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# An Approach to the Relative Type Oriented Growth Analysis of Differential Polynomials

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**Abstract :** The aim of this paper is to investigate the comparative growth analysis of composite entire and meromorphic functions on the basis of relative type and relative lower type of differential polynomials generated by entire and meromorphic functions.

**Keywords :** Entire function, meromorphic function, relative type, relative lower type, differential polynomials.

**AMS Subject Classification:** 30D30, 30D35, 30D20.

## 1 Introduction

Let $f$ be an entire function defined in the open complex plane $\mathbb{C}$. The function $M_f(r)$ on $|z| = r$ known as maximum modulus function corresponding to $f$ is defined as follows:

$$M_f(r) = \max_{|z|=r} |f(z)| \ .$$

When $f$ is meromorphic, $M_f(r)$ can not be defined as $f$ is not analytic. In this situation one may define another function $T_f(r)$ known as Nevanlinna's Characteristic function of $f$, playing the same role as $M_f(r)$ in the following manner:

$$T_f(r) = N_f(r) + m_f(r) \ .$$

And given two meromorphic functions $f$ and $g$ the ratio $\frac{T_f(r)}{T_g(r)}$ as $r \to \infty$ is called the growth of $f$ with respect to $g$ in terms of their Nevanlinna's Characteristic function.
When $f$ is entire function, the Nevanlinna's Characteristic function $T_f(r)$ of $f$ is defined as

$$T_f(r) = m_f(r) \ .$$

We called the function $N_f(r,a) \left(\bar{N}_f(r,a)\right)$ as counting function of $a$-points (distinct $a$-points) of $f$. In many occasions $N_f(r,\infty)$ and $\bar{N}_f(r,\infty)$ are denoted by $N_f(r)$ and $\bar{N}_f(r)$ respectively. We put

$$N_f(r,a) = \int_0^r \frac{n_f(t,a) - n_f(0,a)}{t} dt + \bar{n}_f(0,a) \log r \ ,$$

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where we denote by $n_f(r,a)\left(\bar{n}_f(r,a)\right)$ the number of $a$-points (distinct $a$-points) of $f$ in $|z|\leq r$ and an $\infty$ -point is a pole of $f$ and the quantity $\Theta(a;f)$ of a meromorphic function $f$ is defined as follows

$$\Theta(a;f)=1-\limsup_{r\to\infty}\frac{\bar{N}(r,a;f)}{T(r,f)}.$$

Also we denote by $n_p(r,a;f)$ denotes the number of zeros of $f-a$ in $|z|\leq r$,where a zero of multiplicity $<p$ is counted according to its multiplicity and a zero of multiplicity $\geqslant p$ is counted exactly $p$ times.

Accordingly, $N_p(r,a;f)$ is defined in terms of $n_p(r,a;f)$ in the usual way and we set for any $a\in\mathbb{C}\cup\{\infty\}$

$$\delta_p(a;f)=1-\limsup_{r\to\infty}\frac{N_p(r,a;f)}{T(r,f)}\quad\{\text{ cf. [6]}\},$$

On the other hand, $m\left(r,\frac{1}{f-a}\right)$ is denoted by $m_f(r,a)$ and we mean $m_f(r,\infty)$ by $m_f(r)$ , which is called the proximity function of $f$. We also put

$$m_f(r)=\frac{1}{2\pi}\int_0^{2\pi}\log^+\left|f\left(re^{i\theta}\right)\right|d\theta,\quad\text{where}$$

$$\log^+x=\max(\log x,0)\ \text{ for all } x\geqslant 0\ .$$

Further for any non-constant meromorphic function $f$, $b\equiv b(z)$ is called small with respect to $f$ if $T_b(r)=S_f(r)$ where $S_f(r)=o\{T_f(r)\}$ i.e., $\frac{S_f(r)}{T_f(r)}\to 0$ as $r\to\infty$. Moreover for any non-constant meromorphic function $f$ , we call $M_j[f]=A_j(f)^{n_{0j}}\left(f^{(1)}\right)^{n_{1j}}\ldots\ldots.\left(f^{(k)}\right)^{n_{kj}}$ where $T_{A_j}(r)=S_f(r)$, to be a differential monomial generated by it where $n_{0j}$, $n_{1j}$,......,$n_{kj}$ $(k\geq 1)$ be non-negative integers such that for each $j,\sum_{i=0}^{k}n_{ij}\geq 1$ . In this connection the numbers $\gamma_{M_j}=\sum_{i=0}^{k}n_{ij}$ and $\Gamma_{M_j}=\sum_{i=0}^{k}(i+1)\,n_{ij}$ are called respectively the degree and weight of $M_j[f]$ $\{[2],[8]\}$. The expression $P[f]=\sum_{j=1}^{s}M_j[f]$ is called a differential polynomial generated by $f$. The numbers $\gamma_P=\max_{1<j<s}\gamma_{M_j}$ and $\Gamma_P=\max_{1<j<s}\Gamma_{M_j}$ are called respectively the degree and weight of $P[f]$ $\{[2],[8]\}$. Also we call the numbers $\underline{\gamma}_P=\min_{1<j<s}\gamma_{M_j}$ and $k$ (the order of the highest derivative of $f$ ) the lower degree and the order of $P[f]$ respectively. If $\underline{\gamma}_P=\gamma_P$, $P[f]$ is called a homogeneous differential polynomial. Throughout the paper we consider only the non-constant differential polynomials and we denote by $P_0[f]$ a differential polynomial not containing $f$ i.e. for which $n_{0j}=0$ for $j=1,2,......,s$. We consider only those $P[f],P_0[f]$ singularities of whose individual terms do not cancel each other.

