International Journal of Mathematics And its Applications
Volume 3, Issue 2 (2015), 139–147.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Contra Regular Generalized Semipre Continuous Mapping in Intuitionistic Fuzzy Topological Spaces

Research Article

V.Vaishnavy[1]* and D.Jayanthi[2],

1 MSc Mathematics, Avinashilingam University, Coimbatore, Tamil Nadu, India.

2 Department of Mathematics, Avinashilingam University, Coimbatore, Tamil Nadu, India.

Abstract: In this paper we introduce the notion of intuitionistic fuzzy contra regular generalized semipre continuous mappings. Furthermore we provide some properties of intuitionistic fuzzy contra regular generalized semipre continuous mappings and discuss some interesting theorems.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy closed sets, Intuitionistic fuzzy regular generalized semipre closed sets, Intuitionistic fuzzy regular generalized semipre open sets, Intuitionistic fuzzy contra regular generalized semipre continuous mappings.

1. Introduction and Preliminaries

The notion of intuitionistic fuzzy sets by Atanassov [1] was a breakthrough towards the evolution of intuitionistic fuzzy topology. Using this notion, Coker [3] constructed the basic concepts of intuitionistic fuzzy topological spaces. Later this was followed by the introduction of intuitionistic fuzzy regular generalized semipreclosed sets by Vaishnavy, V and Jayanthi, D [10] in 2015 which was simultaneously followed by the introduction of intuitionistic fuzzy regular generalized semipre continuous mappings [12] by the same authors. We now extend our idea towards intuitionistic fuzzy contra regular generalized semipre continuous mappings and discuss some of their properties.

2. Preliminaries

Definition 2.1 ([1])**.** *An intuitionistic fuzzy set (IFS in short) A is an object having the form* $A = \{\langle x,\ \mu_A(x),\ \ \nu_A(x)\rangle : x \in X\}$*, where the function* $\mu_A : X \to [0,1]$ *and* $\nu_A : X \to [0,1]$ *denote the degree of membership (namely* $\mu_A(x)$*) and the degree of non membership (namely* $\nu_A(x)$*) of each element* $x \in X$ *to the set A, respectively, and* $0 \le \mu_A(x) + \nu_A(x) \le 1$ *for each* $x \in X$*. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X. An intuitionistic fuzzy set A in X is simply denoted by* $A = \langle x, \mu_A, \nu_A\rangle$ *instead of denoting* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

* *E-mail: siingam@yahoo.com*

Definition 2.2 ([1]). *Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then*

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$

(c) $A^c = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$

The intuitionistic fuzzy sets $0 \sim= \langle x, 0, 1\rangle$ and $1 \sim= \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3 ([2]). *An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:*

(i) $0 \sim, 1 \sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called the intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The compliment A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4 ([2]). *Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by*

$$int(A) = \cup\{G/G \text{ is an } IFOS \text{ in } X \text{ and } G \subseteq A\}$$

$$cl(A) = \cap\{K/K \text{ is an } IFCS \text{ in } X \text{ and } A \subseteq K\}$$

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5 ([4]). *An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an*

(i) intuitionistic fuzzy pre closed set (IFPCS in short) if $cl(int(A)) \subseteq A$

(ii) intuitionistic fuzzy closed set (IF$_\alpha$ CS in short) if $cl(int(cl(A))) \subseteq A$.

The respective complements of the above IFCSs are called their respective IFOSs. The family of all IFPCSs, IF$_\alpha$CSs and (respectively IFPOSs, IF$_\alpha$OSs) of an IFTS (X, τ) are respectively denoted by IFPC(X), IF$_\alpha$C(X) (respectively IFPO(X), IF$_\alpha$O(X)).

Definition 2.6 ([13]). *An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an*

(i) intuitionistic fuzzy semi-pre closed set (IFSPCS in short) if there exists an IFPCS B such that $int(B) \subseteq A \subseteq B$

(ii) intuitionistic fuzzy semi-pre open set (IFSPOS in short) if there exists an IFPOS B such that $B \subseteq A \subseteq cl(B)$.

