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# On $\ddot{g}$-Interior and $\ddot{g}$-Closure in Fuzzy Topological Spaces

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**Abstract :** In this paper, we introduce fuzzy $\ddot{g}$-interior and fuzzy $\ddot{g}$-closure and study some of its basic properties.

**Keywords :** Fuzzy topological spaces, fuzzy $\ddot{g}$-interior, fuzzy $\ddot{g}$-closure.

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## 1 Introduction

G. Balasubramanian and P. Sundram[2] introduced fuzzy generalized closed sets in general fuzzy topology. This concept was found to be useful and many results in general fuzzy topology were improved. Many researchers like Sudha [12] introduced f$\omega$-closed sets in fuzzy topological spaces.After the advent of these notions, many fuzzy topologists introduced various types of fuzzy generalized closed sets and studied their fundamental properties. Quite Recently, jeyaraman etal.[6] introduced and studied $f\ddot{g}$-closed sets in general fuzzy topology as another fuzzy generalization of closed sets and proved that the class of $f\ddot{g}$-closed sets properly lies between the class of closed sets and the class of fsg-closed sets. In this paper, the notion of fuzzy $\ddot{g}$-interior (briefly $f\ddot{g}$-*int*) is defined and some of its basic properties are studied. Also we introduce the concept of fuzzy $\ddot{g}$-closure (briefly $f\ddot{g}$-*closure*) in fuzzy topological spaces using the notions of fuzzy $\ddot{g}$-closed sets and we obtain some related results. For any $A \leq X$, it is proved that the complement of $f\ddot{g}$-interior of A is the $f\ddot{g}$-closure of the complement of A.

## 2 Preliminaries

Throughout this paper (X, $\tau$) and (Y, $\sigma$) (or X and Y) represent fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a fuzzy subset A of a space (X, $\tau$), cl(A), int(A) and $A^c$ denote the closure of A, the interior of A and the complement of A respectively.
We recall the following definitions which are useful in the sequel.

**Definition 2.1.** *A fuzzy subset A of a space (X, $\tau$) is called:*

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*(1). fuzzy semi-open set [1] if $A \leq cl(int(A))$.*

*(2). fuzzy $\alpha$-open set [4] if $A \leq int(cl(int(A)))$.*

*(3). fuzzy semi-preopen set [13] if $A \leq cl(int(cl(A)))$.*

*(4). fuzzy regular open set [1] if A=int(cl(A)).*

*The complements of the above mentioned fuzzy open sets are called their respective fuzzy closed sets.*

The fuzzy semi-closure [15] (resp. fuzzy $\alpha$-closure [7], fuzzy semi-preclosure [13]) of a fuzzy subset A of X, denoted by scl(A) (resp. $\alpha$cl(A), spcl(A)) is defined to be the intersection of all fuzzy semi-closed (resp. fuzzy $\alpha$-closed, fuzzy semi-preclosed) sets of (X, $\tau$) containing A. It is known that scl(A) (resp. $\alpha$cl(A), spcl(A)) is a fuzzy semi-closed (resp. fuzzy $\alpha$-closed, fuzzy semi-preclosed) set.

**Definition 2.2.** *A fuzzy subset A of a space (X, $\tau$) is called:*

*(1). a fuzzy generalized closed (briefly fg-closed) set [2] if cl(A)$\leq$U whenever A$\leq$U and U is fuzzy open in (X, $\tau$). The complement of fuzzy g-closed set is called fuzzy g-open set;*

*(2). a fuzzy semi-generalized closed (briefly fsg-closed) set [3] if scl(A)$\leq$U whenever A$\leq$U and U is fuzzy semi-open in (X, $\tau$). The complement of fsg-closed set is called fsg-open set;*

*(3). a fuzzy generalized semi-closed (briefly fgs-closed) set [10] if scl(A)$\leq$U whenever A$\leq$U and U is fuzzy open in (X, $\tau$). The complement of fgs-closed set is called fgs-open set;*

*(4). a fuzzy $\alpha$-generalized closed (briefly f$\alpha$g-closed) set [11] if $\alpha$cl(A)$\leq$U whenever A$\leq$U and U is fuzzy open in (X, $\tau$). The complement of f$\alpha$g-closed set is called f$\alpha$g-open set;*