The *order* of a meromorphic function $f$ which is generally used in computational purpose is defined in terms of the growth of $f$ with respect to the exponential function as

$$\rho_f=\limsup_{r\to\infty}\frac{\log T_f(r)}{\log T_{\exp z}(r)}=\limsup_{r\to\infty}\frac{\log T_f(r)}{\log\left(\frac{r}{\pi}\right)}=\limsup_{r\to\infty}\frac{\log T_f(r)}{\log(r)+O(1)}\ .$$

Lahiri and Banerjee [7] introduced the *relative order* (respectively *relative lower order*) of a meromorphic function with respect to an entire function to avoid comparing growth just with $\exp z$. To compare the relative growth of two meromorphic functions having same non zero finite *relative order* with respect to another entire function, Datta and Biswas [3] introduced the notion of *relative type* of meromorphic functions with respect to an entire function. Extending these notions of *relative type* as cited in the reference, Datta, Biswas and Hoque [4] gave the definition of *relative type* of *differential polynomials* generated by entire and meromorphic functions.

For entire and meromorphic functions, the notion of their growth indicators such as *order* and *type* are classical in complex analysis and during the past decades, several researchers have already been continuing their studies in the area of comparative growth properties of composite entire and meromorphic functions in different directions using the same. But at that time, the concept of *relative order* and consequently *relative type* of entire and meromorphic functions with respect to another entire function was mostly unknown to complex analysts and they are not aware of the technical advantages of using the relative growth indicators of the functions. Therefore the growth of composite entire and meromorphic functions needs to be modified on the basis of their *relative order* and *relative type* some of which has been explored in this paper. Actually in this paper we establish some newly developed results based on the growth properties of *relative type* of *differential polynomials* generated by entire and meromorphic functions.

## 2 Notation and Preliminary Remarks

We use the standard notations and definitions of the theory of entire and meromorphic functions which are available in [5] and [9]. Henceforth, we do not explain those in details. Now we just recall some definitions which will be needed in the sequel.

**Definition 2.1.** *The* order $\rho_f$ *and* lower order $\lambda_f$ *of a meromorphic function* $f$ *are defined as*

$$\rho_f = \limsup_{r\to\infty} \frac{\log T_f(r)}{\log r} \text{ and } \lambda_f = \liminf_{r\to\infty} \frac{\log T_f(r)}{\log r} .$$

The notion of *type* ( *lower type*) to determine the relative growth of two meromorphic functions having same non zero finite order is classical in complex analysis and is given by

**Definition 2.2.** *The* type $\sigma_f$ *and* lower type $\overline{\sigma}_f$ *of a meromorphic function* $f$ *are defined as*

$$\sigma_f = \limsup_{r\to\infty} \frac{T_f(r)}{r^{\rho_f}} \quad \text{and} \quad \overline{\sigma}_f = \liminf_{r\to\infty} \frac{T_f(r)}{r^{\rho_f}}, \quad 0 < \rho_f < \infty.$$

Given a non-constant entire function $f$ defined in the open complex plane $\mathbb{C}$, its Nevanlinna's Characteristic function is strictly increasing and continuous. Hence there exists its inverse function $T_g^{-1} : (T_g(0), \infty) \to (0, \infty)$ with $\lim_{s\to\infty} T_g^{-1}(s) = \infty$.

Lahiri and Banerjee [7] introduced the definition of *relative order* of a meromorphic function $f$ with respect to an entire function $g$ , denoted by $\rho_g(f)$ as follows:

$$\begin{aligned} \rho_g(f) &= \inf\{\mu > 0 : T_f(r) < T_g(r^\mu) \text{ for all sufficiently large } r\} \\ &= \limsup_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log r}. \end{aligned}$$

The definition coincides with the classical one [7] if $g(z) = \exp z$.

In the case of relative order, it therefore seems reasonable to define suitably the relative type of a meromorphic function with respect to an entire function to determine the relative growth of two meromorphic functions having same non zero finite relative order with respect to an entire function. Datta and Biswas [3] gave such definitions of relative type of a meromorphic function $f$ with respect to an entire function $g$ which is as follows:

**Definition 2.3** ([3])**.** *The relative type* $\sigma_g(f)$ *of a meromorphic function* $f$ *with respect to an entire function* $g$ *are defined as*

$$\sigma_g(f) = \limsup_{r\to\infty} \frac{T_g^{-1} T_f(r)}{r^{\rho_g(f)}}, \quad \text{where } 0 < \rho_g(f) < \infty.$$

*Likewise,one can define the lower relative type* $\overline{\sigma}_g(f)$ *in the following way:*

$$\overline{\sigma}_g(f) = \liminf_{r\to\infty} \frac{T_g^{-1} T_f(r)}{r^{\rho_g(f)}}, \quad \text{where } 0 < \rho_g(f) < \infty.$$

**Definition 2.4** ([1])**.** $P[f]$ *is said to be admissible if*

*(i)* $P[f]$ *is homogeneous, or*

*(ii)* $P[f]$ *is non homogeneous and* $m(r,f) = S(r,f)$ *.*

# 3 Some Examples

In this section we present some examples in connection with definitions given in the previous section.

