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$\star$-$A_{\mathcal{I}}^{\star}$-sets and Decompositions of $\star$-$A_{\mathcal{I}}^{\star}$-continuity

Research Article

O.Ravi[1]*, G.Selvi[2], S.Murugesan[3] and S.Vijaya[4]

1 Department of Mathematics, P. M. Thevar College, Usilampatti, Madurai District, Tamil Nadu, India.

2 Department of Mathematics, Vickram College of Engineering, Enathi, Sivagangai District, Tamil Nadu, India.

3 Department of Mathematics, Sri S. Ramasamy Naidu Memorial College, Sattur, Tamil Nadu, India.

4 Department of Mathematics, Sethu Institute of Technology, Kariapatti, Virudhunagar District, Tamil Nadu, India.

Abstract: The aim of this paper is to introduce and study the notions of $\star$-$A_{\mathcal{I}}^{\star}$-sets and $\star$-$C_{\mathcal{I}}$-sets in ideal topological spaces. Properties of $\star$-$A_{\mathcal{I}}^{\star}$-sets and $\star$-$C_{\mathcal{I}}$-sets are investigated. Moreover, decompositions of $\star$-$A_{\mathcal{I}}^{\star}$-continuous functions via $\star$-$A_{\mathcal{I}}^{\star}$-sets and $\star$-$C_{\mathcal{I}}$-sets in ideal topological spaces are established.

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1. Introduction and Preliminaries

In this paper, $\star$-$A_{\mathcal{I}}^{\star}$-sets and $\star$-$C_{\mathcal{I}}$-sets in ideal topological spaces are introduced and studied. The relationships and properties of $\star$-$A_{\mathcal{I}}^{\star}$-sets and $\star$-$C_{\mathcal{I}}$-sets are investigated. Furthermore, decompositions of $\star$-$A_{\mathcal{I}}^{\star}$-continuous functions via $\star$-$A_{\mathcal{I}}^{\star}$-sets and $\star$-$C_{\mathcal{I}}$-sets in ideal topological spaces are provided.

Throughout this paper (X, τ), (Y, σ) (or simply X, Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, the closure and interior of A with respect to τ are denoted by cl(A) and int(A) respectively.

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies

(1) $A\in\mathcal{I}$ and $B\subseteq A \Rightarrow B\in\mathcal{I}$ and

(2) $A\in\mathcal{I}$ and $B\in\mathcal{I} \Rightarrow A\cup B\in\mathcal{I}$ [9].

If $\mathcal{I}$ is an ideal on X and $X\notin\mathcal{I}$, then $\mathcal{F} = \{X\backslash G : G\in\mathcal{I}\}$ is a filter [8]. Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X and if $\wp(X)$ is the set of all subsets of X, a set operator $(.)^{\star} : \wp(X)\rightarrow\wp(X)$, called a local function [9] of A with respect to τ and $\mathcal{I}$ is defined as follows: for $A\subseteq X$, $A^{\star}(\mathcal{I},\tau)=\{x\in X \mid U\cap A\notin\mathcal{I}$ for every $U\in\tau(x)\}$ where $\tau(x)=\{U\in\tau \mid x\in U\}$. A Kuratowski

* E-mail: siingam@yahoo.com

closure operator $cl^{\star}(.)$ for a topology $\tau^{\star}(\mathcal{I},\tau)$, called the $\star$-topology, finer than τ is defined by $cl^{\star}(A)=A\cup A^{\star}(\mathcal{I},\tau)$ [8]. When there is no chance for confusion, we will simply write $A^{\star}$ for $A^{\star}(\mathcal{I},\tau)$ and $\tau^{\star}$ for $\tau^{\star}(\mathcal{I},\tau)$. $int^{\star}(A)$ will denote the interior of A in $(X,\tau^{\star},\mathcal{I})$.

Remark 1.1 ([8]). *The $\star$-topology is generated by τ and by the filter $\digamma$. Also the family $\{H\cap G : H\in\tau,\ G\in\digamma\}$ is a basis for this topology.*

Definition 1.2. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be*

(1) pre-$\mathcal{I}$-open [1] if $A \subseteq int(cl^{\star}(A))$.

