International Journal of Mathematics And its Applications
Volume 3, Issue 2 (2015), 133–137.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Connected Edge Domination in Fuzzy Graphs

Research Article

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Abstract: In this paper we discuss the concepts of connected edge domination and total connected edge domination in fuzzy graph. We determine the connected edge domination number $\gamma'_c(G)$ and the total edge domination number $\gamma'_t(G)$ for several classes of fuzzy graph and obtain bounds for the same. We also obtain Nordhaus Gaddum type resuts for these parameters.

MSC: 05C.

Keywords: Fuzzy graph; Domination; connected Edge Domination; Total connected edge Domination.
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1. Introduction

The study of domination set in graphs was begun by Ore and Berge. Rosenfield introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. A.Somasundram and S.Somasundram discussed domination in Fuzzy graphs. V.R. Kulli and D.K. Patwari discussed the total edge domination number of graph. They defined domination using effective edges in fuzzy graph. In this paper we discuss the connected edge domination number of fuzzy graph using fuzzy edge cardinality and establish the relationship with other parameter which is also investigated.

2. Preliminaries

Definition 2.1. *A fuzzy graph $G = (\sigma, \mu)$ is a set with two functions, $\sigma : V \to [0,1]$ and $\mu : E \to [0,1]$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y)\ \forall x, y \in V$.*

Definition 2.2. *Let $\mu = (\sigma, \mu)$ be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(x) = \sigma(x)$ for all $x \in V_1$ and μ_1 on the collection E_1 of two element subsets of V_1 by $\mu_1(xy) = \mu(xy)$ for all $x, y \in V_1$. Then (σ_1, μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $\langle V_1 \rangle$.*

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Definition 2.3. *The order p and size q of a fuzzy graph $G = (\sigma, \mu)$ are defined to be $p = \sum_{x \in V} \sigma(x)$ and $q = \sum_{xy \in E} \mu(xy)$.*

Definition 2.4. *Let $\sigma : V \to [0, 1]$ be a fuzzy subset of V then the complete fuzzy graph on σ is defined on $G = (\sigma, \mu)$ where $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$ and is denoted by K_σ.*

Definition 2.5. *The complement of a fuzzy graph G denoted by $\bar{G}$ is defined to be $\bar{G} = (\sigma, \bar{\mu})$ where $\bar{\mu}(xy) = \sigma(x) \wedge \sigma(y) - \mu(xy)$.*

Definition 2.6. *Let $G = (\sigma, \mu)$ be a fuzzy graph on V and $S \subseteq V$. Then the fuzzy cardinality of S is defined to be $\sum_{v \in S} \sigma(v)$.*

Definition 2.7. *Let $G = (\sigma, \mu)$ be a fuzzy graph on E and $D \subseteq E$ then the fuzzy edge cardinality of D is defined to be $\sum_{e \in D} \mu(e)$.*

Definition 2.8. *An edge $e = xy$ of a fuzzy graph is called an effective edge if $\mu(xy) = \sigma(x) \wedge \sigma(y)$. $N(x) = \{y \in V \mid \mu(xy) = \sigma(x) \wedge \sigma(y)\}$ is called the neighbourhood of x and $N[x] = N(x) \cup \{x\}$ is the closed neighbourhood of x.*

Definition 2.9. *The effective degree of a vertex u is defined to be sum of the weights of the effective edges incident of u and is denoted by $dE(u)$. $\sum_{v \in N(v)} \sigma(v)$ is called the neighbourhood of u and is denoted by $dN(u)$.*

Definition 2.10. *The minimum effective degree $\delta_E(G) = \min\{dE(u) \mid u \in V(G)\}$ and the maximum effective degree $\Delta_E(G) = \max\{dE(u) \mid u \in V(G)\}$.*

Definition 2.11. *The effective edge degree of an edge $e = uv$, is defined to be $d_E(e) = dE(u) + dE(v)$. The minimum edge effective degree and the maximum edge effective degree are $\delta_E(G) = \min\{d_E(e) \mid e \in X\}$ and $\Delta_E(G) = \max\{d_E(e) \mid e \in X\}$ respectively. $N(e)$ is the set of all effective edges incident with the vertices of e. In a similar way minimum neighbourhood degree and the maximum neighbourhood degree denoted by δ'_N and Δ'_N respectively can also be defined.*

