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Disease Control in Eco-epidemic Prey-Predator Model: An Algebraic Study on Alternative Food Mechanism in Predator

Research Article*

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Abstract: In this paper, the complex dynamic behavior of a discrete time non-linear mathematical prey – predator model with a disease in the prey population is analyzed. The existence, the boundedness and the stability of equilibrium points are studied algebraically. The main objective of this work is to provide a mathematical framework to study the response of a prey – predator model to a disease in the prey population and to understand the role of supplying alternative food to predator as disease controller in the eco-epidemiological system.

Keywords: Mathematical modelling, Infected prey, Alternative food, stability, eco-epidemiology.

1. Introduction

Mathematical modelling allows us to identify the key parameters that determine the rich dynamics of ecological systems. In the development of quantitative theory for prey-predator interactions, mathematical and experimental ecology are both important. Predator-prey models with disease are a major concern and are now becoming an interesting field of study known as eco-epidemiology. Epidemiology is the study of the patterns, causes and effects of health and disease conditions in defined populations. Anderson and May [1] were the first who merged the above two fields and formulated a prey-predator model where prey species were infected by some disease. In the subsequent time, many researchers have proposed and studied different prey-predator models in the presence of disease [2-5, 17-18].Exploitation of biological resources and harvesting of the species is a common practice in fishery, forestry, agriculture and wild life management. The mathematical model in this area was first introduced by C.W. Clark [20].Harvesting or constant quota of harvesting has been studied by manyresearchers in prey-predator models [21-24].

Most important initiatives of the 20^{th} century in the field of applied ecology has been the control of populations of economically damaging species, particularly of agricultural weed and insect pests [6,7]. A major portion of the literature dealing with biological control aspects assumes the role of pest for the prey. There are several chemical control measures for the eradication of infectious diseases such as vaccination, treatment, isolation, insecticide etc. But there are many problems associated with their continued deployment including increasing pressure to reduce chemical use in the environment in

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general, development of pesticide resistance in many pathogens and decreasing availability of active ingredients. Hence, a non-chemical method of disease control continues to gain significance. Haque and Greenhalgh were the first who introduced the first eco-epidemic model with alternative food for the predator [8]. The consequences of providing a predator with alternative food and the corresponding effects on the prey-predator dynamics and its utility in biological control have been an interesting topic of study for many researchers, due to its eco-friendly nature [9-11].Sahoo et.al proposed a food chain model with seasonal effects on additional food and discussed the extinction criteria of species in a system depending on the interaction functions and supply of the quantity of additional food [12-13, 19].

The main objective of this paper is to investigate the role of supplying alternative food to the predators for controlling disease in an epidemic model. The paper is structured as follows. In section 2 an epidemic model representing the dynamics of prey-predator system in presence of alternative food to predator is proposed. Section 3 contains the conditions for the boundedness of the system. The conditions for the existence of the system for various equilibria are determined in section 4. Section 5 presents the local stability analysis of various equilibria that the model exhibits. Finally, Section 6 devotes to conclusion and further research.

2. Model Formulation

A Mathematical model is proposed and analyzed to study the response of a predator – prey model to a disease in the prey population. We impose the following assumptions to formulate the mathematical model.

(1). It is assumed that a parasite is infectious and it spreads among preys. In the presence of disease the prey population consists of two sub classes, namely, the susceptible prey $X_1(T)$ and infected prey $X_2(T)$ and the density of the predator is denoted by $Y(T)$ at time T.

(2). In the presence of disease, the susceptible prey population grows according to logistic law having carrying capacity K and intrinsic birth rate a.

$$\frac{dX_1}{dT} = a\ X_1\left(1 - \frac{X_1}{K}\right)$$

(3). The Susceptible prey population becomes infected when it comes in contact with the infected prey and this contact process is assumed to follow the simple mass action kinetics with α as the rate of conversion.

(4). The infected prey is removed with death rate D_2 or by predation before the possibility of reproducing.