*(5). a fuzzy generalized semi-preclosed (briefly fgsp-closed) set [9] if spcl(A)$\leq$U whenever A$\leq$U and U is fuzzy open in (X, $\tau$). The complement of fgsp-closed set is called fgsp-open set;*

*(6). a fuzzy $\ddot{g}$-closed set [6] if cl(A)$\leq$U whenever A$\leq$U and U is fgs-open in (X, $\tau$). The complement of fuzzy $\ddot{g}$-closed set is called fuzzy $\ddot{g}$-open set (briefly $f\ddot{g}$-open).*

*The collection of all f-closed sets, fuzzy $\ddot{g}$-closed set and fsg-closed sets of a fuzzy topological spaces (X, $\tau$) are denoted by FC(X), F$\ddot{G}$C(X) and FSGC(X) respectively. The collection of all coressponding open sets are denoted by FO(X), F$\ddot{G}$O(X) and FSGO(X) respectively.*

**Proposition 2.1.** *In a fuzzy topological space $(X, \tau)$, fuzzy closedness implies fuzzy $\ddot{g}$-closedness implies fsg-closedness.*

*Proof.* In a fuzzy topological space, if A is fuzzy closed then A is fuzzy $\ddot{g}$-closed[6]. Let A be fuzzy $\ddot{g}$-closed in $(X, \tau)$ and U be any fuzzy semi-open set such that $A \leq U$. then by (i) of definition 2.2, U is fsg-open in $(X, \tau)$. Since A is fuzzy $\ddot{g}$-closed, $cl(A) \leq U$ and $s - cl(A) \leq cl(A) \leq U$ which means A is fsg-closed in $(X, \tau)$. Thus fuzzy $\ddot{g}$-closedness $\Rightarrow$ fsg-closedness. Hence fuzzy-closedness $\Rightarrow$ fuzzy $\ddot{g}$-closedness $\Rightarrow$ fsg-closedness. □

For further discussion we consider.

**Example 2.2.** *Let X={a, b} and $\alpha$, $\beta$:X $\rightarrow$[0, 1]with $\tau$={$0_X$, $\alpha$, $\beta$, $\alpha\vee\beta$, $1_X$} where $\alpha$, $\beta$ are fuzzy sets in X defined by $\alpha$(a)=0.6, $\alpha$(b)=0 and $\beta$(a)=0, $\beta$(b)=0.3. Then (X, $\tau$) is a fuzzy topological space. we have the following class of fuzzy subsets.*

*(1).* $F.S.C(X) = \{(\frac{a}{u}, \frac{b}{v}) | u \in [0.6, 1], v \in [0.3, 0.7]$ *and* $u \in [0, 0.4], v \in [0.3, 1]$ *and* $u = 0, v = 0$ *and* $u = 1, v = 1\}$.

*(2).* $F.S.O(X) = \{(\frac{a}{u}, \frac{b}{v}) | u \in [0, 0.4], v \in [0.3, 0.7]$ *and* $u \in [0.6, 1], v \in [0, 0.7]$ *and* $u = 0, v = 0$ *and* $u = 1, v = 1\}$.

*(3).* $FSGC(X) = \{(\frac{a}{u}, \frac{b}{v}) | u = 0, v \in [0, 1]$ *and* $u \in [0, 0.4], v \in [0, 1]$ *and* $u \in [0.4, 1], v \in [0.3, 1]\}$.

*(4).* $FSGO(X) = \{(\frac{a}{u}, \frac{b}{v}) | u = 1, v \in [0, 1]$ *and* $u \in [0.6, 1], v \in [0, 1]$ *and* $u \in [0, 0.6], v \in [0, 0.7]\}$.

*(5).* $F\ddot{G}C(X) = \{(\frac{a}{u}, \frac{b}{v}) | u = 0, v = 0$ *and* $u \in [0, 1], v = 1$ *and* $u = 0.4, v = 0.7$ *and* $u = 1, v = 0.7\}$.

