International Journal of Mathematics And its Applications
Volume 3, Issue 2 (2015), 185–195.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

A Nonagonal Fuzzy Number and Its Arithmetic Operation

Research Article

A.Felix[1]*, S.Christopher[2] and A.Victor Devadoss[3]

1 PG & Research Department of Mathematics, Loyola College, Chennai, Tamilnau, India.

2 Department of Mathematics, Surya Group of Institution, Vikravandi, Villupram, Tamilnadu, India.

3 PG & Research Department of Mathematics, Loyola College, Chennai, Tamilnau, India.

Abstract: In many real life cases, the decision data of human judgments with preferences are often vague so that the usual ways of using crisp values are inadequate also using fuzzy numbers such as triangular, trapezoidal are not suitable in few cases where the uncertainties arises in nine different points. Therefore, Nanogonal Fuzzy Number (NFN) and its arithmetic operations based on extension principle of fuzzy sets and $\alpha-$cut are introduced.

Keywords: Fuzzy Arithmetic, Linguistic values, Nonagonal Fuzzy Number (NFN), α-cut.

1. Introduction

Fuzzy sets were introduced in their modern form by Zadeh, L.A. in 1965 [12]. It provides natural way of dealing with problems in which the source of imprecision and vagueness occurs and it can be applied in many fields such as artificial intelligence, control system, decision making, expert system etc. The concept of fuzzy number has been defined as a fuzzy subset of real line by Dubois, D and Prade, H [1]. A fuzzy number is a quantity whose values are precise, rather than exact as in the case with single valued numbers [2, 5]. To deal imprecise in real life situation, many researchers used triangular and trapezoidal fuzzy number [8, 10, 11, 14]. Also hexagonal, octagonal, decagonal fuzzy numbers have been introduced to clear the vagueness [4, 6, 7, 9]. Most of the researcher has focused on uncertain linguistic term in group decision making processes [3, 11, 13]. In decision making problem experts may provide uncertain linguistic term to express their opinion when they have no clear idea and lack of information. The uncertain linguistic term is frequently used as input in decision analysis activities. So far linguistic values are usually represented as fuzzy numbers such as triangular, trapezoidal. But it is complex to restrict the membership functions to take triangular, trapezoidal when vagueness arises in nine different points. Therefore, in this paper a new form of Nonagonal Fuzzy Number(NFN) is explored under uncertain linguistic environment. This paper is organized as follows: section one presents introduction. Basic definition of fuzzy number and linguistic terms are given in section two, section three provides nonagonal fuzzy number and it linguistic values. New operation for addition,

* *E-mail: afelixphd@gmail.com*

multiplication, division of nonagonal fuzzy number on the basis of fuzzy extension principle and $\alpha-$cut are proposed in section four. Section five gives a numerical example of arithmetic operation on NFN. Finally, conclusion and future directions are given.

2. Basic Definitions and Notations

In this section, some basic definitions of fuzzy set theory and fuzzy numbers are reviewed.

Definition 2.1. *A fuzzy set $\widetilde{A}$ in X is characterized by a membership function $\mu_{\widetilde{A}}(x)$ which associates each point in X, to a real number in the interval $[0,1]$. The value of $\mu_{\widetilde{A}}(x)$ represents "grade of membership" of $x \in \mu_{\widetilde{A}(x)}$. More general representation for a fuzzy set is $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x))\}$*

Definition 2.2. *The $\alpha-$cut of the fuzzy set $\widetilde{A}$ of the universe of discourse X is defined as $\widetilde{A}_\alpha = \{x \in X/\mu_{\widetilde{A}}(x) \geq \alpha\}$, where $\alpha \in [0,1]$.*

Definition 2.3. *A fuzzy set $\widetilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be a fuzzy number if its membership function $\widetilde{A} : \mathbb{R} \to [0,1]$ has the following characteristics.*

(i) $\widetilde{A}$ is convex

$\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\widetilde{A}}(x_1),\, \mu_{\widetilde{A}}(x_2)),\ \forall x \in [0,1],\ \lambda \in [0,1]$

(ii) $\widetilde{A}$ is normal

i.e. there exists an $x \in \mathbb{R}$ such that if $\max \mu_{\widetilde{A}}(x) = 1$.

(iii) $\widetilde{A}$ is piecewise continuous.

Definition 2.4. *A triangular fuzzy number $\widetilde{A}$ denoted by (a_1, a_2, a_3), and the membership function is defined as*

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & \textit{for } a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & \textit{for } a_2 \leq x \leq a_3 \\ 0, & \textit{elsewhere.} \end{cases}$$

Definition 2.5. *A trapezoidal fuzzy number $\widetilde{A}$ can be defined as (a_1, a_2, a_3, a_4), and the membership function is defined as*

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \dfrac{(a_4-x)}{(a_4-a_3)}, & a_3 \leq x \leq a_4 \\ 0, & \textit{otherwise.} \end{cases}$$

Definition 2.6. *A linguistic variable/term is a variable whose value is not crisp number but word or sentence in a natural language.*

