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Decompositions of $\tilde{g}$-continuity

Research Article

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Abstract: The aim of this paper is to give decompositions of a weaker form of continuity, namely $\tilde{g}$-continuity, by providing the concepts of $\tilde{g}_t$-sets, $\tilde{g}_\alpha*$-sets, $\tilde{g}_t$-continuity and $\tilde{g}_\alpha*$-continuity.

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1. Introduction

Levine [7], Mashhour et. al. [8] and Njastad [9] introduced the topological notions of semi-open sets, preopen sets and α-open sets respectively. The concept of g-closed sets was introduced and studied by Levine [5]. As a generalization, Jafari et. al. introduced and studied the notions of $\tilde{g}$-closed sets [3] and $\tilde{g}_\alpha$-closed sets [4] in topological spaces and Ganesan et. al. [1] introduced and studied the class of $\tilde{g}_p$-closed sets in topological spaces. In 1961, Levine [6] obtained a decomposition of continuity which was later improved by Rose [13]. Tong [15] decomposed continuity into α-continuity and A-continuity and showed that his decomposition is independent of Levine's. Hatir et. al. [2] also obtained a decomposition of continuity. Ravi et. al. [12] obtained decomposition of α-continuity and $\tilde{g}_\alpha$-continuity. In this paper we introduce $\tilde{g}_t$-continuity and $\tilde{g}_\alpha*$-continuity to obtain decompositions of $\tilde{g}$-continuity in topological spaces.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (simply, X and Y) denote topological spaces on which no separation axioms are assumed. Let A be a subset of a space X. The closure of A and the interior of A are denoted by cl(A) and int(A), respectively. The following definitions, Remarks, Proposition and Theorems are useful in the sequel.

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Definition 2.1. *A subset A of a topological space (X, τ) is said to be semi-open [7] (resp. preopen [8], α-open [9]) if $A \subseteq cl(int(A))$ (resp. $A \subseteq int(cl(A))$, $A \subseteq int(cl(int(A)))$). The complement of semi-open (resp. preopen, α-open) set is called semi-closed (resp. preclosed, α-closed) set.*

Definition 2.2. *A subset A of a topological space (X, τ) is said to be*

(1) a t-set [16] if $int(A) = int(cl(A))$.

(2) an α^*-set [2] if $int(A) = int(cl(int(A)))$.

Remark 2.3 ([2])**.**

(1) Every t-set is an α^*-set, but not conversely.

(2) An open set need not be an α^*-set.

(3) The union of two α^*-sets need not be an α^*-set.

(4) Arbitrary intersection of α^*-sets is an α^*-set.

Definition 2.4. *A subset A of a topological space (X, τ) is called*

(1) a g-closed [5] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X.

(2) a $\hat{g}$-closed [17] or ω-closed [14] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in X.

The complement of a g-closed (resp. $\hat{g}$-closed) set is called g-open (resp. $\hat{g}$-open). For a subset A of a topological space X, the α-closure (resp. semi-closure, pre-closure) of A, denoted by $\alpha cl(A)$ (resp. $scl(A)$, $pcl(A)$), is the intersection of all α-closed (resp. semi-closed, preclosed) subsets of X containing A. Dually, the α-interior (resp. semi-interior, pre-interior) of A, denoted by $\alpha int(A)$ (resp. $sint(A)$, $pint(A)$), is the union of all α-open (resp. semi-open, preopen) subsets of X contained in A.

Proposition 2.5 ([10])**.** *Let A and B be subsets of a topological space X. If B is an α^*-set, then $\alpha int(A \cap B) = \alpha int(A) \cap int(B)$.*

Definition 2.6. *A subset A of a topological space (X, τ) is called*

*(1) a *g-closed [18] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is $\hat{g}$-open in (X, τ). The complement of *g-closed set is *g-open.*

*(2) a $^\sharp gs$-closed [19] if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is *g-open in (X, τ). The complement of $^\sharp gs$-closed set is $^\sharp gs$-open.*

(3) a $\tilde{g}$-closed [3] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is $^\sharp gs$-open in (X, τ).