**Remark 2.3.** *As illustrated below, none of the implications in proposition 2.1 is reversible.*

**Example 2.4.** *In example 2.2, α=(0.2, 1) is* $f\ddot{g}$*-closed but not fuzzy-closed.*

**Example 2.5.** *In example 2.2, λ=(0, 0.5) is fsg-closed but not* $f\ddot{g}$*-closed.*

**Theorem 2.6.** *The class of all fuzzy $\ddot{g}$-closed sets properly lies between the class of all fuzzy-closed sets and the class of all fsg-closed sets of a fuzzy topological spaces* $(X, \tau)$.

*Proof.* By proposition 2.1 in a fuzzy topological spaces $(X, \tau)$, fuzzy-closedness ⇒ fuzzy $\ddot{g}$-closedness ⇒ fsg-closedness. Hence $FC(X) \leq F\ddot{G}C(X) \leq FSGC(X)$. By examples 2.4 and 2.5, fsg-closedness ⇏ $f\ddot{g}$-closedness ⇏ fuzzy-closedness. Hence $FC(X) < F\ddot{G}C(X) < FSGC(X)$ and this proves the theorem. □

**Proposition 2.7** ([6])**.** *For any two fuzzy $\ddot{g}$-closed subsets A and B in the fuzzy topological spaces* $(X, \tau)$*,* $A \vee B$ *is fuzzy $\ddot{g}$-closed.*

**Remark 2.8.** *The intersection of two fuzzy $\ddot{g}$-closed sets in a fuzzy topological spaces (X, τ), is not necessarily fuzzy $\ddot{g}$-closed as seen from the example.*

**Example 2.9.** *In example 2.2,* $\lambda = (0.3, 1)$ *and μ=(1, 0.7) are fuzzy $\ddot{g}$-closed, λ∧μ=(0.3, 0.7) is not fuzzy $\ddot{g}$-closed in* $(X, \tau)$.

**Theorem 2.10** ([6])**.** *If A is fuzzy $\ddot{g}$-closed in* $(X, \tau)$ *and* $A \leq B \leq cl(A)$*, then B is fuzzy $\ddot{g}$-closed in* $(X, \tau)$.

**Remark 2.11.** *In a fuzzy topological spaces* $(X, \tau)$*, fsg-open sets and fuzzy $\ddot{g}$-closed sets are independent of each other as seen from the following examples.*

**Example 2.12.** *In example 2.2,* $\lambda = (1, 0.5)$ *is fsg-open but not fuzzy $\ddot{g}$-closed in* $(X, \tau)$.

**Example 2.13.** *In example 2.2,* $\lambda = (0.3, 1)$ *is fuzzy $\ddot{g}$-closed but not fsg-open in* $(X, \tau)$.

**Theorem 2.14.** *In a fuzzy topological spaces* $(X, \tau)$*, if a fuzzy subset A is fsg-open and fuzzy $\ddot{g}$-closed, then A is fuzzy-closed.*

*Proof.* Let $A \leq A$ where A is fsg-open. Since A is fuzzy $\ddot{g}$-closed, $cl(A) \leq A$. Hence $cl(A) = A$ and A is fuzzy-closed. □

**Proposition 2.15.** *In a fuzzy topological spaces* $(X, \tau)$*, fuzzy-openness ⇒ fuzzy $\ddot{g}$-openness ⇒ fsg-openness.*

*Proof.* Considering the compliments of fuzzy-closed, fuzzy $\ddot{g}$-closed and fsg-closed sets, proof follows from proposion 2.1. □

**Remark 2.16.** *As illustrated below, none of the implications in proposition 2.15 is reversible.*

**Example 2.17.** *In example 2.2,* $\lambda = (0.8, 0)$ *is fuzzy* $\ddot{g}$*-open but not fuzzy-open.*

**Example 2.18.** *In example 2.2,* $\mu = (1, 0.5)$ *is fsg-open but not fuzzy* $\ddot{g}$*-open.*

**Theorem 2.19.** *The class of all fuzzy* $\ddot{g}$*-open sets properly lies between the class of all fuzzy-open sets of a fuzzy topological spaces* $(X, \tau)$*.*

*Proof.* The proof follows from proposition 2.15, example 2.17, and example 2.18. □

**Proposition 2.20.** *For any two fuzzy* $\ddot{g}$*-open subsets* $A$ *and* $B$ *in a fuzzy topological spaces* $(X, \tau)$*,* $A \wedge B$ *is fuzzy* $\ddot{g}$*-open.*