(5). We have considered Holling type-II functional response for the predation of susceptible prey and since infected preys are easier to catch, Holling type-I is chosen for the predation of infected prey.

(6). Predators are provided with alternative food (additional food) of constant biomass F which is distributed uniformly in the habitat.

(7). The predator population suffers loss due to death at constant rate D_3.

(8). The number of encounters per predator with the alternative food is proportional to the density of the alternative food. The proportionality constant characterizes the ability of the predator to identify the alternative food [11].

Taking into account the aforementioned considerations, an epidemic mathematical model isformulated as follows:

$$\begin{aligned}
\frac{dX_1}{dT} &= aX_1\left(1-\frac{X_1}{K}\right) - \alpha X_1X_2 - \frac{P_1X_1Y}{(S+X_1+\lambda\mu F)} - D_1X_1 \\
\frac{dX_2}{dT} &= \alpha X_1X_2 - \frac{P_2X_2Y}{S+\lambda\mu F} - D_2X_2 \\
\frac{dY}{dT} &= \frac{P_1C_1\left(X_1+\mu F\right)Y}{S+\lambda\mu F+X_1} + \frac{P_2C_2X_2Y}{S+\lambda\mu F} - D_3Y
\end{aligned} \tag{1}$$

With the initial conditions $X_1(0)$, $X_2(0)$and $Y(0) > 0$ and parameters are all positive. Model parameters are described below.

Parameters	Biological Description
a	Logistic growth of susceptible prey
K	Environmental carrying capacity
α	Rate of transformation from infected prey to susceptible prey
β	Rate of transformation from infected prey to susceptible prey
P_1	Predation rate on S. prey
P_2	Predation rate on I. prey
C_1	Conversion efficiency on susceptible prey
C_2	Conversion efficiency on infected prey
S	Half saturation constant
D_1	Natural death rate of Susceptible Prey
D_2	Natural death rate of Infected Prey
D_3	Natural death rate of predator
h_1	Handling time of the predator per prey
h_2	Handling time of the predator per unit quantity of alternative food
$\lambda = \frac{h_1}{h_2}$	Quality of the alternative food
a_1	Ability of the predator to detect the prey item
a_2	Ability of the predator to detect the alternative food
$\mu F = \left(\frac{a_1}{a_2}\right) F$	Quantity of the alternative food supplied to predator

To reduce the number of parameters and to determine which combinations of parameters control the behavior of the system, we non-dimensionalize the system (1) using $x_1 = \frac{X_1}{K}$, $x_2 = \frac{X_2}{K}$, $y = \frac{Y}{K}$ and $t = aT$.

$$\begin{aligned}
\frac{dx_1}{dt} &= x_1\left(1-x_1\right) - \beta x_1x_2 - \frac{qx_1y}{1+ux_1+\lambda v} - d_1x_1 \\
\frac{dx_2}{dt} &= \beta x_1x_2 - \frac{rx_2y}{1+vF} - d_2x_2 \\
\frac{dy}{dt} &= \frac{\gamma_1\left(x_1+e_1v\right)y}{(1+\lambda v+e_2x_1)} + \frac{\gamma_2x_2y}{1+\lambda v} - d_3y
\end{aligned} \tag{2}$$

Where $\beta = \frac{\alpha K}{a}$, $q = \frac{P_1K}{as}$, $r = \frac{P_2K}{as}$, $d_1 = \frac{D_1}{a}$, $u = \frac{K}{S}$, $v = \frac{\mu F}{S}$, $\gamma_1 = \frac{P_1C_1K}{as}$, $e_1 = \frac{S}{K}$, $e_2 = \frac{K}{S}$, $\gamma_2 = \frac{P_2C_2K}{as}$, $d_3 = \frac{D_3}{a}$ and $x_1(t) \geq 0$, $x_2(t) \geq 0$, $y(t) \geq 0$ and $0 < \gamma_1 < q$, $0 < \gamma_2 < r$, $0 < d_3 < d_2$ and $0 < e_1 < 1$.