3. Nonagonal Fuzzy Number (NFN)

Definition 3.1. *A Nonagonal fuzzy number* $\widetilde{N}$ *denoted as* $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$, *and the membership function is defined as*

$$\mu_{\widetilde{N}}(x) = \begin{cases} \frac{1}{4}\frac{(x-a_1)}{(a_2-a_1)}, & a_1 \le x \le a_2 \\ \frac{1}{4}+\frac{1}{4}\frac{(x-a_2)}{(a_3-a_2)}, & a_2 \le x \le a_3 \\ \frac{1}{2}+\frac{1}{4}\frac{(x-a_3)}{(a_4-a_3)}, & a_3 \le x \le a_4 \\ \frac{3}{4}+\frac{1}{4}\frac{(x-a_4)}{(a_5-a_4)}, & a_4 \le x \le a_5 \\ 1-\frac{1}{4}\frac{(x-a_5)}{(a_6-a_5)}, & a_5 \le x \le a_6 \\ \frac{3}{4}-\frac{1}{4}\frac{(x-a_6)}{(a_7-a_6)}, & a_6 \le x \le a_7 \\ \frac{1}{2}-\frac{1}{4}\frac{(x-a_7)}{(a_8-a_7)}, & a_7 \le x \le a_8 \\ \frac{1}{4}\frac{(a_9-x)}{(a_9-a_8)}, & a_8 \le x \le a_9 \\ 0, & \textit{otherwise.} \end{cases}$$

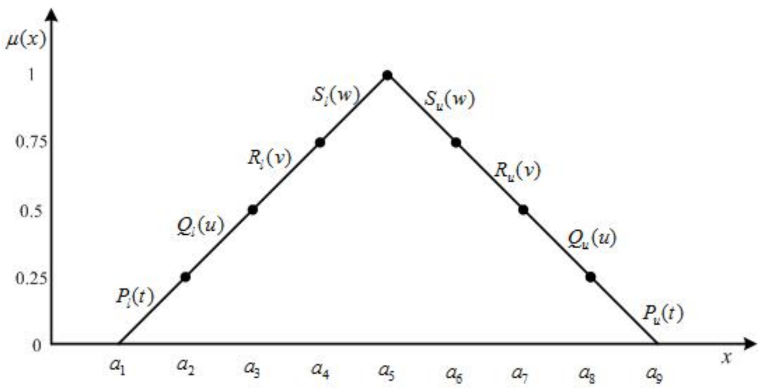

Figure 1. **The Nonagonal fuzzy number**

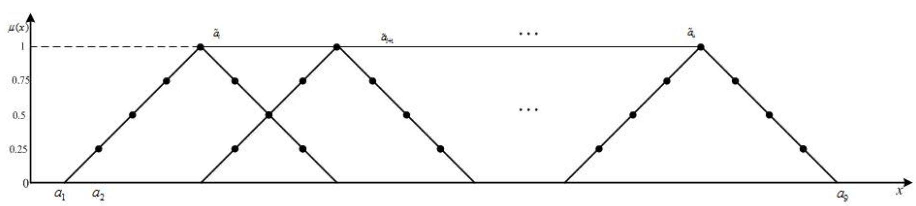

Figure 2. **The Nonagonal fuzzy number from uncertain linguistic term** $[s_l, s_u]$

Furthermore, in light of the research work by Kaufman, A [5] and Zadeh, LA [13], we provide the operations of uncertain linguistic terms in the following.

Table 1. **Nonagonal Fuzzy linguistic scale**

Linguistic terms	Linguistic values
No influence	(0, 0, 0, 0, 0, 0.04, 0.08, 0.12, 0.16)
Very Low influence	(0, 0.04, 0.08, 0.12, 0.16, 0.20, 0.24, 0.28)
Low influence	(0.16, 0.2, 0.24, 0.28, 0.32, 0.36, 0.4, 0.44, 0.48)
Medium	(0.32, 0.36, 0.4, 0.44, 0.48, 0.52, 0.56, 0.6, 0.64)
High influence	(0.48, 0.52, 0.56, 0.6, 0.64, 0.68, 072, 0.76, 0.8)
Very High influence	(0.64, 0.68, 072, 0.76, 0.8, 0.84, 0.88, 0.92, 0.96)
Very Very High influence	(0.8, 0.84, 0.88, 0.92, 0.96, 1, 1, 1, 1)

4. Arithmetic Operations of Nonagonal Fuzzy Numbers (NFN)

In this section, arithmetic operations of NFN based on fuzzy extension principle and α-cut method.

4.1. Arithmetic Operation on Extension Principle

Definition 4.1. *Let $f : X \to Y$ be a mapping from a set X to a set Y, then the extension principle define the Fuzzy Set $\widetilde{B}$ in Y and induced by the fuzzy set $\widetilde{A}$ in X through as follows,*

$$\widetilde{B} = \{(y, \mu_{\widetilde{B}}(y)|y = f(x), x \in X)\}$$

$$\mu_{\widetilde{B}}(y) = \begin{cases} Sup_{y=f(x)}\mu_{\widetilde{A}}(x) & : \ f^{-1}(y) \neq \emptyset \\ 0 & : \ f^{-1}(y) = \emptyset \end{cases}$$

