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Special Pythagorean Triangles and 6 Digit Dhuruva Number

Research Article

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Abstract: Pythagorean triangles, each with a leg represented by a 6-digit Dhuruva numbers are obtained. A few interesting results are given.

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Keywords: Pythagorean triangles, 6-digit Dhuruva number.

1. Introduction

The fascinating branch of mathematics is the theory of numbers where in Pythagorean triangles have been a matter of interest to various mathematicians and to the lovers of mathematics, because it is a treasure house in which the search for many hidden connection is a treasure hunt. For a rich variety of fascinating problems one may refer [1]-[17]. A careful observer of patterns may note that there is a one to one correspondence between the polygonal numbers and the number of sides of the polygon. Apart from the above patterns we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers. These numbers have been presented in [18]-[21]. In [22]-[25], special Pythagorean triangles connected with polygonal numbers and Nasty numbers are obtained. Recently in [26], special Pythagorean triangles in connection with Hardy Ramanujan number 1729 are exhibited. Thus the objective of this paper is to find out the special Pythagorean triangles in connection with 6 digit Dhuruva number 631764. In this communication we have presented Pythagorean triangles each with a leg represented by 6 digit Dhuruva number 631764 Also, a few special Pythagorean triangles in connection with 631764 are obtained.

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2. Main Result

Definition 2.1. *The ternary quadratic Diophantine equation given by $x^2+y^2=z^2$ is known as Pythagorean equation where x, y, z are natural numbers. The above equation is also referred to as Pythagorean triangle and denote it by T(x,y,z). Also, in Pythagorean triangle $T(x,y,z): x^2+y^2+z^2$, x and y are called its legs and z its hypotenuse.*

Definition 2.2. *Most cited solution of the Pythagorean equation is $x=2mn, y=m^2-n^2, z=m^2+n^2$, where $m>n>0$. This solution is called primitive, if m,n are of opposite parity and $gcd(m,n)=1$.*

Definition 2.3 (Dhuruva numbers)**.** *The numbers which do not change when we perform a single operation or a sequence of operations are known as Dhuruva numbers.*

3. Method of analysis

In this section, we exhibit Pythagorean triangles, each with a leg represented by the 6-digit Dhuruva number 631764 and denote this number by N. To start with, it is noted that the leg y can not be represented by N as y is odd and N is even. And z cannot be written as sum of two squares. Since a positive integer P can be written as a sum of two integer squares iff the canonical prime factorization $P=p_1^{r_1}p_2^{r_2}\dots p_r^{r_r}$, (where p_i are distinct primes) satisfies the condition if $p_i\equiv 3(mod\ 4)$ then r_i is even. A prime $p\equiv 1(mod\ 4)$ can be written as $p=a^2+b^2$. Now, consider $x=N\Rightarrow x=2mn$, which is a binary quadratic Diophantine equation. Solving the above equation for m,n we get 24 integer solutions and thus, we have 24 Pythagorean triangles, each having the leg x to be represented by the six digit Dhuruva number $N=631764$ as shown in the table below:

m	n	x	y	z	A	P
315882	1	631764	99781437923	99781437925	31519160173993100	199563507612
157941	2	631764	24945359477	24945359485	7879790042313710	49891350726
105294	3	631764	11086826427	11086826445	3502128905413610	22174284636
52647	6	631764	2771706573	2771706645	875532215692386	5544044982
45126	7	631764	2036355827	2036355925	643248151344414	4073343516
35098	9	631764	1231869523	1231869685	389125408664286	2464370972
22563	14	631764	509088773	509089165	160811979792786	1018809702
17549	18	631764	307967077	307967725	97281256216914	616566566
15042	21	631764	226261323	226262205	71471879231886	453155292
13734	23	631764	188622227	188623285	59582366309214	377877276
7521	42	631764	56563677	56567205	17867447418114	113362646
6867	43	631764	47153573	47157805	14894964946386	94943142
5014	63	631764	25136227	25144165	7940081657214	50912156
4578	69	631764	20953323	20962845	6618777575886	42547932
2898	109	631764	8386523	8410285	2649151658286	17428572
2507	126	631764	6269173	6300925	1980318905586	13201862
2289	138	631764	5220477	5258565	1649054715714	11110806
1962	161	631764	3823523	3875365	1207782092286	8330652
1526	207	631764	2285827	2371525	722051604414	5289116
1449	218	631764	2052077	2147125	648214186914	4830966
981	322	631764	858677	1066045	271240608114	2556486
966	327	631764	826227	1040085	260990237214	2498076
654	483	631764	194427	661005	61415989614	1487196
763	414	631764	410773	753565	129755796786	1796102