3. Boundedness of the System

For the system to be biologically valid and well behaved in a theoretical eco-epidemiology, all its solution must be within a certain region of confinement. This will only happen if the following theorem is satisfied.

Theorem 3.1. *All the solutions of the system* (2) *are uniformly bounded within* R_+^3.

Proof. Let $\{x_1(t), x_2(t), y(t)\}$ be any solution of the system (2). Define a positive definite function W as $W = x_1 + x_2 + y$. From (2), $\frac{d}{dt}(x_1 + x_2 + y) \le x_1(1 - x_1) - d_1x_1 - d_2x_2 - d_3y$. For arbitrarily chosen$\eta$, this simplifies to $\frac{d}{dt}(W) + \eta W \le x_1(1 - x_1 - \eta)$. Applying the theorem of differential inequalities, the above equation has the solution $W \le \frac{x_1}{\eta}(1 - x_1 - \eta)(1 - e^{-\eta t})$. As $t \longrightarrow \infty$, $W \le \frac{x_1}{\eta}(1 - x_1 - \eta)$. This implies that the solution is bounded for $0 \le W \le \frac{x_1}{\eta}(1 - x_1 - \eta)$. This shows that all the solutions of the system (2) in R_+^3 are uniformly bounded in the region $\Gamma = \{(x_1, x_2, y) \in R_+^3 : W \le \frac{x_1}{\eta}(1 - x_1 - \eta) + \epsilon\}$ for all $\epsilon > 0$ and $t \longrightarrow \infty$. This shows that we can sufficiently study the dynamics of the system (2) within Γ and hence consider the system (2) to epidemiologically and mathematically well-formed within Γ. □

4. Existence of Equilibrium States of the System (2)

In this section, the conditions for the existence of all possible equilibrium points of the system (2) are discussed. It is easy to check that the system (2) possesses the following equilibrium points.

a). The trivial equilibrium point $E_0(0, 0, 0)$ always exists.

b). The axial equilibrium point $E_1(1 - d_1, 0, 0)$ always exists.

c). The disease free boundary equilibrium point $E_2(\overline{x_1}, 0, \overline{y})$ where $\overline{x_1} = \frac{d_3 + \lambda v d_3 - \gamma_1 e_1 v}{\gamma_1 - d_3 e_2}$ and $\overline{y} = \frac{(1-d_1) + (u - 1 - \gamma v)\overline{x} - u\overline{x}^2}{q}$ exists only when $\overline{x_1} < 1$, $\gamma_1 > d_3e_2$ and $(1 - d_1) + (u - 1 - \lambda v)\overline{x_1} - u\overline{x_1}^2 > 0$.

d). The predator free boundary equilibrium point $E_3(\frac{d_2}{\beta}, \frac{1}{\beta^2}(\beta - d_2 - \beta d_1), 0)$ exists if $\beta > \frac{d_2}{1-d_1}$ and $d_1 < 1$.

e). The endemic equilibrium point $E_4(x_1^*, x_2^*, y^*)$ where $y^* = \frac{g_2}{q}(\beta x_1^* - d_2)$, $x_2^* = \frac{g_1}{\beta}\left(d_3 - \frac{\gamma_1(x_1^* + e_1 v)}{(1 + \lambda v + e_2 x_1^*)}\right)$ and x_1^* is the positive root of the equation $Q_1{x_1^*}^3 + Q_2{x_1^*}^2 + Q_3x_1^* + Q_4 = 0$ where $Q_1 = e_2u$, $Q_2 = e_2(1 - u + ug_1d_3 + \lambda v + \beta g_2 + ud_1) + u(1 + \lambda v + \gamma_1 g_1)$, $Q_3 = 1 + e_2(g_1d_3 - 1 - \lambda v + g_1d_3\lambda v - d_2g_2 + d_1 + \lambda v d_1) + \gamma v(2 - u + ug_1d_3 + 1 + \lambda v - \gamma_1 g_1 + \beta g_2 + ud_1) + u(g_1d_3 - 1 - \gamma_1 g_1 e_1 v + d_1) - \gamma_1 g_1 + \beta g_2$, $Q_4 = \lambda v(2g_1d_3 - 2 - \lambda v + \lambda v g_1 d_3 - \gamma_1 g_1 e_1 v - d_2 g_2 + 2d_1 + \lambda v d_1) + g_1d_3 - \gamma_1 g_1 e_1 v - d_2g_2 + d_1 - 1$ exists only when $\frac{d_2}{\beta}x_1^* < \frac{d_3 + \lambda v d_3 - \gamma_1 e_1 v}{\gamma_1 - d_3 e_2}$.