**Example 3.1** (Order)**.** *Given any natural number* $n$*, let* $f(z) = \exp z^n$*. Then* $M_f(r) = \exp r^n$*. Therefore putting* $R = 2$ *in the inequality* $T_f(r) \leq \log M_f(r) \leq \frac{R+r}{R-r} T_f(R)$ $\{$*cf. [5]*$\}$ *we get that* $T_f(r) \leq r^n$ *and* $T_f(r) \geq \frac{1}{3}\left(\frac{r}{2}\right)^n$*. Hence*

$$\rho_f = \limsup_{r\to\infty} \frac{\log T_f(r)}{\log r} = n \ .$$

*Further if we take* $g = \exp^{[2]} z$*, then* $T_g(r) \sim \frac{\exp r}{(2\pi^3 r)^{\frac{1}{2}}} (r \to \infty)$*. Therefore*

$$\rho_f = \infty \ .$$

**Example 3.2** (Type (lower type))**.** *Let us consider* $f = \exp z$*. Then* $T_f(r) = \frac{r}{\pi}$*. and* $\rho_f = 1$*. So*

$$\sigma_f = \limsup_{r\to\infty} \frac{T_f(r)}{r^{\rho_f}} = \frac{\frac{r}{\pi}}{r} = \frac{1}{\pi} \quad \textit{and} \quad \overline{\sigma}_f = \liminf_{r\to\infty} \frac{T_f(r)}{r^{\rho_f}} = \frac{\frac{r}{\pi}}{r} = \frac{1}{\pi} \ .$$

*Similarly, if we consider* $g = \frac{1}{1+\exp z}$*, then we can also see that*

$$\sigma_g = \overline{\sigma}_g = \frac{1}{\pi} \ .$$

**Example 3.3** (Relative order)**.** *Suppose* $f = g = \exp^{[2]} z$ *then* $T_f(r) = T_g(r) \sim \frac{\exp r}{(2\pi^3 r)^{\frac{1}{2}}} (r \to \infty)$*. Now we obtain that*

$$\begin{aligned} T_g(r) &\leq \log M_g(r) \leq 3T_g(2r) \quad \{\textit{cf. [5]}\} \\ \textit{i.e.}, T_g(r) &\leq \exp r \leq 3T_g(2r) \ . \end{aligned}$$

*Therefore*

$$\begin{aligned} T_g^{-1} T_f(r) &\geq \log\left(\frac{\exp r}{(2\pi^3 r)^{\frac{1}{2}}}\right) \\ \textit{i.e.}, \liminf_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log r} &\geq 1 \end{aligned}$$

*and*

$$\begin{aligned} T_g^{-1} T_f(r) &\leq 2\log\left(\frac{3\exp r}{(2\pi^3 r)^{\frac{1}{2}}}\right) \\ \textit{i.e.}, \limsup_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log r} &\leq 1 \ . \end{aligned}$$

*Hence*

$$\rho_g(f) = \lambda_g(f) = 1 \ .$$

**Example 3.4** (Relative type ( relative lower type))**.** *Let* $f = g = \exp z$*. Now* $T_f(r) = T_g(r) = T_{\exp z}(r) = \frac{r}{\pi}$*. Therefore*

$$T_g^{-1} T_f(r) = T_g^{-1}\left(\frac{r}{\pi}\right) = r \ .$$

*So*

$$\rho_g(f) = \limsup_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log r} = 1.$$

*Hence*

$$\sigma_g(f) = \limsup_{r\to\infty} \frac{T_g^{-1} T_f(r)}{r^{\rho_g(f)}} = 1 \textit{ and } \overline{\sigma}_g(f) = \liminf_{r\to\infty} \frac{T_g^{-1} T_f(r)}{r^{\rho_g(f)}} = 1.$$

# 4 Lemmas

In this section we present a lemma which will be needed in the sequel.

**Lemma 4.1** ([4]). *If $f$ be a meromorphic function either of finite order or of non-zero lower order such that $\Theta(\infty; f) = \sum_{a\neq\infty} \delta_p(a; f) = 1$ or $\delta(\infty; f) = \sum_{a\neq\infty} \delta(a; f) = 1$ and $g$ be an entire function of regular growth having non zero finite order and $\Theta(\infty; g) = \sum_{a\neq\infty} \delta_p(a; g) = 1$ or $\delta(\infty; g) = \sum_{a\neq\infty} \delta(a; g) = 1$. Then the relative order of $P_0[f]$ with respect to $P_0[g]$ are same as those of $f$ with respect to $g$ where $P_0[f]$ and $P_0[g]$ are homogeneous.*

**Lemma 4.2** ([4]). *If $f$ be a meromorphic function either of finite order or of non-zero lower order such that $\sum_{a\neq\infty} \Theta(a; f) = 2$ and $g$ be an entire function of regular growth having non zero finite order and $\sum_{a\neq\infty} \Theta(a; g) = 2$. Then the relative order of $P_0[f]$ with respect to $P_0[g]$ are same as those of $f$ with respect to $g$ where $P_0[f]$ and $P_0[g]$ are admissible.*

