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A New Decagonal Fuzzy Number under Uncertain Linguistic Environment

A. Felix[†,1] **and A. Victor Devadoss**[†]

[†] *PG & Research Department of Mathematics, Loyola College, Chennai-34*

Abstract : Emerging research has focused on uncertain linguistic term in group decision making processes. The uncertain linguistic term is an important and frequently used as input in decision analysis activities. Linguistic values are usually represented as fuzzy number. Therefore, in this paper a new operation on Decagonal Fuzzy Number under uncertain linguistic environment is presented. The main objective of this paper is to introduce operation for addition, subtraction and multiplication of decagonal fuzzy number on the basis of alpha cut.

Keywords : Fuzzy Arithmetic, Linguistic values, Decagonal Fuzzy Number (DFN), alpha cut.

1 Introduction

Fuzzy sets theory was introduced in their modern form by Zadeh, L.A. in 1965 [11]. It provides natural way of dealing with problems in which the source of imprecision and vagueness occurs and it can be applied in many fields such as artificial intelligence, control system, decision making, expert system etc. The concept of fuzzy number has been defined as a fuzzy subset of real line by Dubois, D and Prade, H [1]. A fuzzy number is a quantity whose values are precise, rather than exact as in the case with single valued numbers [2, 4]. To deal imprecise in real life situation, many researchers used triangular and trapezoidal fuzzy number [7, 9, 10, 13]. Also hexagonal, octagonal fuzzy numbers have been introduced to clear the vagueness [5, 6, 8]. Most of the researcher has focused on uncertain linguistic term in group decision making processes [3, 10, 12].

In decision making problem experts may provide uncertain linguistic term to express their opinion when they have no clear idea and lack of information. The uncertain linguistic term is frequently used as input in decision analysis activities. So far linguistic values are usually represented as fuzzy numbers such as triangular, trapezoidal. But it is complex to restrict the membership functions to take triangular, trapezoidal when vagueness arises in ten different points. Therefore, in this paper a new form of decagonal fuzzy number is explored under uncertain linguistic environment.

[1]`Corresponding author E-Mail: afelixphd@gmail.com (A. Felix)`

This paper is organized as follows: section one presents introduction. Basic definition of fuzzy number and linguistic terms are given in section two, section three provides decagonal fuzzy number and it linguistic values, new operation for addition, subtraction and multiplication of decagonal fuzzy number on the basis of alpha cut is proposed in section four. Finally, conclusion and future directions are given.

2 Basic Definitions and Notations

In this section, some basic definitions of fuzzy set theory and fuzzy numbers are reviewed.

Definition 2.1. *A fuzzy set $\widetilde{A}$ in X is characterized by a membership function $\mu_{\widetilde{A}}(x)$ which associates each point in X, to a real number in the interval $[0,1]$. The value of $\mu_{\widetilde{A}}(x)$ represents "grade of membership" of $x \in \mu_{\widetilde{A}(x)}$. More general representation for a fuzzy set is $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x))\}$*

Definition 2.2. *The $\alpha-$cut of the fuzzy set $\widetilde{A}$ of the universe of discourse X is defined as $\widetilde{A}_\alpha = \{x \in X/\mu_{\widetilde{A}}(x) \geq \alpha\}$, where $\alpha \in [0,1]$.*

Definition 2.3. *A fuzzy set $\widetilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be a fuzzy number if its membership function $\widetilde{A} : \mathbb{R} \to [0,1]$ has the following characteristics.*

(i) $\widetilde{A}$ is convex $\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2))$, $\forall x \in [0,1]$, $\lambda \in [0,1]$

(ii) $\widetilde{A}$ is normal i.e. there exists an $x \in \mathbb{R}$ such that if $\max \mu_{\widetilde{A}}(x) = 1$.

(iii) $\widetilde{A}$ is piecewise continuous.