Note that there are 16 primitive and 8 non-primitive triangles. Also the relation $\frac{4A}{P} - y + z$ represent a 6-digit Dhuruva number for each of the above Pythagorean triangles, where A and P represents the area and perimeter of the Pythagorean triangle. In a similar manner, it is seen that there are 72 Pythagorean triangles wherein, each of the following expressions $\frac{2A}{P}, \frac{1}{2}(x + y - z)$ represent 631764 as shown in the table below.

m	n	x	y	z	A	P
631765	1	1263530	399127015224	399127015226.00	252154478772990000	798255293980
315884	2	1263536	99782701452	99782701460	63039517730927100	199566666448
210591	3	1263546	44348569272	44348569290	28018228654679300	88698402108
157945	4	1263560	24946623009	24946623041	15760777484626000	49894509610
105300	6	1263600	11088089964	11088090036	7005455239255200	22177443600
90259	7	1263626	8146687032	8146687130	5147182773749020	16294637788
70205	9	1263690	4928741944	4928742106	3114200953606680	9858747740
52659	12	1263816	2772970137	2772970425	1752262013331400	5547204378
45140	14	1263920	2037619404	2037619796	1287693958551840	4076503120
35116	18	1264176	1233133132	1233133780	779448655139616	2467531088
30105	21	1264410	906310584	906311466	572974082757720	1813886460
27491	23	1264586	755754552	755755610	477858312947736	1512774748
22591	28	1265096	510352497	510354065	322822451272356	1021971658
17585	36	1266120	309230929	309233521	195761731912740	619730570
15084	42	1267056	227525292	227528820	144143643190176	456321168
13780	46	1267760	189886284	189890516	120365117701920	381044560
10091	63	1271466	101824312	101832250	64733075340696	204928028
9225	69	1273050	85095864	85105386	54165644832600	171474300
7605	84	1277640	57828969	57843081	36942301976580	116949690
6959	92	1280456	48419217	48436145	30999338461476	98135818
5905	109	1287290	34857144	34880906	22435626449880	71025340
5140	126	1295280	26403724	26435476	17100107811360	54134480
4716	138	1301616	22221612	22259700	14462002862496	45782928
4085	161	1315370	16661304	16713146	10957889721240	34689820
3259	207	1349226	10578232	10663930	7136212824216	22591388
3116	218	1358576	9661932	9756980	6563234464416	20777488
2565	276	1415880	6503049	6655401	4603768509060	14574330
2759	252	1390536	7548577	7675585	5248284033636	16614698
2284	322	1470896	5112972	5320340	3760325031456	11904208
2259	327	1477386	4996152	5210010	3690622509336	11683548
1940	414	1606320	3592204	3934996	2885114564640	9133520
1885	436	1643720	3363129	3743321	2764021199940	8750170
1791	483	1730106	2974392	3440970	2573006722776	8145468
1625	644	2093000	2225889	3055361	2329392838500	7374250
1620	654	2118960	2196684	3052116	2327342764320	7367760
1591	763	2427866	1949112	3113450	2366091377496	7490428
1591	828	2634696	1845697	3216865	2431425251556	7697258
1620	966	3129840	1691244	3557556	2646661560480	8378640
1625	981	3188250	1678264	3602986	2675362599000	8469500
1791	1308	4685256	1496817	4918545	3506485415076	11100618
1885	1449	5462730	1453624	5652826	3970377716760	12569180
1940	1526	5920880	1434924	6092276	4248006406560	13448080
2259	1932	8728776	1370457	8835705	5981206085316	18934938
2284	1962	8962416	1367212	9066100	6126761352096	19395728
2565	2289	11742570	1339704	11818746	7865783999640	24901020
2759	2507	13833626	1327032	13897130	9178832189016	29057788
3116	2898	18060336	1311052	18107860	11839019816736	37479248
3259	3052	19892936	1306377	19935785	12993837026436	41135098