**Lemma 4.3** ([4]). *If $f$ be a meromorphic function either of finite order or of non-zero lower order such that $\Theta(\infty; f) = \sum_{a\neq\infty} \delta_p(a; f) = 1$ or $\delta(\infty; f) = \sum_{a\neq\infty} \delta(a; f) = 1$ and $g$ be an entire function of regular growth having non zero finite type and $\Theta(\infty; g) = \sum_{a\neq\infty} \delta_p(a; g) = 1$ or $\delta(\infty; g) = \sum_{a\neq\infty} \delta(a; g) = 1$. Then the relative type and relative lower type of $P_0[f]$ with respect to $P_0[g]$ are $\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}}$ times that of $f$ with respect to $g$ if $\rho_g(f)$ is positive finite and $P_0[f]$ and $P_0[g]$ are homogeneous.*

**Lemma 4.4** ([4]). *If $f$ be a meromorphic function either of finite order or of non-zero lower order such that $\sum_{a\neq\infty} \Theta(a; f) = 2$ and $g$ be an entire function of regular growth having non zero finite type and $\sum_{a\neq\infty} \Theta(a; g) = 2$, then the relative type and relative lower type of $P_0[f]$ with respect to $P_0[g]$ are $\left(\frac{\Gamma_{P_0[f]}}{\Gamma_{P_0[g]}}\right)^{\frac{1}{\rho_g}}$ times that of $f$ with respect to $g$ if $\rho_g(f)$ is positive finite and $P_0[f]$ and $P_0[g]$ are admissible.*

# 5 Theorems

In this section, we present the main results of the paper. In the paper, it is needless to mention that the admissibility and homogeneity of $P_0[f]$ will be needed as per the requirements of the theorems.

**Theorem 5.1.** *Suppose $f$ be a meromorphic function either of finite order or of non-zero lower order such that $\Theta(\infty; f) = \sum_{a\neq\infty} \delta_p(a; f) = 1$ or $\delta(\infty; f) = \sum_{a\neq\infty} \delta(a; f) = 1$. Also let $h$ be an entire function of regular growth having non zero finite type with $\Theta(\infty; h) = \sum_{a\neq\infty} \delta_p(a; h) = 1$ or $\delta(\infty; h) = \sum_{a\neq\infty} \delta(a; h) = 1$ and $g$ be any entire function such that $0 < \overline{\sigma}_h(f\circ g) \le \sigma_h(f\circ g) < \infty$, $0 < \overline{\sigma}_h(f) \le \sigma_h(f) < \infty$ and $\rho_h(f\circ g) = \rho_h(f)$. Then*

$$\frac{\overline{\sigma}_h(f\circ g)}{\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(f)} \le \liminf_{r\to\infty} \frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} \le \frac{\overline{\sigma}_h(f\circ g)}{\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(f)} \le \limsup_{r\to\infty} \frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} \le \frac{\sigma_h(f\circ g)}{\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(f)}.$$

*Proof.* From the definition of $\sigma_h(f)$, $\overline{\sigma}_h(f\circ g)$ and in view of Lemma 4.1, Lemma 4.3 we have for arbitrary positive $\varepsilon$ and for all sufficiently large values of $r$ that

$$T_h^{-1} T_{f\circ g}(r) \geqslant (\overline{\sigma}_h(f\circ g) - \varepsilon)(r)^{\rho_h(f\circ g)} \tag{5.1}$$

and

$$T_{P_0[h]}^{-1} T_{P_0[f]}(r) \le \left(\sigma_{P_0[h]}(P_0[f]) + \varepsilon\right)(r)^{\rho_{P_0[h]}(P_0[f])}$$

$$i.e.,\ T^{-1}_{P_0[h]}T_{P_0[f]}(r) \leq \left(\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(f) + \varepsilon\right)(r)^{\rho_h(f)} . \quad (5.2)$$

Now from (5.1), (5.2) and in view of the condition $\rho_h(f \circ g) = \rho_h(f)$, it follows for all large values of $r$ that

$$\frac{T_h^{-1}T_{f\circ g}(r)}{T^{-1}_{P_0[h]}T_{P_0[f]}(r)} \geqslant \frac{(\overline{\sigma}_h(f\circ g) - \varepsilon)}{\left(\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(f) + \varepsilon\right)} .$$

As $\varepsilon(> 0)$ is arbitrary , we obtain from above that

$$\liminf_{r\to\infty}\frac{T_h^{-1}T_{f\circ g}(r)}{T^{-1}_{P_0[h]}T_{P_0[f]}(r)} \geqslant \frac{\overline{\sigma}_h(f\circ g)}{\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(f)} . \quad (5.3)$$

Again for a sequence of values of $r$ tending to infinity ,

$$T_h^{-1}T_{f\circ g}(r) \leq (\overline{\sigma}_h(f\circ g) + \varepsilon)(r)^{\rho_h(f\circ g)} \quad (5.4)$$

and for all sufficiently large values of $r$ ,

$$T^{-1}_{P_0[h]}T_{P_0[f]}(r) \geqslant \left(\overline{\sigma}_{P_0[h]}(P_0[f]) - \varepsilon\right)(r)^{\rho_{P_0[h]}(P_0[f])}$$

$$i.e.,\ T^{-1}_{P_0[h]}T_{P_0[f]}(r) \geq \left(\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(f) - \varepsilon\right)(r)^{\rho_h(f)} . \quad (5.5)$$