*Proof.* Considering the compliments in proposition 2.7, the proof follows. □

**Remark 2.21.** *The union of two fuzzy* $\ddot{g}$*-open sets in a fuzzy topological spaces* $(X, \tau)$ *is not necessarily fuzzy* $\ddot{g}$*-open as seen from the following example.*

**Example 2.22.** *In example 2.2,* $\mu = (0.7, 0)$ *and* $\lambda = (0, 0.3)$ *are fuzzy* $\ddot{g}$*-open, but* $\mu \vee \lambda = (0.7, 0.3)$ *is not fuzzy* $\ddot{g}$*-open. using (ii) of definition 2.2 for fuzzy* $\ddot{g}$*-closedness of a fuzzy subset, a definition for fuzzy* $\ddot{g}$*-openness is given as a theorem.*

**Theorem 2.23.** *A fuzzy subset* $A$ *of a fuzzy topological spaces* $(X, \tau)$ *is fuzzy* $\ddot{g}$*-open if only if* $F \leq int(A)$ *whenever* $1 - F$ *is fsg-open and* $F \leq A$*.*
*Necessity: Assume that* $A$ *is fuzzy* $\ddot{g}$*-open in* $(X, \tau)$*. let* $1 - F$ *be fsg-open such that* $F \leq A$*. Then* $1 - A \leq 1 - F$ *where* $1 - A$ *is fuzzy* $\ddot{g}$*-closed. Hence* $cl(1 - A) \leq 1 - F$ *and* $F \leq 1 - cl(1 - A) = int(A)$*.*
*Sufficiency: To prove that* $A$ *is fuzzy* $\ddot{g}$*-open under the given conditions, we prove* $1 - A$ *is fuzzy* $\ddot{g}$*-closed in* $(X, \tau)$*. Let* $U$ *be any fsg-open set such that* $1 - A \leq U$*. Then* $1 - U \leq A$*. Taking* $F = 1 - U$*, we have* $F \leq A$ *where* $1 - F$ *is fuzzy* $\ddot{g}$*-open. By assumption* $F \leq int(A)$ *which implies* $1 - U \leq int(A)$ *and hence* $1 - int(A) \leq U$ *thus* $cl(1 - A) \leq U$ *which proves that* $1 - A$ *is fuzzy* $\ddot{g}$*-closed and* $A$ *is fuzzy* $\ddot{g}$*-open.*

**Theorem 2.24.** *If* $A$ *is a fuzzy* $\ddot{g}$*-open subset of* $(X, \tau)$ *such that* $int(A) \leq B \leq A$*, then* $B$ *is fuzzy* $\ddot{g}$*-open in* $(X, \tau)$*.*

*Proof.* Suppose $int(A) \leq B \leq A$ implies $1 - A \leq 1 - B \leq 1 - int(A) = cl(1 - A)$ where $1 - A$ is fuzzy $\ddot{g}$-closed in $(X, \tau)$. By theorem 2.10, $1 - B$ is fuzzy $\ddot{g}$-closed and hence B fuzzy $\ddot{g}$-open in $(X, \tau)$. □

## 3 Properties of Fuzzy $\ddot{g}$-interior

We introduce the definition of fuzzy $\ddot{g}$-interior in fuzzy topological spaces and study the relationships of such sets.

**Definition 3.1.** *For any* $A \leq X$*, fuzzy* $\ddot{g}$*-interior(A)(briefly* $f\ddot{g} - int(A)$*) is defined as the union of all fuzzy* $\ddot{g}$*-open sets contained in A. i.e.,* $f\ddot{g} - int(A) = \vee\{G : G \leq A \text{ and } G \text{ is } f\ddot{g}\text{-open}\}$*.*

**Remark 3.1.** *The union of fuzzy* $\ddot{g}$*-open sets is not necessarily fuzzy* $\ddot{g}$*-open in a fuzzy topological spaces by example 2.24. Hence for a fuzzy subset A of a fuzzy topological spaces (X,* $\tau$*),* $f\ddot{g}$*-int(A) is not necessarily* $f\ddot{g}$*-open. This is illustrated in the following example.*

**Example 3.2.** *In example 2.2, for a fuzzy subset $A = (0.7, 0.3)$ is $f\ddot{g} - int(A) = (0.7, 0.3)$ which is not fuzzy $\ddot{g}$-open in $(X, \tau)$.*