4085	3924	32059080	1289449	32085001	20669274323460	65433530
4716	4578	43179696	1282572	43198740	27690534529056	87661008
5140	5014	51543920	1279404	51559796	32972748711840	104383120
5905	5796	68450760	1275409	68462641	43651357680420	138188810
6959	6867	95574906	1271992	95583370	60785257916376	192430268
7605	7521	114394410	1270584	114401466	72673853517720	230066460
9225	9156	168928200	1268289	168932961	107124888924900	339129450
10091	10028	202385096	1267497	202389065	128261251012356	406041658
13780	13734	378509040	1265644	378511156	239528847710880	758285840
15084	15042	453787056	1265292	453788820	287086565830176	908841168
17585	17549	617198330	1264824	617199626	390323630271960	1235662780
22591	22563	1019441466	1264312	1019442250	644446039380696	2040148028
27491	27468	1510245576	1264057	1510246105	954518246030916	3021755738
30105	30084	1811357640	1263969	1811358081	1144749952436580	3623979690
35116	35098	2465002736	1263852	2465003060	1557699318949540	4931269648
45140	45126	4073975280	1263724	4073975476	2574190168371360	8149214480
52659	52647	5544676746	1263672	5544676890	3503326376485660	11090617308
70205	70196	9856220360	1263609	9856220441	6227204376439620	19713704410
90259	90252	16292110536	1263577	16292110585	10293168077373600	32585484698
105300	105294	22174916400	1263564	22174916436	14009713033024800	44351096400
157945	157941	49891982490	1263544	49891982506	31520357561672300	99785228540
210591	210588	88695875016	1263537	88695875025	56035259915045800	177393013578
315884	315882	199564139376	1263532	199564139380	126077838077018000	399129542288
631765	631764	798252766920	1263529	798252766921	504307760166830000	1596506797370

Note that there are 32 primitive and 40 non-primitive triangles. Also, it is observed that there are 36 Pythagorean triangles where in each, the expressions $y - \frac{2A}{p}$, $\frac{1}{2}(z + y - x)$ is represented by 631764 as shown in the table below:

m	n	x	y	z	A	P
631764	631763	798250239864	1263527	798250239865	504305365412320000	1596501743256
315882	315880	199561612320	1263524	199561612324	126075443322508000	399124488168
210588	210585	88693347960	1263519	88693347969	56032865160535600	177387959448
157941	157937	49889455434	1263512	49889455450	31517962807162100	99780174396
105294	105288	22172389344	1263492	22172389380	14007318278514600	44346042216
90252	90245	16289583480	1263479	16289583529	10290773322863500	32580430488
70196	70187	9853693304	1263447	9853693385	6224809621929440	19708650136
52647	52635	5542149690	1263384	5542149834	3500931621975480	11085562908
45126	45112	4071448224	1263332	4071448420	2571795413861180	8144159976
35098	35080	2462475680	1263204	2462476004	1555304564439360	4926214888
30084	30063	1808830584	1263087	1808831025	1142355197926400	3618924696
27468	27445	1507718520	1262999	1507719049	952123491520740	3016700568
22563	22535	1016914410	1262744	1016915194	642051284870520	2035092348
17549	17513	614671274	1262232	614672570	387928875761784	1230606076
15042	15000	451260000	1261764	451261764	284691811320000	903783528
13734	13688	375981984	1261412	375984100	237134093200704	753227496
10028	9965	199858040	1259559	199862009	125866496502180	400979608
9156	9087	166401144	1258767	166405905	104730134414724	334065816
7521	7437	111867354	1256472	111874410	70279099007544	224998236
6867	6775	93047850	1255064	93056314	58390503406200	187359228
5796	5687	65923704	1251647	65935585	41256603170244	133110936
5014	4888	49016864	1247652	49032740	30577994201664	99297256
4578	4440	40652640	1244484	40671684	25295780018880	82568808
3924	3763	29532024	1237607	29557945	18274519813284	60327576
3052	2845	17365880	1220679	17408729	10599082516260	35995288

2898	2680	15533280	1216004	15580804	9444265306560	32330088
2507	2255	11306570	1200024	11370074	6784077678840	23876668
2289	2013	9215514	1187352	9291690	5471029489464	19694556
1962	1640	6435360	1159844	6539044	3732006841920	14134248
1932	1605	6201720	1156599	6308649	3586451575140	13666968
1526	1112	3393824	1092132	3565220	1853251896384	8051176
1449	1013	2935674	1073432	3125770	1575623206584	7134876
1308	825	2158200	1030239	2391489	1111730904900	5579928
981	337	661194	848792	1075930	280608088824	2585916
966	312	602784	835812	1030500	251907050304	2469096
828	65	107640	681359	689809	36670741380	1478808

Note that there are 16 primitive and 20 non-primitive triangles.

4. Conclusion

One may search for the connections between Pythagorean triangles and other Dhuruva numbers.

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