Therefore, the existence conditions of the equilibrium points E_2, E_3 and E_4 depend on the parameters λ and v.

5. Stability Analysis

In this section, we obtain the sufficient conditions of local asymptotically stable for each equilibrium point.The conditions for the stability of E_1 and E_2 are derived by using the next generation matrix approach introduced in [14] and the conditions for the stability of E_0, E_3 and E_4 are obtained by applying linearization approach.

5.1. The Next Generation Matrix Method

The non-linear vector function $f(x_1, x_2, y)$ for the system (2) is $f = \mathcal{F} - \mathcal{V}$ where the matrix $\mathcal{F}$ represents the transmission matrix and $\mathcal{V}$ represents the transition matrix. The transmission constitutes all epidemiological events that involve new infection and all other events are incorporated in $\mathcal{V}$. Hence we have $\mathcal{F} = \begin{bmatrix} 0 \\ \beta x_1 x_2 \\ 0 \end{bmatrix}$ and $\mathcal{V} = \begin{bmatrix} x_1^2 - x_1 + \frac{q x_1 y}{1+u x_1+\lambda v} + d_1 x_1 \\ \frac{r x_2 y}{1+vF} + d_2 x_2 \\ \frac{-\gamma_1 (x_1+e_1 v) y}{1+\lambda v+e_2 x_1} - \frac{\gamma_2 x_2 y}{1+\lambda v} + d_3 y \end{bmatrix}$.

The Jacobian matrices of the functions $F = D\mathcal{F}$ and $v = D\mathcal{V}$ are obtained as below. $F = [f_{ij}]_{3\times 3} = \begin{bmatrix} 0 & \beta x_2 & 0 \\ 0 & \beta x_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and

$$V = [v_{ij}]_{3\times 3} = \begin{bmatrix} 2x_1 - 1 + \frac{qy}{1+ux_1+\lambda v} - \frac{q x_1 y u}{(1+ux_1+\lambda v)^2} + d_1 & 0 & \frac{q x_1}{1+u x_1+\lambda v} \\ 0 & \frac{ry}{1+vF} + d_2 & \frac{r x_2}{1+vF} \\ \frac{-\gamma_1 y}{1+\lambda v+e_2 x_1} + \frac{\gamma_1 x_1 y e_2}{(1+\lambda v+e_2 x_1)^2} & \frac{-\gamma_2 y}{1+\lambda v} & \frac{-\gamma_1 (x_1+e_1 v)}{1+\lambda v+e_2 x_1} - \frac{\gamma_2 x_2}{1+\lambda v} + d_3 \end{bmatrix}$$

We call, FV^{-1}, the next generation matrix for the model and set $\mathcal{R}_0 = \rho(FV^{-1})$, where $\rho(A)$ denotes the spectral radius of a matrix A. By applying the Theorem 2 in [12], if x_0 is a disease free equilibrium (DFE) of the model, then x_0 is locally asymptotically stable if $\mathcal{R}_0 < 1$, but unstable if $\mathcal{R}_0 > 1$.

Proposition 5.1. *Let $R_{01} = \frac{\beta(1-d_1)}{d_2}$. The equilibrium point E_1 of system (2) is locally asymptotically stable if $R_{01} < 1$, otherwise,unstable.*

Proof. The Jacobian matrices F and V for the equilibrium E_1 are as follows.