**Lemma 3.3.** *In a fuzzy topological space $(X, \tau)$,*

*(1) for a fuzzy subset A, $f - int(A) \leq f\ddot{g} - int(A) \leq A$.*

*(2) for the fuzzy subsets A and B, if $A \leq B$, then $f\ddot{g} - int(A) \leq f\ddot{g} - int(B)$.*

*(3) for a fuzzy subset A, $f\ddot{g} - intf\ddot{g} - int(A)) = f\ddot{g} - int(A)$.*

*Proof.*

(1) By proposition 2.15, a fuzzy-open subset of A is a fuzzy $\ddot{g}$-open subset of A. Hence the proof follows directly. That is $f - int(A) \leq f\ddot{g} - int(A)$. By Definition 3.1, $f\ddot{g} - int(A) \leq A$. Thus $f - int(A) \leq f\ddot{g} - int(A) \leq A$.

(2) Since $A \leq$B, a fuzzy $\ddot{g}$-open subset of A is also a fuzzy $\ddot{g}$-open subset of B, we have $f\ddot{g} - int(A) \leq f\ddot{g} - int(B)$.

(3) Any fuzzy $\ddot{g}$-open subset of $f\ddot{g} - int(A)$ is a fuzzy $\ddot{g}$-open subset of A for $f\ddot{g} - int(A) \leq A$ by (1). Again any fuzzy $\ddot{g}$-open subset of A is also a fuzzy $\ddot{g}$-open subset of $f\ddot{g} - int(A)$ by definition of $f\ddot{g} - int(A)$. Hence the family of fuzzy $\ddot{g}$-open subsets of A as well as $f\ddot{g} - int(A)$ are the same and thus we have $f\ddot{g} - intf\ddot{g} - int(A)) = f\ddot{g} - int(A)$.

□

**Remark 3.4.** *Equality in (1) of lemma 3.3 does not necessarily hold as seen from the following example.*

**Example 3.5.** *In example 2.2, for $A = (0.7, 0.8)$, $f - int(A) = (0.6, 0.3)$ and $f\ddot{g} - int(A) = (0.7, 0.3)$. Hence we have $f - int(A) < f\ddot{g} - int(A) < A$.*

**Remark 3.6.** *In (2) of lemma 3.3, even if $A < B$, we have $f\ddot{g} - int(A) = f\ddot{g} - int(B)$.*

**Example 3.7.** *In example 2.2, for $A = (0.7, 0.7)$ and $B = (0.7, 0.8)$ we have $A < B$. But $f\ddot{g} - int(A) = (0.7, 0.3) = f\ddot{g} - int(B)$.*

**Proposition 3.8.** *In a fuzzy topological space $(X, \tau)$ if a fuzzy subset A is fuzzy $\ddot{g}$-open, then $f\ddot{g} - int(A) = A$.*

*Proof.* $A \leq A$ and A is fuzzy $\ddot{g}$-open in $(X, \tau)$. Hence by definition of $f\ddot{g} - int(A)$, $A \leq f\ddot{g} - int(A)$. Thus $f\ddot{g} - int(A) = A$. □

**Remark 3.9.** *Converse of proposition 3.8 is not true as illustrated in the following example.*

**Example 3.10.** *In example 2.2, for $A = (0.7, 0.3)$, $f\ddot{g} - int(A) = A$. But A is not fuzzy $\ddot{g}$-open in $(X, \tau)$.*

**Proposition 3.11.** *For any two fuzzy subsets A and B of the fuzzy topological space $(X, \tau)$*

*(1) $f\ddot{g} - int(A) \vee f\ddot{g} - int(B) \leq f\ddot{g} - int(A \vee B)$.*

*(2) $f\ddot{g} - int(A) \wedge f\ddot{g} - int(B) \leq f\ddot{g} - int(A \wedge B)$.*

*Proof.*

(1) $A \leq A \vee B$ and $B \leq A \vee B \Rightarrow f\ddot{g} - int(A) \leq f\ddot{g} - int(A \vee B)$ and $f\ddot{g} - int(B) \leq f\ddot{g} - int(A \vee B)$ by (ii) of lemma 3.3. Hence $f\ddot{g} - int(A) \vee f\ddot{g} - int(B) \leq f\ddot{g} - int(A \vee B)$.