The family of all IFSPCSs (respectively IFSPOSs) of an IFTS (X,τ) is denoted by IFSPC(X) (respectively IFSPO(X)). Every IFSCS (respectively IFSOS) and every IFPCS (respectively IFPOS) is an IFSPCS (respectively IFSPOS). But the separate converses need not hold in general.

Definition 2.7 ([7]). *Let A be an IFS in an IFTS (X,τ). Then the semi-pre interior and the semi-pre closure of A are defined as*

$$spint(A) = \cup\{G|G \text{ is an } IFSPOS \text{ in } X \text{ and } G \subseteq A\}$$

$$spcl(A) = \cap\{K|K \text{ is an } IFSPCS \text{ in } X \text{ and } A \subseteq K\}$$

Note that for any IFS A in (X,τ), we have $spcl(A^c) = (spint(A))^c$ and $spint(A^c) = (spcl(A))^c$.

Definition 2.8 ([8]). *An IFS A is an*

(i) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = cl(int(A))$

(ii) intuitionistic fuzzy regular open set (IFROS in short) if $A = int(cl(A))$.

Definition 2.9 ([3]). *An intuitionistic fuzzy point (IFP in short), written as $p_{(\alpha,\beta)}$, is defined to be an intuitionistic fuzzy set of X given by*

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta), & \text{if } x = p, \\ (0,1), & \text{otherwise.} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.10 ([8]). *Two IFSs are said to be q-coincident ($A_q B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.*

Definition 2.11 ([10]). *An IFS A in an IFTS (X,τ) is said to be an intuitionistic fuzzy regular generalized semipreclosed set (IFRGSPCS in short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in (X,τ).*

Definition 2.12 ([11]). *The complement A^c of an IFRGSPCS A in an IFTS (X,τ) is called an intuitionistic fuzzy regular generalized semipreopen set (IFRGSPOS in short) in X.*

Definition 2.13 ([11]). *If every IFRGSPCS in (X,τ) is an IFSPCS in (X,τ), then the space can be called as an intuitionistic fuzzy regular semipre $T_{1/2}$ space (IFRSPT$_{1/2}$ in short).*

Definition 2.14 ([4]). *Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an intuitionistic fuzzy continuous (IF continuous in short) mapping if $f^{-1}(B) \in$ IFO(X) for every $B \in \sigma$.*

Definition 2.15 ([12]). *A mapping $f : (X,\tau) \to (Y,\sigma)$ is called an intuitionistic fuzzy regular generalized semipre continuous (IFRGSP continuous in short) mapping if $f^{-1}(V)$ is an IFRGSPCS in (X,τ) for every IFCS V in (Y,σ).*

Definition 2.16 ([6]). *Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an*

(i) intuitionistic fuzzy contra continuous mapping (IFC continuous mapping in short) if $f^{-1}(B) \in$IFO(X) for each IFCS B in Y

(ii) intuitionistic fuzzy contra α-continuous mapping ($IF_\alpha C$ continuous mapping in short) if $f^{-1}(B) \in IF_\alpha O(X)$ for each IFCS B in Y

(iii) intuitionistic fuzzy contra pre continuous mapping (IFPC continuous mapping in hort) if $f^{-1}(B) \in IFPO(X)$ for each IFCS B in Y

Corollary 2.17. *Let $A, A_i(i \in J)$ be intuitionistic fuzzy sets in X and $B, B_j(j \in K)$ be intuitionistic fuzzy sets in Y and $f : X \to Y$ be a function. Then*