Definition 4.2. *The arithmetic operation (*) of two Fuzzy Numbers is a mapping an input vector $X = [x_1, x_2]^T$ define in the cartesian product space $R \times R$ onto an output y define in the real space R. If $\widetilde{A}_1$ and $\widetilde{A}_2$ are Fuzzy Numbers then their outcome of arithmetic operation is also a fuzzy number determined with the formula.*
$(\widetilde{A}_1 * \widetilde{A}_2)(y) = \left\{(y, sup_{y=x_1*x_2}\left[\min(\mu_{\widetilde{A}_1}, \mu_{\widetilde{A}_1})\right], \quad \forall x_1, x_2, y \in R\right\}$

4.2. Arithmetic Operation on α-cut

Definition 4.3. *A Nonagonal fuzzy number $\widetilde{N}$ can also be defined as $\widetilde{N} = P_l(t), Q_l(u), R_l(v), S_l(w), P_u(t), Q_u(u), R_u(v), S_u(w)$, $t \in [0, 0.25]$, $u \in [0.25, 0.5]$, $v \in [0.5, 0.75]$ and $w \in [0, 1]$, where*

$$P_1(t) = \frac{1}{4}\frac{(x - a_1)}{(a_2 - a_1)},\ Q_1(u) = \frac{1}{4} + \frac{1}{4}\frac{(x - a_2)}{(a_3 - a_2)},\ R_1(v) = \frac{1}{2} + \frac{1}{4}\frac{(x - a_3)}{(a_4 - a_3)},\ S_1(w) = \frac{3}{4} + \frac{1}{4}\frac{(x - a_4)}{(a_5 - a_4)},$$

$$S_2(w) = 1 - \frac{1}{4}\frac{(x - a_5)}{(a_6 - a_5)},\ R_2(v) = \frac{3}{4} - \frac{1}{4}\frac{(x - a_6)}{(a_7 - a_6)},\ Q_2(u) = \frac{1}{2} - \frac{1}{4}\frac{(x - a_7)}{(a_8 - a_7)},\ P_2(t) = \frac{1}{4}\frac{(a_9 - x)}{(a_9 - a_8)}$$

Here,

- $P_1(t), Q_1(u), R_1(v), S_1(w)$, *is bounded and continuous incresing function over* $[0, 0.25]$, $[0.25, 0.5]$, $[0.5, 0.75]$ *and* $[0, 1]$ *respectively.*
- $P_2(t), Q_2(u), R_2(v), S_2(w)$, *is bounded and continuous decresing function over* $[0, 0.25]$, $[0.25, 0.5]$, $[0.5, 0.75]$ *and* $[0, 1]$ *respectively.*

Definition 4.4. *The α-cut of the fuzzy set of the universe of discourse X is defined as $\widetilde{D}_\alpha = \{x \in X/\mu_{\widetilde{A}}(x) \geq \alpha\}$, where $\alpha \in [0,1]$.*

$$\widetilde{D}_\alpha = \begin{cases} [P_1(\alpha), P_2(\alpha)], & \textit{for } \alpha \in [0, 0.25) \\ [Q_1(\alpha), Q_2(\alpha)], & \textit{for } \alpha \in [0.25, 0.5) \\ [R_1(\alpha), R_2(\alpha)], & \textit{for } \alpha \in [0.5, 0.75) \\ [S_1(\alpha), S_2(\alpha)], & \textit{for } \alpha \in [0.75, 1] \end{cases}$$

Definition 4.5. *If $P_1(x) = \alpha$ and $P_2(x) = \alpha$, then α cut operations interval $\widetilde{D}_\alpha$ is obtained as*

1. $[P_1(\alpha), P_2(\alpha)] = [4\alpha(a_2 - a_1) + a_1, -4\alpha(a_9 - a_8) + a_9)]$
   Similarly, we can obtain $\alpha-$ cut operations interval $\widetilde{D}_\alpha$ for $[Q_1(\alpha), Q_2(\alpha)]$, $[R_1(\alpha), R_2(\alpha)]$ and $[S_1(\alpha), S_2(\alpha)]$ as follows;

2. $[Q_1(\alpha), Q_2(\alpha)] = [4\alpha(a_3 - a_2) + 2a_2 - a_3, -4\alpha(a_8 - a_7) + 2a_8 - a_7]$

3. $[R_1(\alpha), R_2(\alpha)] = [4\alpha(a_4 - a_3) + 3a_3 - 2a_4, -4\alpha(a_7 - a_6) + 3a_7 - 2a_6]$

4. $[S_1(\alpha), S_2(\alpha)] = [4\alpha(a_5 - a_4) + 4a_4 - 3a_5, -4\alpha(a_6 - a_5) + 4a_6 - 3a_5]$

Hence, α-cut of Decagonal Fuzzy Number

$$\widetilde{D}_\alpha = \begin{cases} [4\alpha(a_2 - a_1) + a_1, -4\alpha(a_9 - a_8) + a_9)], & \textit{for } \alpha \in [0, 0.25) \\ [4\alpha(a_3 - a_2) + 2a_2 - a_3, -4\alpha(a_8 - a_7) + 2a_8 - a_7], & \textit{for } \alpha \in [0.25, 0.5) \\ [4\alpha(a_4 - a_3) + 3a_3 - 2a_4, -4\alpha(a_7 - a_6) + 3a_7 - 2a_6], & \textit{for } \alpha \in [0.5, 0.75) \\ [4\alpha(a_5 - a_4) + 4a_4 - 3a_5, -4\alpha(a_6 - a_5) + 4a_6 - 3a_5], & \textit{for } \alpha \in [0.75, 1] \end{cases}$$