(2) $A \wedge B \leq A$ and $A \wedge B \leq B \Rightarrow f\ddot{g} - int(A \wedge B) \leq f\ddot{g} - int(A)$ and $f\ddot{g} - int(A \wedge B) \leq f\ddot{g} - int(B)$ by (ii) of lemma 3.3. Hence

$$f\ddot{g} - int(A \wedge B) \leq f\ddot{g} - int(A) \wedge f\ddot{g} - int(B). \tag{3.1}$$

Let $f\ddot{g} - int(A) = \vee_{\alpha \in \Delta} A_\alpha \mid A_\alpha \leq A$ and $A_\alpha$ in $f\ddot{g}$-open for each $\alpha \in \Delta$ and $f\ddot{g} - int(B) = \vee_{\beta \in \Gamma} B_\beta \mid B_\beta \leq B$ and $B_\beta$ in $f\ddot{g}$-open for each $\beta \in \Gamma$, then $f\ddot{g} - int(A) \wedge f\ddot{g} - int(B)$

$= (\vee_{\alpha \in \Delta} A_\alpha) \wedge (\vee_{\beta \in \Gamma} B_\beta)$ where $A_\alpha \leq A,\ B_\beta \leq B$ and $A_\alpha,\ B_\beta$ are $f\ddot{g} -$ open for each $\alpha$ & $\beta$.

$= \vee_{\alpha \in \Delta} \vee_{\beta \in \Gamma} (A_\alpha \wedge B_\beta)$ where $A_\alpha \leq A,\ B_\beta \leq B$ and $A_\alpha, B_\beta$ are $f\ddot{g} -$ open for each $\alpha$ & $\beta$.

Since $A_\alpha$, $B_\beta$ are $f\ddot{g}$-open and also $A_\alpha \wedge B_\beta$ is $f\ddot{g}$-open for each $\alpha \in \Delta$ and $\beta \in \Gamma$ by proposition 2.20. Also $(A_\alpha \wedge B_\beta) \leq A \wedge B$ for $\alpha \in \Delta$ and $\beta \in \Gamma$. Hence $\vee_{\alpha \in \Delta} \vee_{\beta \in \Gamma} (A_\alpha \wedge B_\beta) \leq f\ddot{g} - int(A \wedge B)$. Thus we have

$$f\ddot{g} - int(A) \wedge f\ddot{g} - int(B) \leq f\ddot{g} - int(A \wedge B). \tag{3.2}$$

By (3.1) and (3.2) $f\ddot{g} - int(A) \wedge f\ddot{g} - int(B) = f\ddot{g} - int(A \wedge B)$

□

**Remark 3.12.** *Equality does not necessarily hold in (1) of proposition 3.11 as illustrated in the following example.*

**Example 3.13.** *In example 2.2 for* $A = (1, 0)$ *and* $B = (0, 1)$, $A \vee B = 1_X$. *But* $f\ddot{g} - int(A) = (1, 0)$ *and* $f\ddot{g} - int(B) = (0, 0.3)$. *Hence* $f\ddot{g} - int(A) \vee f\ddot{g} - int(B) = (1, 0) \vee (0, 0.3) = (1, 0.3) < f\ddot{g} - int(A \vee B)$. *Thus* $f\ddot{g} - int(A) \vee f\ddot{g} - int(B) \neq f\ddot{g} - int(A \vee B)$.

# 4 Properties of Fuzzy $\ddot{g}$-closure

In this section we define fuzzy $\ddot{g}$-closure of a fuzzy subset a fuzzy topological space and study some of its basic properties.

**Definition 4.1.** *For any* $A \leq X$, *fuzzy* $\ddot{g}$*-closure(A)(briefly* $f\ddot{g} - cl(A)$*) is defined as the intersection of all fuzzy* $\ddot{g}$*-closed sets contained in A. i.e.,* $f\ddot{g} - cl(A) = \wedge\{A : A \leq G \text{ and } G \text{ is } f\ddot{g} - closed\}$.