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

(c) $A \subseteq f^{-1}(f(A))$ *[If f is injective, then* $A = f^{-1}(f(A))$*]*

(d) $f(f^{-1}(B)) \subseteq B$ *[If f is surjective, then* $B = f(f^{-1}(B))$*]*

(e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

(f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

(g) $f^{-1}(0\sim) = 0\sim$

(h) $f^{-1}(1\sim) = 1\sim$

(i) $f^{-1}(B^c) = (f^{-1}(B))^c$

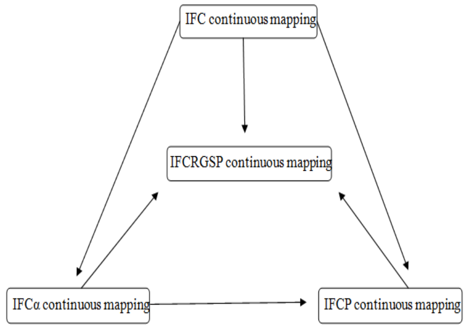
3. Contra Regular Generalized Semipre Continuous Mappings in Intuitionstic Fuzzy Topological Spaces

In this section we introduce the notion of intuitionistic fuzzy contra regular generalized semipre continuous mappings and discuss few of its properties.

Definition 3.1. *A mapping $f : X \to Y$ is said to be an intuitionistic fuzzy contra regular generalized semipre continuous (IFCRGSP continuous in short) mapping if $f^{-1}(A)$ is an IFRGSPCS in X for every IFOS A in Y.*

Example 3.2. *Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ and $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively.*
Then, $IFPC(X) = \{0\sim, 1\sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1]/$ either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, $IFSPC(X) = \{0\sim, 1\sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1]/\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Now $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFOS in Y. Therefore $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$. We have $spcl(f^{-1}(G_2)) = f^{-1}(G_2)$. We have $f^{-1}(G_2) \subseteq G_1$. Hence $spcl(f^{-1}(G_2)) \subseteq G_1$, where G_1 is an IFROS in X. This implies $f^{-1}(G_2)$ is an IFRGSPCS in X. Therefore f is an IFCRGSP continuous mapping.

Remark 3.3. *Every IFC continuous mapping, IFC_α continuous mapping, IFCP continuous mapping is an IFCRGSP continuous mapping but their converses need not hold in general. This can be observed from the following diagram and examples.*



Example 3.4. *Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ and $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively.*

Then, $IFPC(X) = \{0\sim, 1\sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1]/$either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5, \mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b leq1\}$. Therefore, $IFSPC(X) = \{0\sim, 1\sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1]/\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Now $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFOS in Y. Therefore $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$. We have $spcl(f^{-1}(G_2)) = f^{-1}(G_2)$. We have $f^{-1}(G_2) \subseteq G_1$. Hence $spcl(f^{-1}(G_2)) \subseteq G_1$, where G_1 is an IFROS in X. This implies $f^{-1}(G_2)$ is an IFRGSPCS in X. Therefore f is an IFCRGSP continuous mapping. We have $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFOS in Y. But $f^{-1}(G2) = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$ is not an IFCS in X, since $cl(f^{-1}(G_2)) = G_1^c \neq f^{-1}(G_2)$. This implies f is not an IFC continuous mapping. Further we have $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$ is not an IFCS in X, since $cl(int(cl(f^{-1}(G^2)))) = cl(int(G_1^c)) = cl(G_1) = G_1^c \nsubseteq f^{-1}(G_2)$. Therefore f is not an IFC_α continuous mapping. Hence f is an IFCRGSP continuous mapping but it is neither an IFC continuous mapping nor an IFC_α continuous mapping.

Example 3.5. *Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ and $G_2 = \langle y, (0.5, 0.7), (0.5, 0.3)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively.*