Theorem 4.1. *If $\widetilde{A}$=(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9) and $\widetilde{B}$=(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9) are the Nonagonal Fuzzy Numbers, then $\widetilde{C}$=$\widetilde{A} \oplus \widetilde{B}$ is also Nonagonal Fuzzy Numbers $\widetilde{A} \oplus \widetilde{B}$=($a_1 + b_1$, $a_2 + b_2$, $a_3 + b_3$, $a_4 + b_4$, $a_5 + b_5$, $a_6 + b_6$, $a_7 + b_7$, $a_8 + b_8$, $a_9 + b_9$)*

Proof. The membership function of NFN $\widetilde{C}$=$\widetilde{A} \oplus \widetilde{B}$ can be found by $\alpha-$cut method with the transform $z = x + y$, $\alpha-$cut membership function of $\widetilde{A}(x)$ is,

$$x \in \begin{cases} [4\alpha(a_2 - a_1) + a_1, -4\alpha(a_9 - a_8) + a_9)], & \text{for } \alpha \in [0, 0.25) \\ [4\alpha(a_3 - a_2) + 2a_2 - a_3, -4\alpha(a_8 - a_7) + 2a_8 - a_7], & \text{for } \alpha \in [0.25, 0.5) \\ [4\alpha(a_4 - a_3) + 3a_3 - 2a_4, -4\alpha(a_7 - a_6) + 3a_7 - 2a_6], & \text{for } \alpha \in [0.5, 0.75) \\ [4\alpha(a_5 - a_4) + 4a_4 - 3a_5, -4\alpha(a_6 - a_5) + 4a_6 - 3a_5], & \text{for } \alpha \in [0.75, 1] \end{cases}$$

$\alpha-$cut membership function of $\widetilde{B}(x)$ is,

$$y \in \begin{cases} [4\alpha(b_2-b_1)+b_1, -4\alpha(b_9-b_8)+b_9)], & \text{for } \alpha \in [0, 0.25) \\ [4\alpha(b_3-b_2)+2b_2-b_3, -4\alpha(b_8-b_7)+2b_8-b_7], & \text{for } \alpha \in [0.25, 0.5) \\ [4\alpha(b_4-b_3)+3b_3-2b_4, -4\alpha(b_7-b_6)+3b_7-2b_6], & \text{for } \alpha \in [0.5, 0.75) \\ [4\alpha(b_5-b_4)+4b_4-3b_5, -4\alpha(b_6-b_5)+4b_6-3b_5], & \text{for } \alpha \in [0.75, 1] \end{cases}$$

so,

$$z = x + y \in \begin{cases} [4\alpha(a_2-a_1)+a_1, -4\alpha(a_9-a_8)+a_9)]+ & \\ [4\alpha(b_2-b_1)+b_1, -4\alpha(b_9-b_8)+b_9)], & \text{for } \alpha \in [0, 0.25) \\ [4\alpha(a_3-a_2)+2a_2-a_3, -4\alpha(a_8-a_7)+2a_8-a_7]+ & \\ [4\alpha(b_3-b_2)+2b_2-b_3, -4\alpha(b_8-b_7)+2b_8-b_7] & \text{for } \alpha \in [0.25, 0.5) \\ [4\alpha(a_4-a_3)+3a_3-2a_4, -4\alpha(a_7-a_6)+3a_7-2a_6]+ & \\ [4\alpha(b_4-b_3)+3b_3-2b_4, -4\alpha(b_7-b_6)+3b_7-2b_6] & \text{for } \alpha \in [0.5, 0.75) \\ [4\alpha(a_5-a_4)+4a_4-3a_5, -4\alpha(a_6-a_5)+4a_6-3a_5]+ & \\ [4\alpha(b_5-b_4)+4b_4-3b_5, -4\alpha(b_6-b_5)+4b_6-3b_5] & \text{for } \alpha \in [0.75, 1] \end{cases}$$

the membership function of $\widetilde{C}=\widetilde{A} \oplus \widetilde{B}$ is,

$$\mu_{\widetilde{C}}(x) = \begin{cases} \dfrac{z-(a_1+b_1)}{4[(a_2-a_1)+(b_2-b_1)]} & a_1+b_1 \le z \le a_2+b_2 \\ \dfrac{z-2(a_2+b_2)+(a_3+b_3)}{4[(a_3-a_2)+(b_3-b_2)]} & a_2+b_2 \le z \le a_3+b_3 \\ \dfrac{z-3(a_3+b_3)+2(a_4+b_4)}{4[(a_4-a_3)+(b_4-b_3)]} & a_3+b_3 \le z \le a_4+b_4 \\ \dfrac{z-4(a_4+b_4)+3(a_5+b_5)}{4[(a_5-a_4)+(b_5-b_4)]} & a_4+b_4 \le z \le a_5+b_5 \\ \dfrac{z-4(a_6+b_6)+3(a_5+b_5)}{4[(a_5-a_6)+(b_5-b_6)]} & a_5+b_5 \le z \le a_6+b_6 \\ \dfrac{z-3(a_7+b_7)+2(a_6+b_6)}{4[(a_6-a_7)+(b_6-b_7)]} & a_6+b_6 \le z \le a_7+b_7 \\ \dfrac{z-2(a_8+b_8)+(a_7+b_7)}{4[(a_5-a_6)+(b_5-b_6)]} & a_7+b_7 \le z \le a_8+b_8 \\ \dfrac{z-(a_9+b_9)}{4[(a_8-a_9)+(b_8-b_9)]} & a_8+b_8 \le z \le a_9+b_9 \\ 0, & \text{otherwise.} \end{cases}$$