Definition 2.4. *A triangular fuzzy number $\widetilde{A}$ can be defined as (a_1, a_2, a_3), and the membership function is defined as*

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & \textit{for } a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & \textit{for } a_2 \leq x \leq a_3 \\ 0, & \textit{elsewhere.} \end{cases}$$

Definition 2.5. *A trapezoidal fuzzy number $\widetilde{A}$ can be defined as (a_1, a_2, a_3, a_4), and the membership function is defined as*

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \dfrac{(a_4-x)}{(a_4-a_3)}, & a_3 \leq x \leq a_4 \\ 0, & \textit{otherwise.} \end{cases}$$

Definition 2.6. *A linguistic variable/term is a variable whose value is not crisp number but word or sentence in a natural language.*

Definition 2.7. *If $S = s_0, s_1, \ldots, s_g$ be a finite and totally ordered set with odd linguistic terms where s_i denotes the i^{th} linguistic term, $i \in 0, 1 \ldots, g$ then we call set S the linguistic term set and $g+1$ the cardinality of S. It is usually required that set S has the following properties [3]:*

(i) The set is ordered: $s_i' \geq' s_j$ *if* $i \geq j$ *where* $' \geq'$ *denotes 'greater than or equal to'.*

(ii) There is a negation operator: $neg\ (s_i){=}s_j$ *such that* $j = g - i$.

(iii) Maximization operator: $\max\ (s_i, s_j){=}s_i$ *if* $s_i' \geq' s_j$

(iv) Minimization operator: $\min\ (s_i, s_j){=}s_i$ *if* $s_i' \leq' s_j$*, where* $\leq$ *denotes 'less than or equal to'.*

The uncertain linguistic term is a generalization of cognitional expressions to fuzziness and uncertainty. We introduce its definition of uncertain linguistic term below.

Definition 2.8. *Let* $\widetilde{S} = s_l, s_{l+1}, \ldots, s_u$ *where* $s_l, s_{l+1}, \ldots, s_u \in S$*,* $s_l \geq s_u$ s_l *and* s_u *are the lower and upper limits,* $l, u \in 0, 1, \ldots g$ *respectively, Then we call* $\widetilde{S}$ *the uncertain linguistic term.*

For simplicity, we express as $\widetilde{S}$. Here, the greater $u - l$ is, the greater the fuzziness and uncertainty degree of $[s_l, s_u]$ will be. Particularly, if $l = u$, then $\widetilde{S}$ is reduced to a certain linguistic term. For example, in the process of a venture decision , experts may use linguistic term set $S{=}s_0$: No Influence, s_1 : Very Low, s_2 : Low, s_3 : Medium, s_4 : High and s_5 Very High to express his/her opinion on the correlation between anger and violence. One experts judgment may be at least High, which can be expressed by an uncertain linguistic term $[s_3, s_4]$. If his/her judgment is High, then it can be expressed by an uncertain linguistic term $[s_3, s_3]$.

3 Decagonal Fuzzy Number (DFN)

Definition 3.1. *A Decagonal fuzzy number* $\widetilde{D}$ *can be defined as* $(a_{lu}^1, a_{lu}^2, a_{lu}^3, a_{lu}^4, a_{lu}^5, a_{lu}^6, a_{lu}^7, a_{lu}^8, a_{lu}^9, a_{lu}^{10})$*, and the membership function is defined as*

$$\mu_{\widetilde{D}}(x) = \begin{cases} \frac{1}{4}\frac{(x - a_{lu}^1)}{(a_{lu}^2 - a_{lu}^1)}, & a_{lu}^1 \leq x \leq a_{lu}^2 \\ \frac{1}{4} + \frac{1}{4}\frac{(x - a_{lu}^2)}{(a_{lu}^3 - a_{lu}^2)}, & a_{lu}^2 \leq x \leq a_{lu}^3 \\ \frac{1}{2} + \frac{1}{4}\frac{(x - a_{lu}^3)}{(a_{lu}^4 - a_{lu}^3)}, & a_{lu}^3 \leq x \leq a_{lu}^4 \\ \frac{3}{4} + \frac{1}{4}\frac{(x - a_{lu}^4)}{(a_{lu}^5 - a_{lu}^4)}, & a_{lu}^4 \leq x \leq a_{lu}^5 \\ 1, & a_{lu}^5 \leq x \leq a_{lu}^6 \\ 1 - \frac{1}{4}\frac{(x - a_{lu}^6)}{(a_{lu}^7 - a_{lu}^6)}, & a_{lu}^6 \leq x \leq a_{lu}^7 \\ \frac{3}{4} - \frac{1}{4}\frac{(x - a_{lu}^7)}{(a_{lu}^8 - a_{lu}^7)}, & a_{lu}^7 \leq x \leq a_{lu}^8 \\ \frac{1}{2} - \frac{1}{4}\frac{(x - a_{lu}^8)}{(a_{lu}^9 - a_{lu}^8)}, & a_{lu}^8 \leq x \leq a_{lu}^9 \\ \frac{1}{4}\frac{(a_{lu}^{10} - x)}{(a_{lu}^{10} - a_{lu}^9)}, & a_{lu}^9 \leq x \leq a_{lu}^{10} \\ 0, & \text{otherwise.} \end{cases}$$