Then, $IFPC(X) = \{0\sim, 1\sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1]/\mu_b < 0.6$ whenever $\mu_a \geq 0.5, \mu_a < 0.5$ whenever $\mu_b \geq 0.6, \mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, $IFSPC(X) = \{0\sim, 1\sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1]/\mu_b < 0.6$ whenever $\mu_a \geq 0.5, \mu_a < 0.5$ whenever $\mu_b \geq 0.6, \mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Now $G_2 = \langle y, (0.5, 0.7), (0.5, 0.3)\rangle$ is an IFOS in Y. Therefore $f^{-1}(G_2) = \langle x, (0.5, 0.7), (0.5, 0.3)\rangle$. We have $spcl(f^{-1}(G_2)) = f^{-1}(G_2)$. We have $f^{-1}(G_2^c) \subseteq 1\sim$. Hence $spcl(f^{-1}(G_2^c)) \subseteq 1\sim$, where $1\sim$ is an IFROS in X. This implies $f^{-1}(G_2^c)$ is an IFRGSPCS in X. Therefore f is an IFCRGSP continuous mapping. We have $G_2 = \langle y, (0.5, 0.7), (0.5, 0.3)\rangle$ is an IFOS in Y. But since $cl(int(f^{-1}(G_2))) = cl(G_1) = 1\sim \nsubseteq f^{-1}(G_2)$, it is not an IFPCS in X. Hence f is not an IFCP continuous mapping.

Theorem 3.6. *Let $f : X \to Y$ be a mapping. Then f is an IFCRGSP continuous mapping if and only if $f^{-1}(A)$ is an IFRGSPOS in X for every IFCS A in Y.*

Proof. **Necessity:** Let A be an IFCS in Y. Then A^c is an IFOS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFRGSPCS in X. Since, $f^{-1}(A^c) = (f^{-1}(A))^c, f^{-1}(A)$ is an IFRGSPOS in X.

Sufficiency: Let A be an IFOS in Y. Then Ac is an IFCS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFRGSPOS in X. Since,

$f^{-1}(A^c) = (f^{-1}(A))^c, f^{-1}(A)$ is an IFRGSPCS in X. Thus f is an IFCRGSP continuous mapping. □

Theorem 3.7. *Let $f : X \to Y$ be a bijective mapping. Suppose that one of the following properties hold:*

(i) $f^{-1}(cl(B)) \subseteq int(spcl(f^{-1}(B)))$ *for each IFS B in Y*

(ii) $cl(spint(f^{-1}(B))) \subseteq f^{-1}(int(B))$ *for each IFS B in Y*

(iii) $f(cl(spint(A))) \subseteq int(f(A))$ *for each IFS A in X*

(iv) $f(cl(A)) \subseteq int(f(A))$ *for each IFSPOS A in X*

Then f is an IFCRGSP continuous mapping.

Proof. $(i) \Rightarrow (ii)$ is obvious by taking complement of (i).
$(ii) \Rightarrow (iii)$ Let $A \subseteq X$. Put $B = f(A)$ in Y. This implies $A = f^{-1}(f(A)) = f^{-1}(B)$ in X. Now $cl(spint(A)) = cl(spint(f^{-1}(B))) \subseteq f^{-1}(int(B))$ by (ii). Therefore $f(cl(spint(A))) \subseteq f(f^{-1}(int(B))) = int(B) = int(f(A))$.
$(iii) \Rightarrow (iv)$ Let $A \subseteq X$ be an IFSPOS. Then $spint(A) = A$. By hypothesis, $f(cl(spint(A))) \subseteq int(f(A))$. Therefore $f(cl(A)) = f(cl(spint(A))) \subseteq int(f(A))$.
Suppose (iv) holds. Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFS in X and $spint(f^{-1}(A))$ is an IFSPOS in X. Hence by hypothesis, $f(cl(spint(f^{-1}(A)))) \subseteq int(f(spint(f^{-1}(A)))) \subseteq int(f(f^{-1}(A))) = int(A) \subseteq A$. Therefore $cl(spint(f^{-1}(A))) = f^{-1}(f(cl(spint(f^{-1}(A))))) \subseteq f^{-1}(A)$. Now $cl(int(f^{-1}(A))) \subseteq cl(spint(f^{-1}(A))) \subseteq f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFPCS in X and hence an IFRGSPCS in X [10]. Thus f is an IFCRGSP continuous mapping. □

Theorem 3.8. *Let $f : X \to Y$ be a mapping. Suppose that one of the following properties hold:*

(i) $f(spcl(A)) \subseteq int(f(A))$ *for each IFS A in X*

(ii) $spcl(f^{-1}(B)) \subseteq f^{-1}(int(B))$ *for each IFS B in Y*

(iii) $f^{-1}(cl(B)) \subseteq spint(f^{-1}(B))$ *for each IFS B in Y*

Then f is an IFCRGSP continuous mapping.