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \beta(1-d_1) & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } V = \begin{bmatrix} 2\beta(1-d_1) - 1 + d_1 & 0 & \frac{q\beta(1-d_1)}{1+u\beta(1-d_1)+\gamma v} \\ 0 & d_2 & 0 \\ 0 & 0 & \frac{-\gamma_1(\beta(1-d_1)+e_1 v)}{1+\lambda v+e_1\beta(1-d_1)} + d_3 \end{bmatrix}$$

So, $R_{01} = \rho\left(FV^{-1}\right) = \frac{f_{22}}{v_{22}} = \frac{\beta(1-d_1)}{d_2}$. Thus, if $R_{01} < 1$, the equilibrium point E_1 is locally asymptotically stable. Otherwise, E_1 is unstable. □

Proof. Let $R_{02} = \frac{(1+vF)\beta\overline{x}}{(1+vF)d_2+r\overline{y}}$. The equilibrium point E_2 of system (2) is locally asymptotically stable if $R_{02} < 1$, otherwise, unstable. □

Proof. The Jacobian matrices F and V for the equilibrium E_2 are as follows.

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \beta\left(\frac{d_3+\lambda v d_3-\gamma_1 e_1 v}{\gamma_1 - d_3 e_2}\right) & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } V = \begin{bmatrix} 2\overline{x} - 1 + \frac{q\overline{y}}{1+u\overline{x}+\lambda v} - \frac{q\overline{x}\overline{y}u}{(1+u\overline{x}+\lambda v)^2} + d_1 & 0 & \frac{q\overline{x}}{1+u\overline{x}+\lambda v} \\ 0 & \frac{r\overline{y}}{1+vF} + d_2 & 0 \\ \frac{-_1\overline{y}}{1+\lambda v+e_2\overline{x}} + \frac{\gamma_1\overline{x}\overline{y}e_2}{(1+\lambda v+e_2\overline{x})^2} & \frac{-\gamma_2\overline{y}}{1+\lambda v} & \frac{-\gamma_1(\overline{x}+e_1 v)}{1+\lambda v+e_2\overline{x}} + d_3 \end{bmatrix}$$

So, $R_{02} = \rho\left(FV^{-1}\right) = \frac{f_{22}}{v_{22}} = \frac{(1+vF)\overline{x}}{(1+vF)d_2+r\overline{y}}$. Thus, if $R_{02} < 1$, the equilibrium point E_2 is locally asymptotically stable. Otherwise, E_2 is unstable. □

5.2. Linear Stability Analysis

The Jacobian matrix of the system (2) at state variable is given by

$$J = \begin{bmatrix} 1 - 2x_1 - \beta x_2 - \frac{qy}{1+ux_1+\lambda v} + \frac{quyx_1}{(1+ux_1+\lambda v)^2} - d_1 & \beta x_2 & \frac{\gamma_1 y}{1+\lambda v+e_2 x_1} - \frac{\gamma_1 x_1 y e_2}{(1+\lambda v+e_2 x_1)^2} - d_1 \\ -\beta x_1 & \beta x_1 - \frac{ry}{1+vF} - d_2 & \frac{\gamma_2 y}{1+\lambda v} \\ \frac{-q x_1}{1+u x_1+\lambda v} & \frac{-r x_2}{1+vF} & \frac{\gamma_1(x_1+e_1 v)}{1+\lambda v+e_2 x_1} + \frac{\gamma_2 x_2}{1+\lambda v} - d_3 \end{bmatrix}$$

The linearized stability technique for analyzing the local behavior of the non-linear system (2) is given in the following theorem.

Theorem 5.2. *Let $p(\lambda) = \lambda^3 + B\lambda^2 + C + D$. There are atmost three roots of the equation $p(\lambda) = 0$. Then the following statements are true:*

(a). If every root of the equation has absolute value less than one, then the fixed point of the system is locally asymptotically stable and fixed point is called a sink.