Hence, addition rule is proved. Therefore we have $\widetilde{A} \oplus \widetilde{B}$=($a_1+b_1$, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6, a_7+b_7, a_8+b_8 a_9+b_9) is a Nonagonal Fuzzy Number. □

Theorem 4.2. *If* $\widetilde{A}$=(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9) *and* $\widetilde{B}$=(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9) *are the Nonagonal Fuzzy Numbers, then* $\widetilde{P}=\widetilde{A} \otimes \widetilde{B}$ *is also Nonagonal Fuzzy Numbers* $\widetilde{A} \otimes \widetilde{B}$=($a_1b_1$, a_2b_2, a_3b_3, a_4b_4, a_5b_5, a_6b_6, a_7b_7, a_8b_8, a_9b_9)

Proof. The membership function of NFN $\widetilde{P}$=$\widetilde{A} \otimes \widetilde{B}$ can be found by $\alpha-$cut method with the transform $z = x \times y$,

$$z = x \times y \in \begin{cases} [4\alpha(a_2 - a_1) + a_1, -4\alpha(a_9 - a_8) + a_9)]\times & \\ [4\alpha(b_2 - b_1) + b_1, -4\alpha(b_9 - b_8) + b_9)], & \text{for } \alpha \in [0, 0.25) \\ [4\alpha(a_3 - a_2) + 2a_2 - a_3, -4\alpha(a_8 - a_7) + 2a_8 - a_7]\times & \\ [4\alpha(b_3 - b_2) + 2b_2 - b_3, -4\alpha(b_8 - b_7) + 2b_8 - b_7] & \text{for } \alpha \in [0.25, 0.5) \\ [4\alpha(a_4 - a_3) + 3a_3 - 2a_4, -4\alpha(a_7 - a_6) + 3a_7 - 2a_6]\times & \\ [4\alpha(b_4 - b_3) + 3b_3 - 2b_4, -4\alpha(b_7 - b_6) + 3b_7 - 2b_6] & \text{for } \alpha \in [0.5, 0.75) \\ [4\alpha(a_5 - a_4) + 4a_4 - 3a_5, -4\alpha(a_6 - a_5) + 4a_6 - 3a_5]\times & \\ [4\alpha(b_5 - b_4) + 4b_4 - 3b_5, -4\alpha(b_6 - b_5) + 4b_6 - 3b_5] & \text{for } \alpha \in [0.75, 1] \end{cases}$$

So, the membership function of $\widetilde{P}$=$\widetilde{A} \otimes \widetilde{B}$ is,

$$\mu_{\widetilde{P}}(x) = \begin{cases} \dfrac{-B_1 + \sqrt{B_1^2 - 4A_1(a_1b_1 - z)}}{2A_1} & a_1b_1 \le z \le a_2b_2 \\ \dfrac{-B_2 + \sqrt{B_2^2 - 4A_2(4a_2b_2 - 2(a_2b_3 + a_3b_2) + a_3b_3 - z)}}{2A_2} & a_2b_2 \le z \le a_3b_3 \\ \dfrac{-B_3 + \sqrt{B_3^2 - 4A_3(9a_3b_3 - 6(a_3b_4 + a_4b_3) + 4a_4b_4 - z)}}{2A_3} & a_3b_3 \le z \le a_4b_4 \\ \dfrac{-B_4 + \sqrt{B_4^2 - 4A_4(16a_4b_4 - 12(a_4b_5 - a_5b_4) + 9a_5b_5 - z)}}{2A_4} & a_4b_4 \le z \le a_5b_5 \\ \dfrac{B_5 - \sqrt{B_5^2 - 4A_5(16a_5b_5 - 12(a_5b_6 - a_6b_5) + 9a_6b_6 - z)}}{2A_5} & a_5b_5 \le z \le a_6b_6 \\ \dfrac{B_6 - \sqrt{B_6^2 - 4A_6(9a_6b_6 - 6(a_6b_7 + a_7b_6) + 4a_7b_7 - z)}}{2A_3} & a_6b_6 \le z \le a_7b_7 \\ \dfrac{B_7 - \sqrt{B_7^2 - 4A_7(4a_7b_7 - 2(a_7b_8 + a_8b_7) + a_8b_8 - z)}}{2A_2} & a_7b_7 \le z \le a_8b_8 \\ \dfrac{B_8 - \sqrt{B_8^2 - 4A_8(a_9b_9 - z)}}{2A_8} & a_8b_8 \le z \le a_9b_9 \\ 0, & \text{otherwise.} \end{cases}$$