(4) an $\tilde{g}_\alpha$-closed [4] if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is $^\sharp gs$-open in (X, τ).

(5) a $\tilde{g}_p$-closed [1] if $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is $^\sharp gs$-open in (X, τ).

The complement of $\tilde{g}$-closed set (resp. $\tilde{g}_\alpha$-closed set, $\tilde{g}_p$-closed set) is $\tilde{g}$-open (resp. $\tilde{g}_\alpha$-open, $\tilde{g}_p$-open).

Remark 2.7. *The following hold in any topological space:*

(1) Every α-closed set is $\tilde{g}_\alpha$-closed, but not conversely [4].

(2) Every $\tilde{g}_\alpha$-closed set is $\tilde{g}_p$-closed, but not conversely [1].

(3) Every $\tilde{g}$-closed set is $\tilde{g}_\alpha$-closed, but not conversely [4].

(4) Every closed set is α-closed, but not conversely [4].

(5) Every closed set is $\tilde{g}$-closed, but not conversely [3].

Definition 2.8. *A subset S of a topological space (X, τ) is said to be*

(1) $^\sharp gslc^*$-set [11] if $S = U \cap F$, where U is $^\sharp gs$-open and F is closed in (X, τ).

(2) $C\eta^*$-set [12] if $S = U \cap F$, where U is $^\sharp gs$-open and F is α-closed in (X, τ).

(3) $C\eta^{**}$-set [12] if $S = U \cap F$, where U is $\tilde{g}_\alpha$-open and F is a t-set in (X, τ).

Definition 2.9. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be*

(1) α-continuous [12] if for each $V^c \in \sigma$, $f^{-1}(V)$ is an α-closed set in (X, τ).

(2) $\tilde{g}_\alpha$-continuous [12] if for each $V^c \in \sigma$, $f^{-1}(V)$ is an $\tilde{g}_\alpha$-closed set in (X, τ).

(3) $\tilde{g}$-precontinuous [12] if for each $V^c \in \sigma$, $f^{-1}(V)$ is $\tilde{g}_p$-closed set in (X, τ).

(4) $C\eta^*$-continuous [12] if for each $V \in \sigma$, $f^{-1}(V)$ is $C\eta^*$-set in (X, τ).

(5) $C\eta^{**}$-continuous [12] if for each $V \in \sigma$, $f^{-1}(V)$ is $C\eta^{**}$-set in (X, τ).

(6) $C^*\eta^*$-continuous [12] if for each $V^c \in \sigma$, $f^{-1}(V)$ is $C\eta^*$-set in (X, τ).

(7) $\tilde{g}$-continuous [12] if for each $V^c \in \sigma$, $f^{-1}(V)$ is $\tilde{g}$-closed set in (X, τ).

(8) $^\sharp GSLC^*$-continuous [11] if for each $V^c \in \sigma$, $f^{-1}(V)$ is $^\sharp gslc^*$-set in (X, τ).

Recently, the following decompositions have been established in [12].

Theorem 2.10. *A function $f : (X, \tau) \to (Y, \sigma)$ is α-continuous if and only if it is both $\tilde{g}_\alpha$-continuous and $C^*\eta^*$-continuous.*

Theorem 2.11. *A function $f : (X, \tau) \to (Y, \sigma)$ is $\tilde{g}_\alpha$-continuous if and only if it is both $\tilde{g}$-precontinuous and $C\eta^{**}$-continuous.*

3. On $\tilde{g}_t$-sets and $\tilde{g}_\alpha *$-sets

Definition 3.1. *A subset S of a topological space (X, τ) is called*

(1) $\tilde{g}_t$-set if $S = U \cap F$, where U is $\tilde{g}$-open in X and F is a t-set in X,

(2) $\tilde{g}_\alpha*$-set if $S = U \cap F$, where U is $\tilde{g}$-open in X and F is an α^*-set in X.