(2) semi-$\mathcal{I}$-open [7] if $A \subseteq cl^{\star}(int(A))$.

(3) α-$\mathcal{I}$-open [7] if $A \subseteq int(cl^{\star}(int(A)))$.

(4) semi*-$\mathcal{I}$-open [5, 6] if $A \subseteq cl(int^{\star}(A))$.

(5) $\star$-closed [8] if $A^{\star} \subseteq A$ or $A = cl^{\star}(A)$.

The complement of $\star$-closed set is $\star$-open.

Definition 1.3. *The complement of a pre-$\mathcal{I}$-open (resp. α-$\mathcal{I}$-open) set is called pre-$\mathcal{I}$-closed [1](resp. α-$\mathcal{I}$-closed [7]).*

Definition 1.4 ([6]). *The pre-$\mathcal{I}$-closure of a subset A of an ideal topological space $(X, \tau, \mathcal{I})$, denoted by $p_{\mathcal{I}}cl(A)$, is defined as the intersection of all pre-$\mathcal{I}$-closed sets of X containing A.*

Lemma 1.5 ([6]). *For a subset A of an ideal topological space $(X, \tau, \mathcal{I})$, $p_{\mathcal{I}}cl(A) = A \cup cl(int^{\star}(A))$.*

Definition 1.6 ([3]). *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called pre-$\mathcal{I}$-regular if A is pre-$\mathcal{I}$-open and pre-$\mathcal{I}$-closed in $(X, \tau, \mathcal{I})$.*

Definition 1.7 ([2, 3, 10]). *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called $A_{\mathcal{I}}^{\star}$-set if $A = L \cap M$, where L is an open and $M = cl(int^{\star}(M))$.*

Remark 1.8 ([4]). *In any ideal topological space, every open set is $\star$-open but not conversely.*

Definition 1.9 ([3]). *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subseteq X$. A is said to be an $C_{\mathcal{I}}^{\star}$-set if $A = L \cap M$, where L is an open and M is a pre-$\mathcal{I}$-regular set in X.*

Theorem 1.10 ([3]). *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then*

(1) Each $C_{\mathcal{I}}^{\star}$-set in X is a pre-$\mathcal{I}$-open but not conversely.

(2) Every pre-$\mathcal{I}$-open set is $C_{\mathcal{I}}^{\star}$-set but not conversely.

(3) Every pre-$\mathcal{I}$-regular set is $C_{\mathcal{I}}^{\star}$-set but not conversely.

2. $\star$-$A^{\star}_{\mathcal{I}}$-sets and $\star$-$C_{\mathcal{I}}$-sets

Definition 2.1. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be*

(1) an $\star$-$C_{\mathcal{I}}$-set if $A = L \cap M$, where L is an $\star$-open set and M is a pre-$\mathcal{I}$-closed set in X.

(2) an $\star$-$\eta_{\mathcal{I}}$-set if $A = L \cap M$, where L is an $\star$-open set and M is an α-$\mathcal{I}$-closed set in X.

(3) an $\star$-$A^{\star}_{\mathcal{I}}$-set if $A = L \cap M$, where L is an $\star$-open set and $M = cl(int^{\star}(M))$.

Remark 2.2. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subseteq X$. The following diagram holds for A.*

$$\begin{array}{ccccc} & & C^{\star}_{\mathcal{I}}\text{-set} & \longrightarrow & \star\text{-}C_{\mathcal{I}}\text{-set} \\ & & & & \uparrow \\ A^{\star}_{\mathcal{I}}\text{-set} & \longrightarrow & \star\text{-}A^{\star}_{\mathcal{I}}\text{-set} & \longrightarrow & \star\text{-}\eta_{\mathcal{I}}\text{-set} \end{array}$$

The following Examples show that these implications are not reversible in general.