where $A_i = 16(a_{i+1} - a_i)(b_{i+1} - b_i)$, $i = 1, 2 \cdots, 8$ $B_1 = 4[b_1(a_2 - a_1) + a_1(b_2 - b_1)]$, $B_j = 4[(a_{j+1} - a_j)(jb_j - b_{j+1}) + (b_{j+1} - b_j)(ja_j - (j-1)a_{j+1})]$, $j = 2, 3, 4$, $B_5 = -4[(a_6 - a_5)(4b_5 - 3b_6) + (b_6 - b_5)(4a_5 - 3a_6)]$, $B_6 = -4[(a_7 - a_6)(3b_6 - 2b_7) + (b_7 - b_6)(3a_6 - 2a_7)]$, $B_7 = -4[(a_8 - a_7)(2b_7 - b_8) + (b_8 - b_7)(2b_7 - a_8)]$, $B_8 = -4[b_8(a_9 - a_8) + a_8(b_9 - b_8)]$, Hence, multiplication rule is proved. Therefore we have $\widetilde{A} \otimes \widetilde{B}$=$(a_1b_1,\ a_2b_2,\ a_3b_3,\ a_4b_4,\ a_5b_5,\ a_6b_6,\ a_7b_7,\ a_8b_8,\ a_9b_9)$ is a Nonagonal Fuzzy Number.

□

Theorem 4.3. *If $\widetilde{A}$=$(a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6,\ a_7,\ a_8,\ a_9)$ and $\widetilde{B}$=$(b_1,\ b_2,\ b_3,\ b_4,\ b_5,\ b_6,\ b_7,\ b_8,\ b_9)$ are the Nonagonal Fuzzy Numbers, then $\widetilde{D}$=$\widetilde{A} \oslash \widetilde{B}$ is also Nonagonal Fuzzy Numbers $\widetilde{A} \oslash \widetilde{B}$=$\left(\frac{a_1}{b_9}, \frac{a_2}{b_8}, \frac{a_3}{b_7}, \frac{a_4}{b_6}, \frac{a_5}{b_5}, \frac{a_6}{b_4}, \frac{a_7}{b_3}, \frac{a_8}{b_2}, \frac{a_9}{b_1}\right)$*

Proof. The membership function of NFN $\widetilde{D}$=$\widetilde{A} \oslash \widetilde{B}$ can be found by $\alpha-$cut method with the transform $z = x \times y$,

$$z = x \div y \in \begin{cases} [4\alpha(a_2 - a_1) + a_1, -4\alpha(a_9 - a_8) + a_9)]\div & \\ [4\alpha(b_2 - b_1) + b_1, -4\alpha(b_9 - b_8) + b_9)], & \text{for } \alpha \in [0, 0.25) \\ & \\ [4\alpha(a_3 - a_2) + 2a_2 - a_3, -4\alpha(a_8 - a_7) + 2a_8 - a_7]\div & \\ [4\alpha(b_3 - b_2) + 2b_2 - b_3, -4\alpha(b_8 - b_7) + 2b_8 - b_7] & \text{for } \alpha \in [0.25, 0.5) \\ & \\ [4\alpha(a_4 - a_3) + 3a_3 - 2a_4, -4\alpha(a_7 - a_6) + 3a_7 - 2a_6]\div & \\ [4\alpha(b_4 - b_3) + 3b_3 - 2b_4, -4\alpha(b_7 - b_6) + 3b_7 - 2b_6] & \text{for } \alpha \in [0.5, 0.75) \\ & \\ [4\alpha(a_5 - a_4) + 4a_4 - 3a_5, -4\alpha(a_6 - a_5) + 4a_6 - 3a_5]\div & \\ [4\alpha(b_5 - b_4) + 4b_4 - 3b_5, -4\alpha(b_6 - b_5) + 4b_6 - 3b_5] & \text{for } \alpha \in [0.75, 1] \end{cases}$$

So, the membership function of $\widetilde{P}$=$\widetilde{A} \oslash \widetilde{B}$ is,

$$\mu_{\widetilde{D}}(x) = \begin{cases} \dfrac{zb_9 - a_1}{4[(a_2 - a_1) + z(b_9 - b_8)]} & \dfrac{a_1}{b_9} \le z \le \dfrac{a_2}{b_8} \\ \dfrac{z(2b_8 - b_7) - (a_2 - a_3)}{4[(a_3 - a_2) + z(b_8 - b_7)]} & \dfrac{a_2}{b_8} \le z \le \dfrac{a_3}{b_7} \\ \dfrac{z(3b_7 - 2b_6) - (3a_3 - 2a_4)}{4[(a_4 - a_3) + z(b_7 - b_6)]} & \dfrac{a_3}{b_7} \le z \le \dfrac{a_4}{b_6} \\ \dfrac{z(4b_6 - 3b_5) - (4a_4 - 3a_5)}{4[(a_5 - a_4) + z(b_6 - b_5)]} & \dfrac{a_4}{b_6} \le z \le \dfrac{a_5}{b_5} \\ \dfrac{(4b_6 - 3a_5) - z(4b_4 - 3b_5)}{4[(a_6 - a_5) + z(b_5 - b_4)]} & \dfrac{a_1}{b_9} \le z \le \dfrac{a_2}{b_8} \\ \dfrac{(3a_7 - 2a_6) - z(3b_3 - 2b_4)}{4[(a_7 - a_6) + z(b_4 - b_3)]} & \dfrac{a_2}{b_8} \le z \le \dfrac{a_3}{b_7} \\ \dfrac{(2a_8 - a_7) - z(2b_2 - b_3)}{4[(a_8 - a_7) + z(b_3 - b_2)]} & \dfrac{a_3}{b_7} \le z \le \dfrac{a_4}{b_6} \\ \dfrac{a_9 - zb_1}{4[(a_9 - a_8) + z(b_2 - b_1)]} & \dfrac{a_4}{b_6} \le z \le \dfrac{a_5}{b_5} \\ & \\ 0, & \text{otherwise.} \end{cases}$$