Combining (5.4) and (5.5) and in view of the condition $\rho_h(f\circ g) = \rho_h(f)$, we get for a sequence of values of $r$ tending to infinity that

$$\frac{T_h^{-1}T_{f\circ g}(r)}{T^{-1}_{P_0[h]}T_{P_0[f]}(r)} \leq \frac{(\overline{\sigma}_h(f\circ g) + \varepsilon)}{\left(\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(f) - \varepsilon\right)} .$$

Since $\varepsilon(> 0)$ is arbitrary, it follows from above that

$$\liminf_{r\to\infty}\frac{T_h^{-1}T_{f\circ g}(r)}{T^{-1}_{P_0[h]}T_{P_0[f]}(r)} \leq \frac{\overline{\sigma}_h(f\circ g)}{\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(f)} . \quad (5.6)$$

Also for a sequence of values of $r$ tending to infinity that

$$T^{-1}_{P_0[h]}T_{P_0[f]}(r) \leq \left(\overline{\sigma}_{P_0[h]}(P_0[f]) + \varepsilon\right)(r)^{\rho_{P_0[h]}(P_0[f])}$$

$$i.e.,\ T^{-1}_{P_0[h]}T_{P_0[f]}(r) \leq \left(\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(f) + \varepsilon\right)(r)^{\rho_h(f)} . \quad (5.7)$$

Now from (5.1), (5.7) and in view of the condition $\rho_h(f\circ g) = \rho_h(f)$, we obtain for a sequence of values of $r$ tending to infinity that

$$\frac{T_h^{-1}T_{f\circ g}(r)}{T^{-1}_{P_0[h]}T_{P_0[f]}(r)} \geq \frac{(\overline{\sigma}_h(f\circ g) - \varepsilon)}{\left(\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(f) + \varepsilon\right)} .$$

As $\varepsilon(> 0)$ is arbitrary, we get from above that

$$\limsup_{r\to\infty}\frac{T_h^{-1}T_{f\circ g}(r)}{T^{-1}_{P_0[h]}T_{P_0[f]}(r)} \geq \frac{\overline{\sigma}_h(f\circ g)}{\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(f)} . \quad (5.8)$$

Also for all sufficiently large values of $r$ ,

$$T_h^{-1}T_{f\circ g}(r) \leq (\sigma_h(f\circ g) + \varepsilon)(r)^{\rho_h(f\circ g)} . \quad (5.9)$$

As the condition $\rho_h(f \circ g) = \rho_h(f)$, it follows from (5.5) and (5.9) for all sufficiently large values of $r$ that

$$\frac{T_h^{-1} T_{f \circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} \leq \frac{(\sigma_h(f \circ g) + \varepsilon)}{\left(\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(f) - \varepsilon\right)}.$$

Since $\varepsilon (> 0)$ is arbitrary, we obtain that

$$\limsup_{r \to \infty} \frac{T_h^{-1} T_{f \circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} \leq \frac{\sigma_h(f \circ g)}{\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(f)}. \quad (5.10)$$

Thus the theorem follows from (5.3), (5.6), (5.8) and (5.10). □

**Remark 5.2.** *If we take* $\sum_{a \neq \infty} \Theta(a; f) = 2$ *and* $\sum_{a \neq \infty} \Theta(a; h) = 2$ *instead of* $\Theta(\infty; f) = \sum_{a \neq \infty} \delta_p(a; f) = 1$ *or* $\delta(\infty; f) = \sum_{a \neq \infty} \delta(a; f) = 1$ *and* $\Theta(\infty; h) = \sum_{a \neq \infty} \delta_p(a; h) = 1$ *or* $\delta(\infty; h) = \sum_{a \neq \infty} \delta(a; h) = 1$ *respectively in Theorem 5.1 and the other conditions remain the same, then with the help of Lemma 4.2 and Lemma 4.4 one can easily prove that*

$$\frac{\overline{\sigma}_h(f \circ g)}{\left(\frac{\Gamma_{P_0[f]}}{\Gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(f)} \leq \liminf_{r \to \infty} \frac{T_h^{-1} T_{f \circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} \leq \frac{\overline{\sigma}_h(f \circ g)}{\left(\frac{\Gamma_{P_0[f]}}{\Gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(f)} \leq \limsup_{r \to \infty} \frac{T_h^{-1} T_{f \circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} \leq \frac{\sigma_h(f \circ g)}{\left(\frac{\Gamma_{P_0[f]}}{\Gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(f)}.$$

The following theorem can be proved in the line of Theorem 5.1 and so its proof is omitted:

**Theorem 5.3.** *Suppose* $g$ *be an entire function either of finite order or of non-zero lower order such that* $\sum_{a \neq \infty} \Theta(a; g) = 2$. *Also let* $h$ *be an entire function of regular growth having non zero finite type with* $\sum_{a \neq \infty} \Theta(a; h) = 2$ *and* $f$ *be any meromorphic function such that* $0 < \overline{\sigma}_h(f \circ g) \leq \sigma_h(f \circ g) < \infty$, $0 < \overline{\sigma}_h(g) \leq \sigma_h(g) < \infty$ *and* $\rho_h(f \circ g) = \rho_h(g)$. *Then*

$$\frac{\overline{\sigma}_h(f \circ g)}{\left(\frac{\Gamma_{P_0[g]}}{\Gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(g)} \leq \liminf_{r \to \infty} \frac{T_h^{-1} T_{f \circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[g]}(r)} \leq \frac{\overline{\sigma}_h(f \circ g)}{\left(\frac{\Gamma_{P_0[g]}}{\Gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(g)} \leq \limsup_{r \to \infty} \frac{T_h^{-1} T_{f \circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[g]}(r)} \leq \frac{\sigma_h(f \circ g)}{\left(\frac{\Gamma_{P_0[g]}}{\Gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(g)}.$$