Example 2.3. *Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{b\}, \{e\}, \{b, e\}, \{c, d\}, \{b, c, d\}, \{c, d, e\}, \{b, c, d, e\}, \{a, c, d, e\}, X\}$ and $\mathcal{I} = \{\emptyset, \{b\}, \{e\}, \{b, e\}\}$. Then $A = \{a\}$ is $\star$-$A^{\star}_{\mathcal{I}}$-set but not an $A^{\star}_{\mathcal{I}}$-set.*

Example 2.4. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ and $\mathcal{I} = \{\emptyset\}$. Then $A = \{c\}$ is $\star$-$\eta_{\mathcal{I}}$-set but not an $\star$-$A^{\star}_{\mathcal{I}}$-set.*

Example 2.5. *In Example 2.4, $A = \{c\}$ is $\star$-$C_{\mathcal{I}}$-set but not an $C^{\star}_{\mathcal{I}}$-set.*

Example 2.6. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then $A = \{c\}$ is $\star$-$C_{\mathcal{I}}$-set but not an $\star$-$\eta_{\mathcal{I}}$-set.*

Theorem 2.7. *For a subset A of an ideal topological space $(X, \tau, \mathcal{I})$, the following properties are equivalent.*

(1) A is an $\star$-$C_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set in X.

(2) $A = L \cap cl(int^{\star}(A))$ for an $\star$-open set L.

Proof. (1) $\Rightarrow$ (2): Suppose that A is an $\star$-$C_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set in X. Since A is $\star$-$C_{\mathcal{I}}$-set, then we have $A = L \cap M$, where L is an $\star$-open set and M is a pre-$\mathcal{I}$-closed set in X. We have $A \subseteq M$, so $cl(int^{\star}(A)) \subseteq cl(int^{\star}(M))$. Since M is a pre-$\mathcal{I}$-closed set in X, we have $cl(int^{\star}(M)) \subseteq M$. Since A is a semi*-$\mathcal{I}$-open set in X, We have $A \subseteq cl(int^{\star}(A))$. It follows that $A = A \cap cl(int^{\star}(A)) = L \cap M \cap cl(int^{\star}(A)) = L \cap cl(int^{\star}(A))$.

(2) $\Rightarrow$ (1): Let $A = L \cap cl(int^{\star}(A))$ for an $\star$-open set L. We have $A \subseteq cl(int^{\star}(A))$. It follows that A is a semi*-$\mathcal{I}$-open set in X. Since $cl(int^{\star}(A))$ is a closed set, then $cl(int^{\star}(A))$ is a pre-$\mathcal{I}$-closed set in X. Hence, A is an $\star$-$C_{\mathcal{I}}$-set in X. □

Theorem 2.8. *For a subset A of an ideal topological space $(X, \tau, \mathcal{I})$, the following properties are equivalent.*

(1) A is an $\star$-$A^{\star}_{\mathcal{I}}$-set in X.

(2) A is an $\star$-$\eta_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set in X.

(3) A is an $\star$-$C_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set in X.

Proof. (1) ⇒ (2): Suppose that A is an $\star$-$A_{\mathcal{I}}^{\star}$-set in X. It follows that A = L ∩ M, where L is an $\star$-open set and M = cl(int*(M)). This implies A = L ∩ M = L ∩ cl(int*(M)) = int*(L) ∩ cl(int*(M)) ⊆ cl(int*(L)) ∩ cl(int*(M)) ⊆ cl(int*(L ∩ int*(M)) = cl(int*(L ∩ M)) = cl(int*(A)). Thus A ⊆ cl(int*(A)) and hence A is a semi*-$\mathcal{I}$-open set in X. Moreover, Remark 2.2, A is an $\star$-$\eta_{\mathcal{I}}$-set in X.

(2) ⇒ (3): It follows from the fact that every $\star$-$\eta_{\mathcal{I}}$-set is an $\star$-$C_{\mathcal{I}}$-set in X by Remark 2.2.

(3) ⇒ (1): Suppose that A is an $\star$-$C_{\mathcal{I}}$-set and a semi*-$\mathcal{I}$-open set in X. By Theorem 2.7, A = L ∩ cl(int*(A)) for an $\star$-open set L. We have cl(int*(cl(int*(A)))) = cl(int*(A)). It follows that A is an $\star$-$A_{\mathcal{I}}^{\star}$-set in X. □

Remark 2.9.

(1) The notions of $\star$-$\eta_{\mathcal{I}}$-set and semi*-$\mathcal{I}$-open set are independent of each other.