Proof. $(i) \Rightarrow (ii)$ Let $B \subseteq Y$. Then $f^{-1}(B)$ is an IFS in X. By hypothesis, $f(spcl(f^{-1}(B))) \subseteq int(f(f^{-1}(B))) \subseteq int(B)$. Now $spcl(f^{-1}(B)) \subseteq f^{-1}(f(spcl(f^{-1}(B)))) \subseteq f^{-1}(int(B))$.
$(ii) \Rightarrow (iii)$ is obvious by taking complement in (ii).
Suppose (iii) holds. Let A be an IFCS in Y. Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X. Now $f^{-1}(A) = f^{-1}(cl(A)) \subseteq spint(f^{-1}(A)) \subseteq f^{-1}(A)$, by hypothesis. This implies $f^{-1}(A)$ is an IFSPOS in X and hence an IFRGSPOS in X [11]. Therefore f is an IFCRGSP continuous mapping. □

Theorem 3.9. *Let f : X Y be a bijective mapping. Then f is an IFCRGSP continuous mapping if cl(f(A)) f(spint(A)) for every IFS A in X.*

Proof. Let A be an IFCS in Y. Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X. By hypothesis $cl(f(f^{-1}(A))) \subseteq f(spint(f^{-1}(A)))$. Since f is onto, $f(f^{-1}(A)) = A$. Therefore $A = cl(A) = cl(f(f^{-1}(A))) \subseteq f(spint(f^{-1}(A)))$. Now $f^{-1}(A) \subseteq f^{-1}(f(spint(f^{-1}(A)))) = spint(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence $f^{-1}(A)$ is an IFSPOS in X and hence an IFRGSPOS in X [11]. Thus f is an IFCRGSP continuous mapping. □

Theorem 3.10. *If $f : X \to Y$ is an IFCRGSP continuous mapping, where X is an IFRSPT$_{1/2}$ space, then the following conditions hold:*

(i) $spcl(f^{-1}(B)) \subseteq f^{-1}(int(spcl(B)))$ for every IFOS in Y

(ii) $f^{-1}(cl(spint(B))) \subseteq spint(f^{-1}(B))$ for every IFCS B in Y.

Proof.

(i) Let $B \subseteq Y$ be an IFOS. By hypothesis $f^{-1}(B)$ is an IFRGSPCS in X. Since X is an IFRSPT$_{1/2}$ space, $f^{-1}(B)$ is an IFSPCS in X. This implies $spcl(f^{-1}(B)) = f^{-1}(B) \subseteq f^{-1}(int(B)) \subseteq f^{-1}(int(spcl(B)))$.

(ii) can be proved easily by taking the complement of (i).

□

Theorem 3.11. *If $f : X \to Y$ is an IFCRGSP continuous mapping and $g : Y \to Z$ is an IF continuous mapping then $g \circ f : X \to Z$ is an IFCRGSP continuous mapping.*

Proof. Let A be an IFOS in Z. Then $g^{-1}(A)$ is an IFOS in Y, since g is an IF continuous mapping. Since f is an IFCRGSP continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFRGSPCS in X. Therefore $g \circ f$ is an IFCRGSP continuous mapping. □

Theorem 3.12. *If $f : X \to Y$ is an IFCRGSP continuous mapping and $g : Y \to Z$ is an IFC continuous mapping then $g \circ f : X \to Z$ is an IFRGSP continuous mapping.*

Proof. Let A be an IFOS in Z. Then $g^{-1}(A)$ is an IFCS in Y, since g is an IFC continuous mapping. Since f is an IFCRGSP continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFRGSPOS in X. Therefore $g \circ f$ is an IFRGSP continuous mapping. □