The family of all $\tilde{g}_t$-sets (resp. $\tilde{g}_\alpha*$-sets) in a topological space (X, τ) is denoted by $\tilde{g}_t(X, \tau)$(resp. $\tilde{g}_\alpha*(X, \tau)$).

Proposition 3.2. *Let S be a subset of a topological space (X, τ).*

(1) If S is a t-set, then $S \in \tilde{g}_t(X, \tau)$.

(2) If S is an α^*-set, then $S \in \tilde{g}_\alpha*(X, \tau)$.

(3) If S is a $\tilde{g}$-open set in X, then $S \in \tilde{g}_t(X, \tau)$ and $S \in \tilde{g}_\alpha*(X, \tau)$.

Proposition 3.3. *In a topological space X, every $\tilde{g}_t$-set is $\tilde{g}_\alpha*$-set but not conversely.*

Example 3.4. *Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{b\}, \{a, c\}, \{a, b, c\}, X\}$. In (X, τ), the set $\{a, d\}$ is $\tilde{g}_\alpha*$-set but it is not $\tilde{g}_t$-set.*

Remark 3.5. *The following examples show that*

(1) the converse of Proposition 3.2 need not be true.

(2) the concepts of $\tilde{g}_t$-sets and $\tilde{g}_p$-open sets are independent.

(3) the concepts of $\tilde{g}_\alpha*$-sets and $\tilde{g}_\alpha$-open sets are independent.

Example 3.6. *Let X and τ be as in Example 3.4. Then $\{a\}$ is $\tilde{g}_t$-set but not a t-set and the set $\{a, b, c\}$ is $\tilde{g}_\alpha*$-set but not an α^*-set.*

Example 3.7. *Let X and τ be as in Example 3.4. Then $\{d\}$ is both $\tilde{g}_t$-set and $\tilde{g}_\alpha*$-set, but it is not a $\tilde{g}$-open set.*

Example 3.8. *Let X and τ be as in Example 3.4. Then $\{d\}$ is $\tilde{g}_t$-set but not a $\tilde{g}_p$-open set. Also $\{a, b, d\}$ is a $\tilde{g}_p$-open set but not $\tilde{g}_t$-set.*

Example 3.9. *Let X and τ be as in Example 3.4. Then $\{d\}$ is $\tilde{g}_\alpha*$-set but not an $\tilde{g}_\alpha$-open set.*

Example 3.10. *Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{c\}, \{a, c\}, X\}$. In (X, τ), the set $\{b, c\}$ is an $\tilde{g}_\alpha$-open set but not $\tilde{g}_\alpha*$-set.*

Remark 3.11.

(1) The union of two $\tilde{g}_t$-sets need not be $\tilde{g}_t$-set.

(2) The union of two $\tilde{g}_\alpha*$-sets need not be $\tilde{g}_\alpha*$-set.

Example 3.12. *Let X and τ be as in Example 3.10. Then (1) $\{b\}$ and $\{c\}$ are $\tilde{g}_t$-sets but $\{b\} \cup \{c\} = \{b, c\}$ is not $\tilde{g}_t$-set. (2) $\{b\}$ and $\{c\}$ are $\tilde{g}_\alpha*$-sets but $\{b\} \cup \{c\} = \{b, c\}$ is not $\tilde{g}_\alpha*$-set.*

Lemma 3.13.

(1) A subset S of X is $\tilde{g}$-open [3] if and only if $F \subseteq int(S)$ whenever $F \subseteq S$ and F is $^\sharp gs$-closed in X.

(2) A subset S of X is $\tilde{g}_\alpha$-open [4] if and only if $F \subseteq \alpha int(S)$ whenever $F \subseteq S$ and F is $^\sharp gs$-closed in X.