(2) The notions of $\star$-$C_{\mathcal{I}}$-set and semi*-$\mathcal{I}$-open set are independent of each other.

Example 2.10.

(1) In Example 2.4, A = {c} is $\star$-$C_{\mathcal{I}}$-set as well as $\star$-$\eta_{\mathcal{I}}$-set but not semi*-$\mathcal{I}$-open set.

(2) In Example 2.6, A = {a, b} is a semi*-$\mathcal{I}$-open set but it is neither $\star$-$C_{\mathcal{I}}$-set nor $\star$-$\eta_{\mathcal{I}}$-set.

Definition 2.11. *A subset A of an ideal topological space (X, τ, $\mathcal{I}$) is said to be $\star$-gp$_{\mathcal{I}}$-open if N ⊆ p$_{\mathcal{I}}$int(A) whenever N ⊆ A and N is an $\star$-closed set in X where p$_{\mathcal{I}}$int(A) = A ∩ int(cl*(A)).*

Definition 2.12. *A subset A of an ideal topological space (X, τ, $\mathcal{I}$) is said to be $\star$-generalized pre-$\mathcal{I}$-closed ($\star$-gp$_{\mathcal{I}}$-closed) in X if X \ A is $\star$-gp$_{\mathcal{I}}$-open.*

Theorem 2.13. *For a subset A of an ideal topological space (X, τ, $\mathcal{I}$), A is $\star$-gp$_{\mathcal{I}}$-closed if and only if p$_{\mathcal{I}}$cl(A) ⊆ N whenever A ⊆ N and N is an $\star$-open set in (X, τ, $\mathcal{I}$).*

Proof. Let A be an $\star$-gp$_{\mathcal{I}}$-closed set in X. Suppose that A ⊆ N and N is an $\star$-open set in (X, τ, $\mathcal{I}$). Then X \ A is $\star$-gp$_{\mathcal{I}}$-open and X \ N ⊆ X \ A where X \ N is $\star$-closed. Since X \ A is $\star$-gp$_{\mathcal{I}}$-open, then we have X \ N ⊆ p$_{\mathcal{I}}$int(X \ A), where p$_{\mathcal{I}}$int(X \ A) = (X \ A) ∩ int(cl*(X \ A)). Since (X \ A) ∩ int(cl*(X \ A)) = (X \ A) ∩ (X \ cl(int*(A))) = X \ (A ∪ cl(int*(A))), then by Lemma 1.5, (X \ A) ∩ int(cl*(X \ A)) = X \ (A ∪ cl(int*(A))) = X \ p$_{\mathcal{I}}$cl(A). It follows that p$_{\mathcal{I}}$int(X \ A) = X \ p$_{\mathcal{I}}$cl(A). Thus p$_{\mathcal{I}}$cl(A) = X \ p$_{\mathcal{I}}$int(X \ A) ⊆ N and hence p$_{\mathcal{I}}$cl(A) ⊆ N. The converse is similar. □

Theorem 2.14. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and V ⊆ X. Then V is an $\star$-$C_{\mathcal{I}}$-set in X if and only if V = G ∩ p$_{\mathcal{I}}$cl(V) for an $\star$-open set G in X.*

Proof. If V is an $\star$-$C_{\mathcal{I}}$-set, then V = G ∩ M for an $\star$-open set G and a pre-$\mathcal{I}$-closed set M. But then V ⊆ M and so V ⊆ p$_{\mathcal{I}}$cl(V)⊆ M. It follows that V = V ∩ p$_{\mathcal{I}}$cl(V) = G ∩ M ∩ p$_{\mathcal{I}}$cl(V) = G ∩ p$_{\mathcal{I}}$cl(V). Conversely, it is enough to prove that p$_{\mathcal{I}}$cl(V) is a pre-$\mathcal{I}$-closed set. But p$_{\mathcal{I}}$cl(V) ⊆ M, for any pre-$\mathcal{I}$-closed set M containing V. So, cl(int*(p$_{\mathcal{I}}$cl(V))) ⊆ cl(int*(M)) ⊆ M. It follows that cl(int*(p$_{\mathcal{I}}$cl(V))) ⊆ $\bigcap_{V \subseteq M,\ M \text{ is pre-}\mathcal{I}\text{-closed}}$ M = p$_{\mathcal{I}}$cl(V). □

Theorem 2.15. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and A ⊆ X. The following properties are equivalent.*

(1) A is a pre-$\mathcal{I}$-closed set in X.