**Remark 5.4.** *If we consider* $\Theta(\infty; g) = \sum_{a \neq \infty} \delta_p(a; g) = 1$ *or* $\delta(\infty; g) = \sum_{a \neq \infty} \delta(a; g) = 1$ *and* $\Theta(\infty; h) = \sum_{a \neq \infty} \delta_p(a; h) = 1$ *or* $\delta(\infty; h) = \sum_{a \neq \infty} \delta(a; h) = 1$ *instead of* $\sum_{a \neq \infty} \Theta(a; g) = 2$ *and* $\sum_{a \neq \infty} \Theta(a; h) = 2$ *respectively in Theorem 5.3 and the other conditions remain the same then with the help of Lemma 4.1 and Lemma 4.3 it can easily be proved that*

$$\frac{\overline{\sigma}_h(f \circ g)}{\left(\frac{\gamma_{P_0[g]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(g)} \leq \liminf_{r \to \infty} \frac{T_h^{-1} T_{f \circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[g]}(r)} \leq \frac{\overline{\sigma}_h(f \circ g)}{\left(\frac{\gamma_{P_0[g]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(g)} \leq \limsup_{r \to \infty} \frac{T_h^{-1} T_{f \circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[g]}(r)} \leq \frac{\sigma_h(f \circ g)}{\left(\frac{\gamma_{P_0[g]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \overline{\sigma}_h(g)}.$$

**Theorem 5.5.** *Suppose $f$ be a meromorphic function either of finite order or of non-zero lower order such that $\Theta(\infty; f) = \sum_{a \neq \infty} \delta_p(a; f) = 1$ or $\delta(\infty; f) = \sum_{a \neq \infty} \delta(a; f) = 1$. Also let $h$ be an entire function of regular growth having non zero finite type with $\Theta(\infty; h) = \sum_{a \neq \infty} \delta_p(a; h) = 1$ or $\delta(\infty; h) = \sum_{a \neq \infty} \delta(a; h) = 1$ and $g$ be any entire function such that $0 < \sigma_h(f \circ g) < \infty$, $0 < \sigma_h(f) < \infty$ and $\rho_h(f \circ g) = \rho_h(f)$. Then*

$$\liminf_{r\to\infty} \frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} \leq \frac{\sigma_h(f\circ g)}{\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(f)} \leq \limsup_{r\to\infty} \frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} .$$

*Proof.* From the definition of $\sigma_{P_0[h]}(P_0[f])$ and in view of Lemma 4.1 and Lemma 4.3, we get for a sequence of values of $r$ tending to infinity that

$$T_{P_0[h]}^{-1} T_{P_0[f]}(r) \geqslant \left(\sigma_{P_0[h]}(P_0[f]) - \varepsilon\right)(r)^{\rho_{P_0[h]}(P_0[f])}$$

$$i.e.,\ T_{P_0[h]}^{-1} T_{P_0[f]}(r) \geq \left(\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(f) - \varepsilon\right)(r)^{\rho_h(f)} . \tag{5.11}$$

Now from (5.9), (5.11) and in view of the condition $\rho_h(f \circ g) = \rho_h(f)$, it follows for a sequence of values of $r$ tending to infinity that

$$\frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} \leq \frac{(\sigma_h(f\circ g) + \varepsilon)}{\left(\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(f) - \varepsilon\right)} .$$

As $\varepsilon (> 0)$ is arbitrary, we obtain that

$$\liminf_{r\to\infty} \frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} \leq \frac{\sigma_h(f\circ g)}{\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(f)} . \tag{5.12}$$

Again for a sequence of values of $r$ tending to infinity ,

$$T_h^{-1} T_{f\circ g}(r) \geqslant (\sigma_h(f\circ g) - \varepsilon)(r)^{\rho_h(f\circ g)} . \tag{5.13}$$

So combining (5.2) and (5.13) and in view of the condition $\rho_h(f \circ g) = \rho_h(f)$, we get for a sequence of values of $r$ tending to infinity that

$$\frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} \geqslant \frac{(\sigma_h(f\circ g) - \varepsilon)}{\left(\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(f) + \varepsilon\right)} .$$

Since $\varepsilon (> 0)$ is arbitrary, it follows that

$$\limsup_{r\to\infty} \frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} \geqslant \frac{\sigma_h(f\circ g)}{\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(f)} . \tag{5.14}$$