Hence, division rule is proved. Therefore we have $\widetilde{A} \oslash \widetilde{B}$=$\left(\frac{a_1}{b_9}, \frac{a_2}{b_8}, \frac{a_3}{b_7}, \frac{a_4}{b_6}, \frac{a_5}{b_5}, \frac{a_6}{b_4}, \frac{a_7}{b_3}, \frac{a_8}{b_2}, \frac{a_9}{b_1}\right)$ is a Nonagonal Fuzzy Number. $\square$

5. Numerical Example of Arithmetic operation on Nonagonal Fuzzy Number

Here, numerical example of arithmetic operation of NFN based on fuzzy extension principle and on α- cut method is presented.

5.1. Addition of Two NFN by Fuzzy Extension Principle

Let us consider two NFN $\tilde{A} = (1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5)$ and $\tilde{B} = (2, 3, 4, 5, 6, 7, 8, 9, 10)$ and $\tilde{C} = \tilde{A} \oplus \tilde{B}$. Then,

$$\mu_{\tilde{C}}(z) = \sup\{\min\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)\right) : x + y = z\}$$

In order to show the computational procedure involve in the above equation for membership function, first choose a value for z, then evaluate $\min\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right)$ for x and y which add up to 5.5. It is done for certain value for x and y as shown in table-2, where the maximum occurs at $x = 2.5$ and $y = 3$. Hence, $\mu_{\tilde{C}} = 0.5$. By doing the sameway for others values of z, finally we get as $\tilde{C} = \tilde{A} \oplus \tilde{B} = (3.5, 5, 6.5, 8, 9.5, 11, 12.5, 14, 15.5)$ a NFN.

Table 2. **Finding membership function of sum of two NFN**

x	$\mu_{\tilde{A}}(x)$	y	$\mu_{\tilde{B}}(x)$	$\min\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right)$
1.5	0	4	0.5	0
2	0.25	3.5	0.375	0.25
2.5	0.5	3	0.25	0.5
3	0.75	2.5	0.125	0.125
3.45	1	2	0	0
4	0.75	1.5	0	0

Using theorem 4.2, the addition of these two NFN is defined as $\tilde{C} = \tilde{A} \oplus \tilde{B} = (3.5, 5, 6.5, 8, 9.5, 11, 12.5, 14, 15.5)$ with membership function as follows

5.2. Addition of Two NFN by $\alpha-$cut method

Let us consider two NFN $\tilde{A} = (1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5)$ and $\tilde{B} = (2, 3, 4, 5, 6, 7, 8, 9, 10)$. Using theorem 4.6, the addition of these two NFN is defined as $\tilde{A} \oplus \tilde{B} = (3.5, 5, 6.5, 8, 9.5, 11, 12.5, 14, 15.5)$ with membership function as follows

$$\mu_{\tilde{C}}(x) = \begin{cases} \frac{1}{4}\frac{(x-3.5)}{1.5}, & 3.5 \le x \le 5 \\ \frac{1}{4}\frac{(x-3.5)}{1.5}, & 5 \le x \le 6.5 \\ \frac{1}{4}\frac{(x-3.5)}{1.5}, & 6 \le x \le 8 \\ \frac{1}{4}\frac{(x-3.5)}{1.5}, & 8 \le x \le 9.5 \\ \frac{-1}{4}\frac{(x-15.5)}{1.5}, & 9.5 \le x \le 11 \\ \frac{-1}{4}\frac{(x-15.5)}{1.5}, & 11 \le x \le 12.5 \\ \frac{-1}{4}\frac{(x-15.5)}{1.5}, & 12.5 \le x \le 14 \\ \frac{-1}{4}\frac{(x-15.5)}{1.5}, & 14 \le x \le 15.5 \\ 0, & \text{otherwise.} \end{cases}$$

5.3. Multiplication of Two NFN by $\alpha-$cut method

Let us consider two NFN $\tilde{A} = (1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5)$ and $\tilde{B} = (2, 3, 4, 5, 6, 7, 8, 9, 10)$. Using theorem 4.7, the multiplication of these two NFN is defined as $\tilde{A} \otimes \tilde{B} = (3, 6, 10, 15, 21, 28, 36, 45, 55)$ with membership function as follows

$$\mu_{\tilde{P}}(x) = \begin{cases} \frac{1}{4}\frac{(x-3)}{3}, & 3 \le x \le 6 \\ \frac{1}{4}\frac{(x-2)}{4}, & 6 \le x \le 10 \\ \frac{1}{4}\frac{(x)}{5}, & 10 \le x \le 15 \\ \frac{1}{4}\frac{(x+3)}{6}, & 15 \le x \le 21 \\ \frac{-1}{4}\frac{(x-49)}{7}, & 21 \le x \le 28 \\ \frac{-1}{4}\frac{(x-52)}{8}, & 28 \le x \le 36 \\ \frac{-1}{4}\frac{(x-54)}{9}, & 36 \le x \le 45 \\ \frac{-1}{4}\frac{(x-45)}{10}, & 45 \le x \le 55 \\ 0, & \text{otherwise.} \end{cases}$$