(2) A is an $\star$-$C_{\mathcal{I}}$-set and an $\star$-$gp_{\mathcal{I}}$-closed set in X.

Proof. (1) $\Rightarrow$ (2): It follows from the fact that any pre-$\mathcal{I}$-closed set in X is an $\star$-$C_{\mathcal{I}}$-set and an $\star$-$gp_{\mathcal{I}}$-closed set in X.
(2) $\Rightarrow$ (1): Suppose that A is an $\star$-$C_{\mathcal{I}}$-set and an $\star$-$gp_{\mathcal{I}}$-closed set in X. Since A is an $\star$-$C_{\mathcal{I}}$-set, then by Theorem 2.14, A = G $\cap$ $p_{\mathcal{I}}cl(A)$ for an $\star$-open set G in (X, τ, $\mathcal{I}$). Since A $\subseteq$ G and A is $\star$-$gp_{\mathcal{I}}$-closed set in X, then $p_{\mathcal{I}}cl(A) \subseteq G$. It follows that $p_{\mathcal{I}}cl(A) \subseteq G \cap p_{\mathcal{I}}cl(A) = A$. Thus, $A = p_{\mathcal{I}}cl(A)$ and hence A is pre-$\mathcal{I}$-closed. $\square$

Theorem 2.16. *Let (X, τ, $\mathcal{I}$) be an ideal topological space and $A \subseteq X$. If A is an $\star$-$C_{\mathcal{I}}$-set in X, then $p_{\mathcal{I}}cl(A) \setminus A$ is a pre-$\mathcal{I}$-closed set and $A \cup (X \setminus p_{\mathcal{I}}cl(A))$ is a pre-$\mathcal{I}$-open set in X.*

Proof. Suppose that A is an $\star$-$C_{\mathcal{I}}$-set in X. By Theorem 2.14, we have $A = L \cap p_{\mathcal{I}}cl(A)$ for an $\star$-open set L in X. It follows that $p_{\mathcal{I}}cl(A) \setminus A = p_{\mathcal{I}}cl(A) \setminus (L \cap p_{\mathcal{I}}cl(A)) = p_{\mathcal{I}}cl(A) \cap (X \setminus (L \cap p_{\mathcal{I}}cl(A))) = p_{\mathcal{I}}cl(A) \cap ((X \setminus L) \cup (X \setminus p_{\mathcal{I}}cl(A))) = (p_{\mathcal{I}}cl(A) \cap (X \setminus L)) \cup (p_{\mathcal{I}}cl(A) \cap (X \setminus p_{\mathcal{I}}cl(A))) = (p_{\mathcal{I}}cl(A) \cap (X \setminus L)) \cup \phi = p_{\mathcal{I}}cl(A) \cap (X \setminus L)$. Thus $p_{\mathcal{I}}cl(A) \setminus A = p_{\mathcal{I}}cl(A) \cap (X \setminus L)$ and hence $p_{\mathcal{I}}cl(A) \setminus A$ is pre-$\mathcal{I}$-closed set. Moreover, since $p_{\mathcal{I}}cl(A) \setminus A$ is a pre-$\mathcal{I}$-closed set in X, then $X \setminus (p_{\mathcal{I}}cl(A) \setminus A) = (X \setminus (p_{\mathcal{I}}cl(A) \cap (X \setminus A)) = (X \setminus p_{\mathcal{I}}cl(A)) \cup A$ is a pre-$\mathcal{I}$-open set. Thus, $X \setminus (p_{\mathcal{I}}cl(A) \setminus A) = (X \setminus p_{\mathcal{I}}cl(A)) \cup A$ is a pre-$\mathcal{I}$-open set in X. $\square$

3. Decompositions of $\star$-$A^{\star}_{\mathcal{I}}$-continuity

Definition 3.1. *A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be*

(1) $\star$-$C_{\mathcal{I}}$-continuous if $f^{-1}(A)$ is an $\star$-$C_{\mathcal{I}}$-set in X for every open set A in Y.