(3) A subset S of X is $\tilde{g}_p$-open [1] if and only if F ⊆ pint(S) whenever F ⊆ S and F is $^\sharp gs$-closed in X.

Theorem 3.14. *A subset S of X is $\tilde{g}$-open in (X, τ) if and only if it is both $\tilde{g}_\alpha$-open and $\tilde{g}_\alpha*$-set in (X, τ).*

Proof. Necessity. The proof is obvious.

Sufficiency. Let S be an $\tilde{g}_\alpha$-open set and $\tilde{g}_\alpha*$-set. Since S is $\tilde{g}_\alpha*$-set, S = A∩B, where A is $\tilde{g}$-open and B is an α*-set. Assume that F ⊆ S, where F is $^\sharp gs$-closed in X. Since A is $\tilde{g}$-open, by Lemma 3.13(1), F ⊆ int(A). Since S is $\tilde{g}_\alpha$-open in X, by Lemma 3.13(2), F ⊆ αint(S) = S ∩ int(cl(int(S))) = (A∩B)∩int(cl(int(A∩B))) ⊆ A∩B∩int(cl(int(A)))∩int(cl(int(B))) = A∩B∩int(cl(int(A)))∩int(B) ⊆ int(B). Therefore, we obtain F ⊆ int(B) and hence F ⊆ int(A)∩int(B)= int(S). Hence S is $\tilde{g}$-open. □

Theorem 3.15. *A subset S of X is $\tilde{g}$-open in (X, τ) if and only if it is both $\tilde{g}_p$-open and $\tilde{g}_t$-set in (X, τ).*

Proof. Similar to Theorem 3.14. □

Remark 3.16. *We obtain the following diagram by the above discussions and the following Examples, where A ⟶ B (resp. A $\ddagger$ B) represents A implies B but not conversely (resp. A and B are independent of each other).*

$$\begin{array}{ccccc}
\textit{closed} & \longrightarrow & \tilde{g}\textit{-closed} & \longrightarrow & \tilde{g}_t\textit{-set} \\
 & & \downarrow & & \\
\downarrow & & \tilde{g}_\alpha*\textit{-set} & & \ddagger \\
 & & \uparrow & & \\
\alpha\textit{-closed} & \longrightarrow & \tilde{g}_\alpha\textit{-closed} & \longrightarrow & \tilde{g}_p\textit{-closed}
\end{array}$$

Example 3.17. *Let X and τ be as in Example 3.10. Then {a} is α-closed but it is neither a $\tilde{g}$-closed set nor a closed set.*

Example 3.18. *Let X and τ be as in Example 3.4. Then {a, d} is an $\tilde{g}_\alpha$-closed but not an α-closed set.*

Example 3.19. *Let X and τ be as in Example 3.10. Then {b, c} is *g-closed set but not an $\tilde{g}_\alpha$-closed set.*

Example 3.20. *Let X and τ be as in Example 3.10. Then (1) {a} is an $\tilde{g}_\alpha$-closed set but not a *g-closed set, (2) {c} is $\tilde{g}_\alpha*$-set but it is neither a $\tilde{g}$-closed nor an $\tilde{g}_\alpha$-closed set.*

Example 3.21. *Let X and τ be as in Example 3.4. Then {a} is $\tilde{g}_p$-closed set but not an $\tilde{g}_\alpha$-closed set.*

Example 3.22. *Let X and τ be as in Example 3.4. Then {a, d} is $\tilde{g}$-closed, but it is neither an α-closed nor a closed set.*

Example 3.23. *Let X and τ be as in Example 3.4. Then (1) {a} is $\tilde{g}_t$-set but not a $\tilde{g}$-closed set, (2) {a, d} is $\tilde{g}_p$-closed set but not $\tilde{g}_t$-set, (3) {b} is $\tilde{g}_t$-set but not a $\tilde{g}_p$-closed set.*