Thus the theorem follows from (5.12) and (5.14). ☐

**Remark 5.6.** *If we take $\sum_{a \neq \infty} \Theta(a; f) = 2$ and $\sum_{a \neq \infty} \Theta(a; h) = 2$ instead of $\Theta(\infty; f) = \sum_{a \neq \infty} \delta_p(a; f) = 1$ or $\delta(\infty; f) = \sum_{a \neq \infty} \delta(a; f) = 1$ and $\Theta(\infty; h) = \sum_{a \neq \infty} \delta_p(a; h) = 1$ or $\delta(\infty; h) = \sum_{a \neq \infty} \delta(a; h) = 1$ respectively in Theorem 5.5 and the other conditions remain the same then with the help of Lemma 4.2 and Lemma 4.4 one can easily prove that*

$$\liminf_{r\to\infty} \frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} \leq \frac{\sigma_h(f\circ g)}{\left(\frac{\Gamma_{P_0[f]}}{\Gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(f)} \leq \limsup_{r\to\infty} \frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[f]}(r)} .$$

The following theorem can be carried out in the line of Theorem 5.5 and therefore we omit its proof.

**Theorem 5.7.** *Suppose $g$ be an entire function either of finite order or of non-zero lower order such that $\sum_{a \neq \infty} \Theta(a; g) = 2$. Also let $h$ be an entire function of regular growth having non zero finite type with $\sum_{a \neq \infty} \Theta(a; h) = 2$ and $f$ be any meromorphic function such that $0 < \sigma_h(f \circ g) < \infty$, $0 < \sigma_h(g) < \infty$ and $\rho_h(f \circ g) = \rho_h(g)$. Then*

$$\liminf_{r\to\infty} \frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[g]}(r)} \leq \frac{\sigma_h(f\circ g)}{\left(\frac{\Gamma_{P_0[g]}}{\Gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}} \cdot \sigma_h(g)} \leq \limsup_{r\to\infty} \frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[g]}(r)} .$$

**Remark 5.8.** *If we consider* $\Theta(\infty;g) = \sum\limits_{a\neq\infty}\delta_p(a;g) = 1$ *or* $\delta(\infty;g) = \sum\limits_{a\neq\infty}\delta(a;g) = 1$ *and* $\Theta(\infty;h) = \sum\limits_{a\neq\infty}\delta_p(a;h) = 1$ *or* $\delta(\infty;h) = \sum\limits_{a\neq\infty}\delta(a;h) = 1$ *instead of* $\sum\limits_{a\neq\infty}\Theta(a;g) = 2$ *and* $\sum\limits_{a\neq\infty}\Theta(a;h) = 2$ *respectively in Theorem 5.7 and the other conditions remain the same then with the help of Lemma 4.1 and Lemma 4.3 one can easily prove that*

$$\liminf_{r\to\infty}\frac{T_h^{-1}T_{f\circ g}(r)}{T_{P_0[h]}^{-1}T_{P_0[g]}(r)} \leq \frac{\sigma_h(f\circ g)}{\left(\frac{\gamma_{P_0[g]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}}\cdot\sigma_h(g)} \leq \limsup_{r\to\infty}\frac{T_h^{-1}T_{f\circ g}(r)}{T_{P_0[h]}^{-1}T_{P_0[g]}(r)}.$$

The following theorem is a natural consequence of Theorem 5.1 and Theorem 5.5:

**Theorem 5.9.** *Suppose $f$ be a meromorphic function either of finite order or of non-zero lower order such that* $\Theta(\infty;f) = \sum\limits_{a\neq\infty}\delta_p(a;f) = 1$ *or* $\delta(\infty;f) = \sum\limits_{a\neq\infty}\delta(a;f) = 1$*. Also let $h$ be an entire function of regular growth having non zero finite type with* $\Theta(\infty;h) = \sum\limits_{a\neq\infty}\delta_p(a;h) = 1$ *or* $\delta(\infty;h) = \sum\limits_{a\neq\infty}\delta(a;h) = 1$ *and $g$ be any entire function such that* $0 < \overline{\sigma}_h(f\circ g) \leq \sigma_h(f\circ g) < \infty$, $0 < \overline{\sigma}_h(f) \leq \sigma_h(f) < \infty$ *and* $\rho_h(f\circ g) = \rho_h(f)$*. Then*

$$\liminf_{r\to\infty}\frac{T_h^{-1}T_{f\circ g}(r)}{T_{P_0[h]}^{-1}T_{P_0[f]}(r)} \leq \min\left\{A\cdot\frac{\overline{\sigma}_h(f\circ g)}{\overline{\sigma}_h(f)}, A\cdot\frac{\sigma_h(f\circ g)}{\sigma_h(f)}\right\}$$
$$\leq \max\left\{A\cdot\frac{\overline{\sigma}_h(f\circ g)}{\overline{\sigma}_h(f)}, A\cdot\frac{\sigma_h(f\circ g)}{\sigma_h(f)}\right\} \leq \limsup_{r\to\infty}\frac{T_h^{-1}T_{f\circ g}(r)}{T_{P_0[h]}^{-1}T_{P_0[f]}(r)}$$

*where* $A = \frac{1}{\left(\frac{\gamma_{P_0[f]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}}}$.