(b). If at-least one of the roots of the equation has absolute value greater than one, then the fixed point of the system is unstable and fixed point is called saddle.

(c). If every root of the equation has absolute value greater than one, then the system is source.

(d). The fixed point of the system is called hyperbolic if no root of the equation has absolute value equal to one. If there exists a root of the equation with absolute value equal to one, then the fixed point is called Non-hyperbolic [15].

- **Stability of Equilibrium E_0**: The Jacobian matrix $J(E_0)$ at the equilibrium point E_0 is given as follows.

$$J(E_0) = \begin{bmatrix} 1-d_1 & 0 & 0 \\ \beta & -d_2 & 0 \\ 0 & 0 & \frac{\gamma_1 e_1 v}{1+\lambda v} - d_3 \end{bmatrix}$$

The Eigen values are $\lambda_1 = 1 - d_1$, $\lambda_2 = -d_2$ and $\lambda_3 = \frac{\gamma_1 e_1 v}{1+\lambda v} - d_3$. Thus, the trivial equilibrium point E_0 of system (2) is locally asymptotically stable if $d_1 > 1$ and $\gamma_1 e_1 v < d_3(1 + \lambda v)$ otherwise, E_0 is unstable. We summarize the result in the following proposition.

Proposition 5.3. *The trivial equilibrium point E_0 of system (2) is locally asymptotically stable if $d_1 > 1$ and $\gamma_1 e_1 v < d_3(1 + \lambda v)$ otherwise, E_0 is unstable.*

- **Dynamical behavior of the system (2) around the equilibrium point E_3**: The Jacobian matrix $J(E_3)$ at the equilibrium point E_3 is given as follows.

$$J(E_3) = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Where $b_{11} = \frac{-d_2}{\beta}$, $b_{12} = 1 - \frac{d_2}{\beta} - d_1$, $b_{13} = 0$, $b_{21} = -d_2$, $b_{22} = -2d_2$, $b_{23} = 0$, $b_{31} = \frac{-qd_2}{\beta+ud_2+\lambda v\beta}$, $b_{32} = \frac{-r(\beta-d_2-\beta d_1)}{\beta^2(1+vF)}$, $b_{33} = \frac{\gamma_1(d_2+e_1\beta v)}{\beta+\beta\lambda v+e_2d_2} + \frac{\gamma_2(\beta-d_2-\beta d_1)}{\beta^2(1+\lambda v)} - d_3$. The Eigen values are $\lambda_{1,2} = \frac{R_1 \pm \sqrt{R_1^2-4R_2}}{2}$, where $R_1 = b_{11} + b_{22}$ and $R_2 = b_{11}b_{22} - b_{12}b_{21}$ and $\lambda_3 = b_{33}$. By Theorem 5.2, E_3 is locally asymptotically stable if and only if $R_1 + \sqrt{R_1^2 - 4R_2} < 2$, $R_1 - \sqrt{R_1^2 - 4R_2} < 2$ and $\beta^2\gamma_1(d_2 + e_1\beta v)(1 + \lambda v) + \gamma_2(\beta - d_2 - \beta d_1)(\beta + \lambda v\beta + e_2d_2) < 1 + d_3\beta^2(1 + \lambda v)[\beta(1 + \lambda v) + e_2d_2]$.

- **Local Stability of the system (2) around the interior equilibrium point $\mathbf{E}_4$**: The Jacobian matrix of system (2) at the equilibrium point $E_4(x_1^*,\ x_2^*,\ y^*)$ is given below.