Remark 3.24. *The concepts of *g-closed sets and $\tilde{g}_\alpha$-closed sets are independent by the Examples 3.19 and 3.20.*

Remark 3.25. *The concepts of $\tilde{g}$-closed sets and α-closed sets are independent by the Examples 3.17 and 3.22.*

Proposition 3.26. *Let (X, τ) be a topological space. Then a subset A of X is closed if and only if it is both $\tilde{g}$-closed and $^\sharp gslc$*-set.*

Proof. Necessity is trivial. To prove the sufficiency, assume that A is both $\tilde{g}$-closed and $^\sharp gs$lc*-set. Then A= U∩V, where U is $^\sharp gs$-open and V is closed in X. Therefore A ⊆ U and A ⊆ V and so by hypothesis, cl(A) ⊆ U and cl(A) ⊆ V, thus cl(A) ⊆ U∩V= A and hence cl(A)= A. Therefore A is closed in X. □

Remark 3.27. *The following Example shows that the concepts of $\tilde{g}$-closed sets and $^\sharp gslc$*-sets are independent.*

Example 3.28. *Let X and τ be as in Example 3.4. Then (1) $\{a\}$ is $^\sharp gslc$*-set but not $\tilde{g}$-closed set. (2) $\{a,\ d\}$ is $\tilde{g}$-closed set but not $^\sharp gslc$*-set.*

4. Decompositions of $\tilde{g}$-continuity

Definition 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be*

(1) $\tilde{g}_t$-continuous if for each $V \in \sigma$, $f^{-1}(V) \in \tilde{g}_t(X, \tau)$.

(2) $\tilde{g}_\alpha *$-continuous if for each $V \in \sigma$, $f^{-1}(V) \in \tilde{g}_\alpha *(X, \tau)$.

Proposition 4.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following implications hold:*

(1) $\tilde{g}$-continuity $\Rightarrow$ $\tilde{g}_t$-continuity;

(2) $\tilde{g}$-continuity $\Rightarrow$ $\tilde{g}_\alpha *$-continuity;

(3) $\tilde{g}$-continuity $\Rightarrow$ $\tilde{g}_\alpha$-continuity $\Rightarrow$ $\tilde{g}$-precontinuity.

The reverse implications in Proposition 4.2 are not true as shown in the following Examples.

Example 4.3. *Let $X = Y = \{a,\ b,\ c,\ d\}$, $\tau = \{\emptyset,\ \{b\},\ \{a,\ c\},\ \{a,\ b,\ c\},\ X\}$ and $\sigma = \{\emptyset,\ \{d\},\ \{b,\ d\},\ \{a,\ c,\ d\},\ Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\tilde{g}_t$-continuous function. However, f is neither $\tilde{g}$-continuous nor $\tilde{g}$-precontinuous.*

Example 4.4. *Let $X = Y = \{a,\ b,\ c,\ d\}$, $\tau = \{\emptyset,\ \{b\},\ \{a,\ c\},\ \{a,\ b,\ c\},\ X\}$ and $\sigma = \{\emptyset,\ \{c,\ d\},\ \{a,\ c,\ d\},\ \{b,\ c,\ d\},\ Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\tilde{g}_\alpha *$-continuous function. However, f is neither $\tilde{g}$-continuous nor $\tilde{g}_\alpha$-continuous.*

Example 4.5. *Example (4.3) and the following Example (4.6) show that $\tilde{g}_t$-continuity and $\tilde{g}$-precontinuity are independent.*

Example 4.6. *Let $X = Y = \{a,\ b,\ c,\ d\}$, $\tau = \{\emptyset,\ \{b\},\ \{a,\ c\},\ \{a,\ b,\ c\},\ X\}$ and $\sigma = \{\emptyset,\ \{a\},\ \{a,\ b,\ d\},\ Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\tilde{g}$-precontinuous function but it is not $\tilde{g}_t$-continuous.*

