0.18,0.22,0.28,0.75)	(0.06,0.08,0.09,0.1,0.4)
1-3-5-7-8	(0.09,0.12,0.14,0.17,0.6)	(0.09,0.14,0.16,0.2,0.75)	(0.05,0.06,0.07,0.09,0.3)
1-4-7-8	(0.13,0.18,0.2,0.3,1)	(0.12,0.16,0.19,0.2,1)	(0.06,0.07,0.09,0.1,0.18)

Table 7: Normalised fuzzy decision matrix

Paths	Time	Cost	Quality
1-2-6-8	(0.12,0.65,0.8,1.14,2.7)	(0.33,0.7,1.03,1.27,5.4)	(0.08,0.66,0.97,1.3,6.7)
1-3-5-6-8	(0.08,0.5,0.7,0.9,3)	(0.24,0.6,0.8,1.14,3.6)	(0.06,0.38,0.5, 0.67,10)
1-3-5-8	(0.1,0.65,0.8,1.14,3)	(0.36,0.8,1.25,1.9,6.75)	(0.06,0.38,0.5,0.67,4)
1-3-5-7-8	(0.09,0.6,0.8,1.14,6)	(0.27,0.66,0.9,1.3,6.75)	(0.05,0.3,0.4,0.6,3)
1-4-7-8	(0.13,0.9,1.2,2,10)	(0.36,0.8,1.08,1.3,9)	(0.06,0.33,0.5,0.67,1.8)

Table 8: Weighted normalised fuzzy decision matrix

From the above table, we get the value of A^+ as (10,10,10,10,10), (9,9,9,9,9) and (10,10,10,10,10) and the value of A^- as (0.08, 0.08,0.08,0.08,0.08), (0.24,0.24,0.24,0.24,0.24) and (0.05,0.05,0.05,0.05,0.05) and by this we compute d_i^+ and d_i^- for all the five paths by using the distance formula and then using these values we calculate the relative closeness coefficient of the project network.

Paths	Time	Cost	Quality
$d(A_1, A^+)$	8.9	7.5	8.4
$d(A_2, A^+)$	9.02	7.8	8.6
$d(A_3, A^+)$	8.9	7.2	8.9
$d(A_4, A^+)$	8.5	7.43	9.2
$d(A_5, A^+)$	8.02	7.3	9.3

Table 9: Distance between paths and A^+ with respect to the criterion

Paths	Time	Cost	Quality
$d(A_1, A^-)$	1.33	2.38	3.07
$d(A_2, A^-)$	1.4	1.58	4.46
$d(A_3, A^-)$	1.44	3.05	1.8
$d(A_4, A^-)$	2.7	2.97	1.4
$d(A_5, A^-)$	4.6	3.97	0.9

Table 10: Distance between paths and A^- with respect to the criterion

Paths	d_i^+	d_i^-	$d_i^+ + d_i^-$	$cc_i = \frac{d_i^-}{d_i^+ + d_i^-}$
1-2-6-8	24.8	6.78	31.58	0.21
1-3-5-6-8	25.42	7.44	32.86	0.23
1-3-5-8	25	6.29	31.29	0.20
1-3-5-7-8	25.13	7.07	32.2	0.22
1-4-7-8	24.62	9.47	34.09	0.28

Table 11: Distances d_i^+, d_i^- and cc_i

Now, we have

$$cc_1 = 0.21$$

$$cc_2 = 0.23$$

$$cc_3 = 0.20$$

$$cc_4 = 0.22$$

$$cc_5 = 0.28$$

According to the closeness coefficient the ranking order of the five Paths are $cc_5 > cc_2 > cc_4 > cc_1 > cc_3$. From, this we could find that the path 1-4-7-8 has the maximum relative closeness coefficient. Therefore, the path 1-4-7-8 is the required fuzzy critical path for the project network. Therefore, in the construction project, the path 1-4-7-8 is said to have no delay in their activities in executing the project.

3. Conclusion

In the fuzzy TOPSIS method, we get the relative closeness coefficient for measuring each alternative and the alternative with the highest score is selected as the fuzzy critical path of the project network. In this paper, the linguistic fuzzy variables in the form of pentagonal fuzzy number could be a more precise activity times in project network. An advantage of this method is to deal with problems of qualitative as well as quantitative data. Thus, the method used in this paper, is very efficient and easy to compute the fuzzy critical path of the project network.

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