Theorem 3.13. *For a mapping $f : X \to Y$, where X is an IFRSPT$_1/2$ space, the following are equivalent:*

(i) f is an IFCRGSP continuous mapping

(ii) For every IFCS A in Y and for any IFP $p(\alpha, \beta) \in X$, if $f(p(\alpha, \beta))_q A$ then $p(\alpha, \beta)_q$ $spint(f^{-1}(A))$

(iii) For every IFCS A in Y and for any $p(\alpha, \beta) \in X$, if $f(p(\alpha, \beta))_q A$ then there exists an IFRGSPOS B such that $p(\alpha, \beta)_q B$ and $f(B) \subset A$.

Proof. $(i) \Rightarrow (ii)$ Let f be an IFCRGSP continuous mapping. Let $A \subset Y$ be an IFCS and let $p(\alpha, \beta) \in X$. Also let $f(p(\alpha, \beta))_q A$ then $p(\alpha, \beta)_q f^{-1}(A)$. By hypothesis $f^{-1}(A)$ is an IFRGSPOS in X. Since X is an IFRSPT$_1/2$ space, $f^{-1}(A)$ is an IFSPOS in X. Hence $spint(f^{-1}(A)) = f^{-1}(A)$. This implies $p(\alpha, \beta)_q$ $spint(f^{-1}(A))$.

$(ii) \Rightarrow (i)$ Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFCS in X. Let $p(\alpha, \beta) \in X$ and let $f(p(\alpha, \beta))_q A$ then $p(\alpha, \beta)_q f^{-1}(A)$. By hypothesis this implies $p(\alpha, \beta)_q$ $spint(f^{-1}(A))$. That is $f^{-1}(A) \subseteq spint(f^{-1}(A))$. But $spint(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $spint(f^{-1}(A)) = f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFSPOS in X and hence an IFRGSPOS in X [11]. This implies f is an IFCRGSP continuous mapping.

(ii) $\Rightarrow$ *(iii)* Let Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFCS in X. Let $p(\alpha,\beta) \in X$. Also let $f(p(\alpha,\beta))_q A$ then $p(\alpha,\beta)_q f^{-1}(A)$. By hypothesis this implies $p(\alpha,\beta)_q$ $spint(f^{-1}(A))$. That is $f^{-1}(A) \subseteq spint(f^{-1}(A))$. But $spint(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $spint(f^{-1}(A)) = f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFSPOS in X and hence an IFRGSPOS in X [11]. Let $f^{-1}(A) = B$. Therefore $p(\alpha,\beta)_q B$ and $f(B) \subseteq f(f^{-1}(A)) \subset A$.

(iii) $\Rightarrow$ *(ii)* Let Let $A \subset Y$ be an IFCS then $f^{-1}(A)$ is an IFCS in X. Let $p(\alpha,\beta) \in X$. Also let $f(p(\alpha,\beta))_q A$ then $p(\alpha,\beta)_q f^{-1}(A)$. By hypothesis there exists an IFRGSPOS B in X such that $p(\alpha,\beta)_q B$ and $f(B) \subseteq A$. Let $B = f^{-1}(A)$. Since X is an IFRSPT$_{1/2}$ space, $f^{-1}(A)$ is an IFSPOS in X. Therefore $p(\alpha,\beta)_q$ $spint(f^{-1}(A))$. □

Theorem 3.14. *A mapping $f : X \to Y$ is an IFCRGSP continuous mapping if $f^{-1}(spcl(B)) \subseteq int(f^{-1}(B))$ for every IFS B in Y.*

Proof. Let $B \subseteq Y$ be an IFCS. Then $cl(B) = B$. Since every IFCS is an IFSPCS [13], $spcl(B) = B$. Now by hypothesis, $f^{-1}(B) = f^{-1}(spcl(B)) \subseteq int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFOS in X and hence an IFRGSPOS in X [11]. Therefore f is an IFCRGSP continuous mapping. □