Definition 2.12. *A fuzzy graph $G = (\sigma, \mu)$ is said to be bipartite if the vertex set V can be partitioned into two non-empty sets V_1 and V_2 such that $\mu(v_1 v_2) = 0$ if $v_1, v_2 \in V_1$ (or) $v_1, v_2 \in V_2$. Further if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u \in V_1$ and $v \in V_2$ than G is called a complete bipartite graph and is denoted by K_{σ_1, σ_2} where σ_1 and σ_2 are, respectively, the restrictions of σ to V_1 and V_2.*

3. Connected edge domination in fuzzy graphs

Definition 3.1. *Let $G = (\sigma, \mu)$ be a fuzzy graph on (V, X). A subset S of X is said to be an edge domination set in G if for every edge in $X - S$ is adjacent to atleast one effective edge in S. The minimum fuzzy cardinality of an edge dominating set is G is called the edge domination number of G and is denoted by $\gamma'(G)$ or γ'.*

Definition 3.2. *Let $G = (\sigma, \mu)$ be a fuzzy graph on (V, X), an edge dominating set F of a fuzzy graph G is connected edge dominating set with $\langle F \rangle$ is connected. The connected edge domination number $\gamma'_c(G)$ or γ'_c is the minimum fuzzy cardinality of connected edge dominating set.*

Remark 3.3. *It is clear that if G has at least one edge, then $0 \leq \gamma'(G) \leq q$. However if a graph G has no effective edges, then $\gamma'(G) = 0$.*

Example 3.4.

(1) Since D is a connected edge dominating set of K_σ with minimum fuzzy edge cardinality, we have $\gamma'_c(K_\sigma) = \min\left\{\sum_{i=1}^{\lfloor \frac{n}{2} \rfloor} \mu(e_i)\right\}$ *where n is the number of vertices of G.*

(2) $\gamma'_c(G) = q$ *if and only if* $\mu(xy) < \sigma(x) \wedge \sigma(y)$ *for all* $xy \in E$. *In particular,* $\gamma'_c(\bar{K}_\sigma) = 0$.

(3) Since D is the connected edge dominating set of G of K_{σ_1,σ_2} with minimum fuzzy cardinality we have $\gamma'_c(K_{\sigma_1,\sigma_2}) = \min\left\{\sum_{i=1}^{r} \mu(e_i)\right\}$ *where* $r = \min\{m, n\}$ *in complete bipartite graph $K_{m,n}$. For the edge domination number γ', the following theorem gives a Nordhaus Gaddum type result.*

Theorem 3.5. *For any fuzzy graph G,* $\gamma'_c + \bar{\gamma}_c \leq 2q$. *where is the connected edge domination number of $\bar{G}$ and equality holds if and only if* $0 < \mu(xy) < \sigma(x) \wedge \sigma(y)$ *for all* $xy \in E$.

Proof. The inequality is trivial. Further $\gamma'_c = q$ if and only if $\mu(xy) < \sigma(x) \wedge \sigma(y)$ for all $xy \in E$ and $\bar{\gamma}_c = q$ if and only if $\sigma(x) \wedge \sigma(y) - \mu(xy) < \sigma(x) \wedge \sigma(y)$ for all $xy \in E$ which is equivalent to $\mu(xy) > 0$. Hence $\gamma'_c + \bar{\gamma}_c = 2q$ if and only $0 < \mu(xy) < \sigma(x) \wedge \sigma(y)$. □

Definition 3.6. *A connected edge dominating set S of a fuzzy graph G is said to be minimal connected edge dominating set if no proper subset S is a connected edge dominating set of G.*

Theorem 3.7. *A connected edge dominating set S is minimal if and only if for each edge $e \in S$, one of the following two conditions holds.*

(a) $N(e) \cap S = \varphi$

(b) There exists an edge $f \in X - S$ such that $N(f) \cap S = \{e\}$ and f is an effective edge.