5.4. Division of Two NFN by $\alpha-$cut method

Let us consider two NFN $\tilde{A} = (1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5)$ and $\tilde{B} = (2, 3, 4, 5, 6, 7, 8, 9, 10)$. Using theorem 4.8, the multiplication of these two NFN is defined as $\tilde{A} \oslash \tilde{B} = (0.15, 0.2, 0.31, 0.43, 0.58, 0.8, 1.13, 1.67, 2.75)$ with membership function as follows

$$\mu_{\tilde{C}}(x) = \begin{cases} \frac{1}{4}\frac{(x-1.5)}{0.3}, & 0.15 \le x \le 0.2 \\ \frac{1}{4} + \frac{1}{4}\frac{(x-0.2)}{0.11}, & 0.2 \le x \le 0.31 \\ \frac{1}{2} + \frac{1}{4}\frac{(x-0.31)}{0.12}, & 0.31 \le x \le 0.43 \\ \frac{3}{4} + \frac{1}{4}\frac{(x-0.43)}{0.15}, & 0.43 \le x \le 0.58 \\ 1 - \frac{1}{4} - \frac{(x-0.58)}{0.22}, & 0.58 \le x \le 0.8 \\ \frac{3}{4} - \frac{1}{4}\frac{(x-0.8)}{0.33}, & 0.8 \le x \le 1.13 \\ \frac{1}{2} - \frac{1}{4}\frac{(x-1.13)}{0.54}, & 1.13 \le x \le 1.67 \\ \frac{1}{4}\frac{(2.75-x)}{1.08}, & 1.67 \le x \le 2.75 \\ 0, & \text{otherwise.} \end{cases}$$

6. Conclusion

In this paper, nonagonal fuzzy number and its arithmetic operations have been proposed on the basis of fuzzy extension principle of and $\alpha-$cut. Also, linguistic value of NFN is introduced. Therefore, decision making techniques such as DEMATEL and TOPSIS method can be extended by representing linguistic variable in to nonagonal fuzzy number under uncertain liguistic environment will be the further research.

Acknowledgement

This research is supported by UGC scheme MANF. Award letter F1-17.1/2012-13/MANF-2012-13-CHR-TAM-11197 / (SA-III/Website).

References

[1] D.Dubois and H.Prade, *Operations on Fuzzy Numbers*, International Journal of Systems Science, 9(6)(1978), 613-626.

[2] P.S.Dwyer, *Fuzzy sets*, Information and Control, 8(1965), 338-353.

[3] Z.P.Fan and Y.Liu, *A method for Group Decision-Making based on Multi-Granularity uncertain Linguistic Information*, Expert Syst Appl, 37(5)(2010), 4000-4008.

[4] A.Felix and A.Victor Devadoss, *A new Decagonal Fuzzy Number under Uncertain Linguistic Environment*, International journal of Mathematics and Its Applications, 3(1)(2015), 89-97.

[5] A.Kauffmann and M.Gupta, *Introduction to Fuzzy Arithmetic: Theory and Applications*, Van Nostrand Reinhold, New York (1980).

[6] S.U.Malini and F.C.Kennedy, *An approach for solving Fuzzy Transportation using Octagonal Fuzzy numbers*, Applied Mathematical Sciences, 54(2013), 2661-2673.

[7] P.Rajarajeshwari, A.S. Sudha and R. Karthika, *A new Operation on Hexagonal Fuzzy Number*, International Journal of Fuzzy Logic Systems, 3(3)(2013), 15-26.

[8] S.Rezvani, *Multiplication Operation on Trapezoidal Fuzzy numbers*, Journal of Physical Sciences, 15(2011), 17-26.

[9] A.Victor Devadoss and A.Felix, *A new Fuzzy DEMATEL method in an Uncertain Linguistic Environment*, Advances in Fuzzy Sets and Systems, 16(2)(2013), 93-123.

[10] A.Victor Devadoss and A.Felix, *A Fuzzy DEMATEL approach to Study cause and effect relationship of Youth Violence*, International Journal of Computing Algorithm, 2(2013), 363-372.

[11] W.L.Suo WL, B.Feng and Z.P.Fan, *Extension of the DEMATEL method in an Uncertain Linguistic Environment*, Soft Comput, 16(2012), 471-483.

[12] L.A.Zadeh, *Fuzzy Sets*, Information and Control, 8(3)(1965), 338-353.

[13] L.A.Zadeh, *The concept of a Linguistic variable and its Application to Approximate Reasoning (Part II)*, Information Science, 8(1975), 301-357.

[14] H.J.Zimmermann, *Fuzzy Set Theory and its Application*, Fourth Edition, Springer, (2011).