Theorem 3.15. *A mapping $f : X \to Y$ is an IFCRGSP continuous mapping, where X is an IFRSPT$_1/2$ space if and only if $f^{-1}(spcl(B)) \subseteq spint(f^{-1}(cl((B)))$ for every IFS B in Y.*

Proof. **Necessity:** Let $B \subseteq Y$ be an IFS. Then $cl(B)$ is an IFCS in Y. By hypothesis, $f^{-1}(cl(B))$ is an IFRGSPOS in X. Since X is an IFRSPT$_{1/2}$ space, $f^{-1}(cl(B))$ is an IFSPOS in X. Therefore $f^{-1}(spcl(B)) \subseteq f^{-1}(cl(B)) = spint(f^{-1}(cl(B)))$.

Sufficiency: Let $B \subseteq Y$ be an IFCS. Then $cl(B) = B$. By hypothesis, $f^{-1}(spcl(B)) \subseteq spint(f^{-1}(cl((B))) = spint(f^{-1}(B))$. But $spcl(B) = B$. Therefore $f^{-1}(B) = f^{-1}(spcl(B)) \subseteq spint(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFSPOS in X and hence an IFRGSPOS in X [11]. Hence f is an IFCRGSP continuous mapping. □

Theorem 3.16. *An IF continuous mapping $f : X \to Y$ is an IFCRGSP continuous mapping if $IFRGSPO(X) = IFRGSPC(X)$.*

Proof. Let $A \subset Y$ be an IFOS. By hypothesis, $f^{-1}(A)$ is an IFOS in X and hence is an IFRGSPOS in X [11]. Thus $f^{-1}(A)$ is an IFRGSPCS in X, since $IFRGSPO(X) = IFRGSPC(X)$. Therefore f is an IFCRGSP continuous mapping. □

References

[1] K.Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, (1986), 87- 96.

[2] D.Coker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy sets and systems, (1997), 81-89.

[3] D.Coker and M.Demirci, *On intuitionistic fuzzy points*, Notes on Intuitionistic Fuzzy Sets 1(1995), 79-84.

[4] H.Gurcay, D.Coker and Es.A.Haydar, *On fuzzy continuity in intuitionistic fuzzy topological spaces*, The J. of Fuzzy Mathematics, (1997), 365-378.

[5] Joung Kon Jeon, Young Bae Jun and Jin Han Park, *Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy pre continuity*, International Journal of Mathematics and Mathematical Sciences, (2005), 3091-3101.

[6] B.Krsteska and E.Ekici, *Intuitionistic fuzzy contra strong precontinuity*, Faculty of Sciences and Mathematics, University of Nis, Siberia, (2007), 273284.

[7] R.Santhi and D.Jayanthi, *Intuitionistic fuzzy generalized semi-pre closed sets*, Tripura math. Soc., (2009), 61-72.

[8] S.S.Thakur and Rekha Chaturvedi, *Regular generalized closed sets in intuitionistic fuzzy topological spaces*, Universitatea Din Bacau, Studii SiCercetariStiintifice, Seria: Matematica, (2006), 257-272.

[9] S.S.Thakur and S.Singh, *On fuzzy semi-pre open sets and fuzzy semi- pre continuity*, Fuzzy sets and systems, (1998), 383-391.

[10] V.Vaishnavy and D.Jayanthi, *On Intuitionistic fuzzy regular generalized semipreclosed sets*, (accepted)

[11] V.Vaishnavy and D.Jayanthi, *Regular generalized semipreopen sets in intuitionistic fuzzy topological spaces*, International Journal of Engineering Research & Technology, 4(02)(2015), 890-892.

[12] V.Vaishnavy and D.Jayanthi, *On intuitionistic fuzzy regular generalized semipre continuous mappings*, (submitted).

[13] Young Bae Jun and Seok- Zun Song, *Intuitionistic fuzzy semi-pre open sets and Intuitionistic semi-pre continuous mappings*, Jour. of Appl. Math & computing, (2005), 467-474.