Example 4.7. *Let $X = Y = \{a,\ b,\ c,\ d\}$, $\tau = \{\emptyset,\ \{b\},\ \{a,\ c\},\ \{a,\ b,\ c\},\ X\}$ and $\sigma = \{\emptyset,\ \{b,\ c\},\ \{a,\ b,\ c\},\ \{b,\ c,\ d\},\ Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\tilde{g}$-precontinuous function but not an $\tilde{g}_\alpha$-continuous.*

Example 4.8. *Let $X = Y = \{a,\ b,\ c,\ d\}$, $\tau = \{\emptyset,\ \{b\},\ \{a,\ c\},\ \{a,\ b,\ c\},\ X\}$ and $\sigma = \{\emptyset,\ \{d\},\ \{a,\ d\},\ \{c,\ d\},\ \{a,\ c,\ d\},\ Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\tilde{g}_\alpha *$-continuous function but not a $\tilde{g}_t$-continuous.*

Example 4.9. *Let $X = Y = \{a,\ b,\ c\}$, $\tau = \{\emptyset,\ \{c\},\ \{a,\ c\},\ X\}$ and $\sigma = \{\emptyset,\ \{b,\ c\},\ Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\tilde{g}$-precontinuous function but not a $\tilde{g}$-continuous.*

Example 4.10. *Let X, Y, τ, σ and f be as in Example 4.9. Then f is $\tilde{g}_\alpha$-continuous function but not a $\tilde{g}$-continuous.*

Remark 4.11. *By the above discussions, we obtain the following diagram, where $A \longrightarrow B$ (resp. $A \ddagger B$) represents A implies B but not conversely (resp. A and B are independent of each other).*

$$
\begin{array}{ccccc}
\tilde{g}_\alpha\text{-continuity} & & \longrightarrow & & \tilde{g}\text{-precontinuity} \\
 & \nwarrow & & \nearrow & \\
\downarrow & & \tilde{g}\text{-continuity} & & \ddagger \\
 & \swarrow & & \searrow & \\
\tilde{g}_\alpha*\text{-continuity} & & \longleftarrow & & \tilde{g}_t\text{-continuity}
\end{array}
$$

Theorem 4.12. *A function $f : (X, \tau) \to (Y, \sigma)$ is $\tilde{g}$-continuous if and only if it is both $\tilde{g}_\alpha$-continuous and $\tilde{g}_\alpha*$-continuous.*

Proof. The proof follows immediately from Theorem 3.14. □

Theorem 4.13. *A function $f : (X, \tau) \to (Y, \sigma)$ is $\tilde{g}$-continuous if and only if it is both $\tilde{g}$-precontinuous and $\tilde{g}_t$-continuous.*

Proof. From Theorem 3.15, the proof is immediate. □

Corollary 4.14. *A function $f : (X, \tau) \to (Y, \sigma)$ is $\tilde{g}$-continuous if and only if it is $\tilde{g}$-precontinuous, $C\eta$**-continuous and $\tilde{g}_\alpha*$-continuous.*

Proof. It follows from Theorems 2.11 and 4.12. □

Remark 4.15. *The following Examples show that the concepts of $\tilde{g}$-continuity and $^\sharp GSLC$*-continuity are independent of each other.*

Example 4.16. *Let $X = Y= \{a, b, c, d\}$, $\tau = \{\emptyset, \{b\}, \{a, c\}, \{a, b, c\}, X\}$ and $\sigma = \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\tilde{g}$-continuous function but not an $^\sharp GSLC$*-continuous.*

Example 4.17. *Let X, Y, τ, σ and f be as in Example 4.4. Then f is $^\sharp GSLC$*-continuous function but not a $\tilde{g}$-continuous.*

Theorem 4.18. *A function $f : (X, \tau) \to (Y, \sigma)$ is continuous if and only if it is both $\tilde{g}$-continuous and $^\sharp GSLC$*-continuous.*

Proof. It follows from Proposition 3.26. □

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