**Remark 4.1.** *The intersection of fuzzy* $\ddot{g}$*-closed sets is not necessarily fuzzy* $\ddot{g}$*-closed in a fuzzy topological spaces by example 2.18. Hence for a fuzzy subset A of* $(X, \tau)$, $f\ddot{g}$*-closure is not necessarily fuzzy* $\ddot{g}$*-closed. This is illustrated in the following example.*

**Example 4.2.** *In example 2.2, for the subset* $A = (0.3, 0.7)$ *and* $f\ddot{g} - cl(A) = (0.3, 0.7)$ *which is not fuzzy* $\ddot{g}$*-closed in* $(X, \tau)$.

**Lemma 4.3.** *In a fuzzy topological spaces* $(X, \tau)$,

*(1) for a fuzzy subset A,* $A \leq f\ddot{g} - cl(A) \leq f - cl(A)$.

*(2) for the fuzzy subsets A and B, if* $A \leq B$, *then* $f\ddot{g} - cl(A) \leq f\ddot{g} - cl(B)$.

*(3) for a fuzzy subset A,* $f\ddot{g} - cl(f\ddot{g} - cl(A)) = f\ddot{g} - cl(A)$.

*Proof.* The proof follows from Lemma 3.3. □

**Remark 4.4.** *Equality in (1) of lemma 4.3 does not necessarily hold as seen from the following example.*

**Example 4.5.** *In example 2.2, for* $A = (0.3, 0.2)$*,* $f - cl(A) = (0.4, 0.7)$ *and* $f\ddot{g} - cl(A) = (0.3, 0.7)$*. Hence we have* $A < f\ddot{g} - cl(A) < f - cl(A)$*.*

**Remark 4.6.** *In (2) of lemma 4.3, even if* $A < B$*, we have* $f\ddot{g} - cl(A) = f\ddot{g} - cl(B)$*.*

**Example 4.7.** *In example 2.2, for* $A = (0.3, 0.3)$ *and* $B = (0.3, 0.2)$ *we have* $A < B$*. But* $f\ddot{g} - cl(A) = (0.3, 0.7) = f\ddot{g} - cl(B)$*.*

**Proposition 4.8.** *In a fuzzy topological space* $(X, \tau)$ *if a fuzzy subset A is fuzzy* $\ddot{g}$*-closed, then* $f\ddot{g} - cl(A) = A$*.*

**Remark 4.9.** *Converse of proposition 4.8 is not true as illustrated in the following example.*

**Example 4.10.** *In example 2.2, for* $A = (0.3, 0.7)$*,* $f\ddot{g} - cl(A) = A$*. But A is not fuzzy* $\ddot{g}$*-closed in* $(X, \tau)$*.*

**Proposition 4.11.** *For any two fuzzy subsets A and B of the fuzzy topological space* $(X, \tau)$

*(1)* $f\ddot{g} - cl(A \vee B) \leq f\ddot{g} - cl(A) \vee f\ddot{g} - cl(B)$*.*

*(2)* $f\ddot{g} - cl(A \wedge B) = f\ddot{g} - cl(A) \wedge f\ddot{g} - cl(B)$*.*

*Proof.* The proof follows from proposition 3.11. □

**Theorem 4.12.** *Let A be any subset of X. then*

*(1)* $f\ddot{g} - int(A))^c = f\ddot{g} - cl(A)^c$*.*

*(2)* $f\ddot{g} - int(A) = (f\ddot{g} - cl(A^c))^c$*.*

*(3)* $f\ddot{g} - cl(A) = (f\ddot{g} - int(A^c))^c$*.*

*Proof.*

(1) $(f\ddot{g} - int(A))^c = \{[\vee A_\alpha]^C \mid A_\alpha \leq A$ and $A_\alpha$ in $f\ddot{g}$-open for each $\alpha \in \Delta\}\{\wedge(A_\alpha)^c \mid A_\alpha^c \geq A^c$ and $A_\alpha^c$ is fuzzy $\ddot{g}$-closed for each $\alpha \in \Delta\} \Rightarrow \{cl(A_\alpha)^c \mid A_\alpha^c \geq A^c$ and $A_\alpha^c$ is fuzzy $\ddot{g}$-closed$\} \Rightarrow f\ddot{g} - cl(A)^c$.

(2) Follows by taking complements in (1).

(3) Follows by replacing A by $A^c$ in (1).

□

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