**Remark 5.10.** *If we take* $\sum\limits_{a\neq\infty}\Theta(a;f) = 2$ *and* $\sum\limits_{a\neq\infty}\Theta(a;h) = 2$ *instead of* $\Theta(\infty;f) = \sum\limits_{a\neq\infty}\delta_p(a;f) = 1$ *or* $\delta(\infty;f) = \sum\limits_{a\neq\infty}\delta(a;f) = 1$ *and* $\Theta(\infty;h) = \sum\limits_{a\neq\infty}\delta_p(a;h) = 1$ *or* $\delta(\infty;h) = \sum\limits_{a\neq\infty}\delta(a;h) = 1$ *respectively in Theorem 5.9 and the other conditions remain the same then with the help of Lemma 4.2 and Lemma 4.4 one can easily prove that*

$$\liminf_{r\to\infty}\frac{T_h^{-1}T_{f\circ g}(r)}{T_{P_0[h]}^{-1}T_{P_0[f]}(r)} \leq \min\left\{B\cdot\frac{\overline{\sigma}_h(f\circ g)}{\overline{\sigma}_h(f)}, B\cdot\frac{\sigma_h(f\circ g)}{\sigma_h(f)}\right\}$$
$$\leq \max\left\{B\cdot\frac{\overline{\sigma}_h(f\circ g)}{\overline{\sigma}_h(f)}, B\cdot\frac{\sigma_h(f\circ g)}{\sigma_h(f)}\right\} \leq \limsup_{r\to\infty}\frac{T_h^{-1}T_{f\circ g}(r)}{T_{P_0[h]}^{-1}T_{P_0[f]}(r)}$$

*where* $B = \frac{1}{\left(\frac{\Gamma_{P_0[f]}}{\Gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}}}$.

Analogously one may state the following theorem without its proof.

**Theorem 5.11.** *Suppose $g$ be an entire function of finite order or of non-zero lower order and* $\sum\limits_{a\neq\infty}\Theta(a;g) = 2$*. Also let $h$ be an entire function of regular growth having non zero finite type with* $\sum\limits_{a\neq\infty}\Theta(a;h) = 2$ *and $f$ be any meromorphic function such that* $0 < \overline{\sigma}_h(f\circ g) \leq \sigma_h(f\circ g) < \infty$, $0 < \overline{\sigma}_h(g) \leq \sigma_h(g) < \infty$ *and* $\rho_h(f\circ g) = \rho_h(g)$*. Then*

$$\liminf_{r\to\infty}\frac{T_h^{-1}T_{f\circ g}(r)}{T_{P_0[h]}^{-1}T_{P_0[g]}(r)} \leq \min\left\{C\cdot\frac{\overline{\sigma}_h(f\circ g)}{\overline{\sigma}_h(g)}, C\cdot\frac{\sigma_h(f\circ g)}{\sigma_h(g)}\right\}$$
$$\leq \max\left\{C\cdot\frac{\overline{\sigma}_h(f\circ g)}{\overline{\sigma}_h(g)}, C\cdot\frac{\sigma_h(f\circ g)}{\sigma_h(g)}\right\} \leq \limsup_{r\to\infty}\frac{T_h^{-1}T_{f\circ g}(r)}{T_{P_0[h]}^{-1}T_{P_0[g]}(r)}$$

*where* $C = \frac{1}{\left(\frac{\Gamma_{P_0[g]}}{\Gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}}}$.

**Remark 5.12.** *If we consider* $\Theta(\infty;g) = \sum\limits_{a\neq\infty}\delta_p(a;g) = 1$ *or* $\delta(\infty;g) = \sum\limits_{a\neq\infty}\delta(a;g) = 1$ *and* $\Theta(\infty;h) = \sum\limits_{a\neq\infty}\delta_p(a;h) = 1$ *or* $\delta(\infty;h) = \sum\limits_{a\neq\infty}\delta(a;h) = 1$ *instead of* $\sum\limits_{a\neq\infty}\Theta(a;g) = 2$ *and* $\sum\limits_{a\neq\infty}\Theta(a;h) = 2$ *respectively in*

*Theorem 5.11 and the other conditions remain the same then with the help of Lemma 4.1 and Lemma 4.3 one can easily prove that*

$$\liminf_{r\to\infty} \frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[g]}(r)} \leq \min\left\{ D\cdot \frac{\overline{\sigma}_h(f\circ g)}{\overline{\sigma}_h(g)}, D\cdot \frac{\sigma_h(f\circ g)}{\sigma_h(g)} \right\}$$

$$\leq \max\left\{ D\cdot \frac{\overline{\sigma}_h(f\circ g)}{\overline{\sigma}_h(g)}, D\cdot \frac{\sigma_h(f\circ g)}{\sigma_h(g)} \right\} \leq \limsup_{r\to\infty} \frac{T_h^{-1} T_{f\circ g}(r)}{T_{P_0[h]}^{-1} T_{P_0[g]}(r)}$$

*where* $D = \frac{1}{\left(\frac{\gamma_{P_0[g]}}{\gamma_{P_0[h]}}\right)^{\frac{1}{\rho_h}}}$.

# 6 Conclusion

Actually this paper deals with the extension of the works on the growth properties *differential polynomials* generated by entire and meromorphic functions on the basis of their *relative types*. These theories can also be modified by the treatment of the notions of *generalized relative type* and *(p,q)-th relative type*. In addition some extensions of the same may be done in the light of slowly changing functions. Moreover the notion of *relative type* of *differential polynomials* generated by entire and meromorphic functions may has a wide range of applications in complex dynamics, factorization theory of entire functions of single complex variable, the solution of complex differential equations etc. which might be a strong and effective area of further research.

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