(2) $\star$-$A^{\star}_{\mathcal{I}}$-continuous if $f^{-1}(A)$ is an $\star$-$A^{\star}_{\mathcal{I}}$-set in X for every open set A in Y.

(3) $\star$-$\eta_{\mathcal{I}}$-continuous if $f^{-1}(A)$ is an $\star$-$\eta_{\mathcal{I}}$-set in X for every open set A in Y.

(4) $A^{\star}_{\mathcal{I}}$-continuous [3] if $f^{-1}(A)$ is an $A^{\star}_{\mathcal{I}}$-set in X for every open set A in Y.

Remark 3.2. *For a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following diagram holds. The reverses of these implications are not true in general as shown in the following Examples.*

$$\begin{array}{ccccc} & & \star\text{-}C_{\mathcal{I}}\text{-continuity} & \longleftarrow & C^{\star}_{\mathcal{I}}\text{-continuity} \\ & & \uparrow & & \\ \star\text{-}\eta_{\mathcal{I}}\text{-continuity} & \longleftarrow & \star\text{-}A^{\star}_{\mathcal{I}}\text{-continuity} & \longleftarrow & A^{\star}_{\mathcal{I}}\text{-continuity} \end{array}$$

Example 3.3. *Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$, $Y = \{p, q, r\}$, $\sigma = \{\emptyset, Y, \{q\}, \{r\}, \{q, r\}\}$, $\mathcal{I} = \{\emptyset, \{a\}\}$ and $\mathcal{J} = \{\emptyset\}$. Define $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ by $f(a) = p$; $f(b) = q$ and $f(c) = r$. Then f is $\star$-$C_{\mathcal{I}}$-continuous but not $\star$-$\eta_{\mathcal{I}}$-continuous.*

Example 3.4. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$, $Y = \{p, q, r, s\}$, $\sigma = \{\emptyset, Y, \{r\}, \{s\}, \{r, s\}\}$, $\mathcal{I} = \{\emptyset\}$ and $\mathcal{J} = \{\emptyset\}$. Define $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ by $f(a) = p$, $f(b) = q$, $f(c) = r$ and $f(d) = s$. Then f is $\star$-$C_{\mathcal{I}}$-continuous but not $C^{\star}_{\mathcal{I}}$-continuous.*

Example 3.5. *In Example 3.4, f is $\star$-$\eta_{\mathcal{I}}$-continuous but not $\star$-$A^{\star}_{\mathcal{I}}$-continuous.*

Example 3.6. *Let* $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{b\}, \{e\}, \{b, e\}, \{c, d\}, \{b, c, d\}, \{c, d, e\}, \{b, c, d, e\}, \{a, c, d, e\}, X\}$, $Y = \{p, q, r, s, t\}$, $\sigma = \{\emptyset, Y, \{p\}\}$, $\mathcal{I} = \{\emptyset, \{b\}, \{e\}, \{b, e\}\}$ *and* $\mathcal{J} = \{\emptyset\}$. *Define* $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ *by* $f(a) = p$, $f(b) = q$, $f(c) = r$, $f(d) = s$ *and* $f(e) = t$. *Then* f *is* $\star$-$A_{\mathcal{I}}^{\star}$-*continuous but not* $A_{\mathcal{I}}^{\star}$-*continuous.*

Definition 3.7 ([3]). *A function* $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ *is said to be semi**-$\mathcal{I}$-*continuous if* $f^{-1}(V)$ *is a semi**-$\mathcal{I}$-*open set in* X *for every open set* V *in* Y.

Theorem 3.8. *The following properties are equivalent for a function* $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$*:*

(1) f *is* $\star$-$A_{\mathcal{I}}^{\star}$-*continuous.*

(2) f *is* $\star$-$\eta_{\mathcal{I}}$-*continuous and semi**-$\mathcal{I}$-*continuous.*

(3) f *is* $\star$-$C_{\mathcal{I}}$-*continuous and semi**-$\mathcal{I}$-*continuous.*

Proof. It follows from Theorem 2.8. □

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