International Journal of Mathematics And Its Applications
Vol.1 No.1 (2013), pp.13-19.
ISSN: 2347-1557(online)

Balance An Unbalanced Transportation Problem By A Heuristic approach

Nigus Girmay* **and Tripti Sharma**[†,1]

*Department of Mathematics, Semera University, Ethiopia.
nigking03@gmail.com.
†Department of Applied Mathematics, Natural Science College,
Arba Minch University, Ethiopia.
bhatt_tripti@rediffmail.com

Abstract : The objective of this paper is to analyze Vogels Approximation Method and its modification due to Shimshak [2] and Goyal [1] in finding an initial solution to an unbalanced transportation problem. This paper suggests a heuristic approach in order to balance the unbalanced transportation problem and improve the Vogels Approximation Method in order to get improved (sometimes) initial solution of unbalanced transportation problem in comparison to usual VAM. The algorithm is supported by numerical illustrations.

Keywords : Transportation Problem, Vogels Approximation Method, Unbalanced Transportation Problem.

---

1 Introduction

In 1941, Hitchcock [4] originally developed the basic transportation problem. In 1953, Charnes et al. [5] developed the stepping stone method which provided an alternative way of determining the simplex method information. In 1963, Dantzig [6] used the simplex method to the transportation problems as

[1]Corresponding author E-Mail: bhatt_tripti@rediffmail.com (Tripti Sharma)

the primal simplex transportation method. Till date, several researchers studied extensively to solve cost minimizing transportation problem in various ways.

A transportation problem is unbalanced if the sum of all available quantities is not equal to the sum of requirements or vice-versa. In regular approach, to balanced the unbalanced transportation problem either a dummy row or dummy column is introduced to the cost matrix such that if total availability is more than the total requirement, a dummy column (destination) is introduced with the requirement to overcome the difference between total availability and total requirement. Cost for dummy row or column is set equal to zero.

Such problem is usually solved by Vogels Approximation Method to find an initial solution. This paper suggests an algorithm which gives improved initial solution than Vogels Approximation Method.

Analysis: Goyal [1] suggested that to assume the largest unit cost of transportation to and from a dummy row or column, present in the given cost matrix rather than assuming to be zero as usual in Vogels Approximation method. He claimed that by this modification, the allocation of units to dummy row or column is automatically given least priority and in addition to this the row or column penalty costs are considered for each interaction. He justified his suggestion by comparing the solution of a numerical problem with VAM and Shimshak [2].

While Shimshak [2] suggested to ignor the penalty cost involved with the dummy row or column. So that to give least priority to the allocation of units in dummy row or column. With this suggestion Shimshak [2] obtained initial solution by Vogels Approximation Method.

Suggestion: For the more than last of the decades no intensive and fruitful approach came into being in this area. This paper suggests a heuristic method to balance an unbalanced transportation problem. In this present method no dummy row or column is needed to use in order to balance the unbalanced transportation problem. In order to find basic initial solution, it is suggested that "instead of using dummy row or column, reduce the demand or supply which one is greater than other heuristically to balance the unbalanced transportation problem". i.e; suppose if the sum of supply is x (say, the sum of demand is y (say), and $x \sim y = c$, then if the sum of supply is more reduce it heuristically by quantity c and if the sum of demand is more reduce it by quantity c so that the unbalanced T.P. will become balanced. The initial solution is to be found by VAM.

The suggested approach is explained by illustrations. The basic initial solution obtained by this approach sometimes gives lesser cost and is less laborious, economical than Vogel, Shimshak [2] and Goyal [1].

2 Formulation of Transportation Problem

Let the transportation problem consist of m origins and n destinations, where
x_{ij} = the amount of goods transported from the i^{th} origin to the j^{th} destination.
c_{ij} = the cost involved in transporting per unit product from the i^{th} origin to the j^{th} destination.
a_i = the number of units available at the i^{th} origin.

b_j = the number of units required at the j^{th} destination.
Consider the linear transportation problem as:

(P0) Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to the constraints

$a_{ij} = \sum_{j=1}^{n} x_{ij}$; for all $i \in I = (1, 2, ..., m)$
$b_{ij} = \sum_{i=1}^{m} x_{ij}$; for all $j \in J = (1, 2, ..., n)$

and $x_{ij} \geq 0$, for all $(i, j) \in I \times J$. For unbalanced T.P. $a_{ij} < b_{ij}$ or $a_{ij} > b_{ij}$.

3 Numerical Examples

Example 3.1. *Consider the following Transportation Problem*

	D_1	D_2	D_3	*Capacity*
O_1	*4*	*8*	*8*	*76*
O_2	*16*	*24*	*16*	*82*
O_3	*8*	*16*	*21*	*77*
Requirement	*72*	*102*	*41*	

Table: 3.1.1

Here $\sum a_i = 235$ and $\sum b_j = 215$
Initial solution by usual VAM:

	D_1	D_2	D_3	Dummy	Capacity
O_1		**35**	**41**		76
	4	8	8	0	
O_2		**62**		**20**	82
	16	24	16	0	
O_3	**72**	**5**			77
	8	16	24	0	
Requirement	72	102	41	20	

Table: 3.1.2

Initial transportation cost $= 35(8) + 41(8) + 62(24) + 20(0) + 72(8) + 5(16) = 2752/-$

Initial solution by present algorithm:

Let us reduce the availability of row 2 by 20.
Now apply regular Vogels Approximation method to find initial solution of the given problem.