$$J(E_4) = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix}$$

Where $H_{11} = 1 - 2x_1^* - \beta x_2^* - \frac{qy^*}{1+ux_1^*+\lambda v} + \frac{quy^*x_1}{(1+ux_1^*+\lambda v)^2} - d_1$, $H_{12} = \beta x_2^*$, $H_{13} = \frac{\gamma_1 y^*}{1+\lambda v+e_2x_1^*} - \frac{\gamma_1 x_1^* y^* e_2}{(1+\lambda v+e_2x_1^*)^2}$, $H_{21} = -\beta x_1^*$, $H_{22} = \beta x_1^* - \frac{ry^*}{1+vF} - d_2$, $H_{23} = \frac{\gamma_2 y^*}{1+\lambda v}$, $H_{31} = \frac{-qx_1^*}{1+ux_1^*+\lambda v}$, $H_{32} = \frac{-x_2^*}{1+vF}$ and $H_{33} = \frac{\gamma_1(x_1^*+e_1v)}{1+\lambda v+e_2x_1^*} + \frac{\gamma_2 x_2^*}{1+\lambda v} - d_3$. The characteristic equation of $J(E_4)$ is $\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0$, where $A_1 = -(H_{11} + H_{22} + H_{33})$, $A_2 = H_{11}H_{22} - H_{12}H_{21} - H_{23}H_{32} + H_{11}H_{33}$ and $A_3 = H_{11}H_{23}H_{32} - H_{11}H_{22}H_{33} + H_{12}H_{21}H_{33}$. According to the Routh-Hurwitz criterion [16], $E_4(x^*, y^*, z^*)$ is locally asymptotically stable if only $A_1 > 0$, $A_3 > 0$ and $A_1A_2 > A_3$. Thus, the sufficient conditions for the existence and the local stability of equilibria for the system (2) are summarized in the following table.

Equilibria	Existence Condition	Stability Condition
$E_0\,(0,0,0)$	Always exists	$d_1 > 1$ and $\gamma_1 e_1 v < d_3(1+\lambda v)$
$E_1(1-d_1,0,0)$	$d_1 < 1$	$R_{01} < 1$
$E_2\,(\overline{x_1},0,\overline{y})$	$\overline{x_1} < 1$, $\gamma_1 > d_3 e_2$ and $(1-d_1)+(u-1-\lambda v)\,\overline{x_1} - u\overline{x_1}^2 > 0$	$R_{02} < 1$
$E_3\left(\frac{d_2}{\beta}, \frac{1}{\beta^2}\,(\beta - d_2 - \beta d_1)\,, 0\right)$	$\beta > \frac{d_2}{1-d_1}$ and $d_1 < 1$	$R_1 + \sqrt{R_1^2 - 4R_2} < 2$, $R_1 - \sqrt{R_1^2 - 4R_2} < 2$ and $\beta^2\gamma_1\,(d_2 + e_1\beta v)\,(1+\lambda v) + \gamma_2\,(\beta - d_2 - \beta d_1)\,(\beta + \lambda v\beta + e_2 d_2) < 1 + d_3\beta^2\,(1+\lambda v)\,[\beta\,(1+\lambda v) + e_2 d_2]$
$E_4\,({x_1}^*, {x_2}^*, y^*)$	$\frac{d_2}{\beta} < x_1^* < \frac{d_3+\lambda v d_3 - \gamma_1 e_1 v}{\gamma_1 - d_3 e_2}$	$A_1 > 0$, $A_3 > 0$ and $A_1 A_2 > A_3$

Therefore, we observe that the stability conditions for every equilibrium point depend on the parameters λ and v.

6. Conclusion and Further Research

Mathematical modelling has been a great tool for understanding disease dynamics as well as disease control policies which allow us to obtain useful biological insights and enable us to make correct decision to obtain disease free system in nature. In this paper, we proposed an epidemic prey-predator model with disease in prey in presence of alternative food to predator. The conditions for the existence and the local stability of various equilibria of the system were obtained algebraically and we observe that these conditions depend on the quality and quantity of alternative food supplied to predator. Therefore, Suitable alternative food to predator has the capability to make the system disease free. This non-chemical method of disease control will be useful for the biological conservation of prey species in real world biological systems.

Analytical findings always remain incomplete without numerical verification of the results. In future, it is interesting to see the dynamical behavior of the system (2) by performing numerical simulation with variation in the infection rate β, the quality of alternative food λ and the quantity of alternative food v within specified range.

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