Proof. Let S be a minimal connected edge dominating set and $e \in S$. Then $S_e = S - \{e\}$ is not an edge connected dominating set and hence there exists $f \in X - S_e$ such that f is not dominated by any element of S_e. If $f = e$ we get (a) and if $f \neq e$ we get (b). The converse is obvious. □

Definition 3.8. *An edge e of a fuzzy graph G is said to be an isolated edge if no effective edges incident with the vertices of e. Thus an isolated edge does not dominate any other edge in G.*

Theorem 3.9. *If G is a fuzzy graph without isolated edges then for every minimal connected edge dominating set S, $X - S$ is also a connected edge dominating set.*

Proof. Let f be any edge in S. Since G has no isolated edges, there is an edge $c \in N(f)$. It follows from Theorem 3.7 that $c \in X - S$. Thus every element of S is dominated by some element of $X - S$. □

Corollary 3.10. *For any fuzzy graph G without isolated edges* $\gamma'_c \leq \frac{q}{2}$.

Proof. Any graph without isolated edges has two disjoint connected edge dominating sets and hence the result follows. □

Corollary 3.11. *Let G be a fuzzy graph such that both G and $\bar{G}$ have no isolated edges. Then* $\gamma'_c + \bar{\gamma}'_c \leq q$, *where $\bar{\gamma}'_c$ is the edge domination number of $\bar{G}$. Further equality holds if and only if* $\gamma'_c = \bar{\gamma}'_c = \frac{q}{2}$.

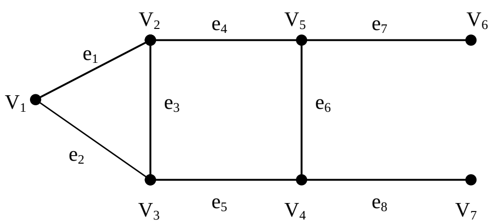
Theorem 3.12. *If G is a fuzzy graph without isolated edges then* $\frac{q}{\Delta'(G)+1} \geq \gamma'_c(G)$.

Proof. Let D be a connected edge dominating set of G. Since,

$$\begin{aligned}|D|\Delta'(G) \leq \sum_{e\in D} d_E(e) &= \sum_{e\in D} |N(e)| \\ &\leq |\bigcup_{e\in D} N(e)| \\ &\leq |E-D| \\ &\leq q-|D|\end{aligned}$$

$\therefore |D|\Delta'(G)+|D| \leq q$. Thus $\gamma'_c \leq \frac{q}{\Delta'(G)+1}$. □

Example 3.13.



$$\begin{aligned}Here\mu(e_i) &= \sigma(V_i) = 0.1 \\ q = 0.8,\ \gamma'_c &= 0.13 \\ \Delta'(G) &= 0.6 \\ \therefore \frac{0.8}{0.6+1} &\geq 0.3 \\ Hence 0.5 &> 0.3\end{aligned}$$

Theorem 3.14. *For any fuzzy graph* $\gamma'_c \geq q - \Delta'(G)$.

Definition 3.15. *Let G be a fuzzy graph without isolated edges. A subset D of E is said to be a total edge dominating set if every edge in E is dominated by an edge in D. The minimum fuzzy cardinality of a total edge dominating set is called the total edge domination number of G and is denoted by* $\gamma'_t(G)$

Theorem 3.16. *If a fuzzy graph G has no isolated edges then* $\gamma'_c(G) \leq \gamma'_t(G)$.

Theorem 3.17. *For any fuzzy graph* $\frac{q}{\Delta'(G)} \geq \gamma'_c(G0)$.

Proof. Let D be a connected edge dominating set with minimum number of fuzzy edge cardinality. Then every edge in D is adjacent to atleast $\Delta'(G)$ fuzzy edge cardinality, therefore, $|D|\ \Delta'(G) \leq q$. Hence $\frac{q}{\Delta'(G)} \geq \gamma'_c(G0)$ □

Acknowledgement

Thanks are due to the referees for their valuable comments and suggestions.

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