	D_1	D_2	D_3	Availability	P_1
O_1	4	8	8	76 0	4
O_2	16	24	16	**82-20=62** 0	0
O_3	**72** 8	16	24	77 0	← 8
Requirement	72	102	41		
P_1	4	8	8		

Table: 3.1.3

	D_2	D_3	Availability	P_2
O_1	**76** 8	8	76	0
O_2	24	16	**62**	8
O_3	16	24	5	8
Requirement	102	41		
P_2	8 ↑	8		

Table: 3.1.4

	D_2	D_3	Availability	P_3
O_2	24	**41** 16	62	← 8
O_3	16	24	5	8
Requirement	26	41		
P_3	8	8		

Table: 3.1.5

	D_2	Availability	P_4
O_2	**21** 24	21	24
O_3	**5** 16	5	16
Requirement	26		
P_4	8		

Table: 3.1.6

$x_{31} = 72, x_{12} = 76, x_{23} = 41, x_{22} = 21$, and $x_{32} = 5$

Total Initial Cost $= 76(8) + 21(24) + 41(16) + 72(8) + 16(5) = 2424/-$

Example 3.2. *Consider the following Transportation Problem*

	I	*II*	*III*	*IV*	*V*	*Available*
1	*5*	*8*	*6*	*6*	*3*	*800*
2	*4*	*7*	*7*	*6*	*5*	*500*
3	*8*	*4*	*6*	*6*	*4*	*900*
Requirement	*400*	*400*	*500*	*400*	*800*	

Table: 3.2.1

Here $\sum a_i = 2200$ and $\sum b_j = 2500$

Initial solution by usual VAM:

	I	II	III	IV	V	
1	 5	 8	200 6	300 6	300 3	800
2	400 4	 7	 7	100 6	 5	500
3	 8	400 4	 6	 6	500 4	900
Dummy	 0	 0	300 0	 0	 0	300
	400	400	500	400	800	

Table: 3.2.2

Initial transportation cost $= 400(4)+400(4)+200(6)+300(0)+300(6)+100(6)+300(3)+500(4) = 9700$

Total initial cost $= 9700$/-

Initial solution by present algorithm:

Let us reduce demand at 2^{nd} and 3^{rd} columns by 150 and 150 from each.

	I	II	III	IV	V	Supply
1	5	**8**	6	6	3	800
2	4	7	7	6	5	500
3	**8**	4	6	6	4	900
Demand	400	400 − 150	500 − 150	400	800	
		=250	**=350**			

Table: 3.2.3

Now $\sum a_i = \sum b_j = 2200$

Now apply regular Vogels Approximation method to find initial solution of the given problem.

	I	II	III	IV	V	Available	P_1
1	5	8	6	6	3	800	2
2	4	7	7	6	5	500	1
3	8	250 4	6	6	4	900	0
Requirement	400	250	350	400	800		
P_1	1	3 ↑	0	0	1		

Table: 3.2.4

	I	III	IV	V	Available	P_2
1	5	6	6	800 3	800	2 ←
2	4	7	6	5	500	1
3	8	6	6	4	650	2
Requirement	400	350	400	800		
P_2	1	0	0	1		

Table: 3.2.5

	I	III	IV	Available	P_3
2	400 4	7	6	500	1
3	8	6	6	900	2
Requirement	400	350	400		
P_3	4 ↑	1	0		

Table: 3.2.6

	III	IV	Available	P_4
2	7	100 6	100	← 1
3	6	6	900	0
Requirement	350	400		
P_4	1	0		

Table: 3.2.7

	III	IV	Available
3	350 6	300 6	650
Requirement	350	300	

Table: 3.2.8

$x_{32} = 250, x_{15} = 800, x_{21} = 400, x_{24} = 100, x_{33} = 350$ and $x_{34} = 300$.
Total Initial Cost = 9500/-

References

[1] S.K. Goyal, *Improving Vam for unbalanced transportion problem*, J. Ope. Res. Soc., 35(12)(1984), 113–114.

[2] D.G. Shimshak, J.A. Kashlik and T.D. Barclay, *A modification of vogels approximation method through the use of heuristics*, Can. J. Opl. Inf. Processing 19(1981), 259–263.

[3] N.V. Reinfeld and W.R. Vogel, *Mathematical Programming*, Prentice-Hall, Englewood Cliffs, New Jersey, (1958).

[4] F. L. Hitchcock, *The distribution of a product from several resources to numerous localities*, J. Math. Phy., 20(1941), 224–230.

[5] A. Charnes, W. W. Cooper and A. Henderson, *OAn introduction to Linear Programming*, Wiley, New Work, (1953).

[6] G.B. Dantzig, *Linear Programming and Extensions*, Princeton University Press, NJ, (1963).