where $l, u \in \{0, 1, \ldots g\}$*. The derived decagonal fuzzy number from the uncertain linguistic term* $[s_l, s_u]$ *is shown in Fig .2. Thus, the aggregation operations of uncertain linguistic terms can be achieved by the operations of decagonal fuzzy numbers.*

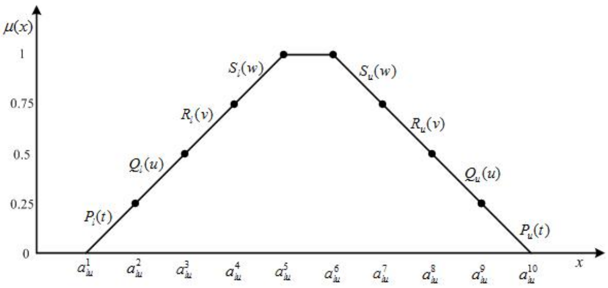

Figure 1: The decagonal fuzzy number

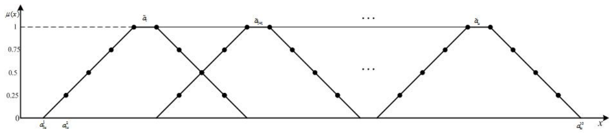

Figure 2: The decagonal fuzzy number from uncertain linguistic term $[s_l, s_u]$

Table 1: Decagonal Fuzzy linguistic scale

Linguistic terms	Linguistic values
No influence	(0, 0, 0, 0, 0, 0.3, 0.6, 0.9, 1.2, 1.5)
Very Low influence	(0.3, 0.6, 0.9, 1.2, 1.5, 1.8, 2.1, 2.4, 2.7, 3.0)
Low influence	(1.8, 2.1, 2.4, 2.7, 3.0, 3.3, 3.6, 3.9, 4.2, 4.5)
Medium	(3.3, 3.6, 3.9, 4.2, 4.5, 4.8, 5.1, 5.4, 5.7, 6.0)
High influence	(4.8, 5.1, 5.4, 5.7, 6.0, 6.3, 6.6, 6.9, 7.2, 7.5)
Very High influence	(6.3, 6.6, 6.9, 7.2, 7.5, 7.8, 7.8, 7.8, 7.8, 7.8)

Furthermore, in light of the research work by Kaufman, A [4] and Zadeh, LA [12], we provide the operations of uncertain linguistic terms in the following.

3.1 Operations of Decagonal Fuzzy Numbers

Definition 3.2. *Let $[s_l, s_u]$ and $[s_\beta, s_\gamma]$ be two arbitrary uncertain linguistic terms , $(a^1_{lu}, a^2_{lu}, a^3_{lu}, a^4_{lu}, a^5_{lu}, a^6_{lu}, a^7_{lu}, a^8_{lu}, a^9_{lu}, a^{10}_{lu})$ and $(a^1_{\beta\gamma}, a^2_{\beta\gamma}, a^3_{\beta\gamma}, a^4_{\beta\gamma}, a^5_{\beta\gamma}, a^6_{\beta\gamma}, a^7_{\beta\gamma}, a^8_{\beta\gamma}, a^9_{\beta\gamma}, a^{10}_{\beta\gamma})$ be their corresponding decagonal fuzzy numbers ; then the addition, subtraction and multiplication operations of $[s_l, s_u]$ and $[s_\alpha, s_\beta]$, denoted as $[s_l, s_u] \oplus [s_\beta, s_\gamma]$, $[s_l, s_u] \ominus [s_\beta, s_\gamma]$ and $[s_l, s_u] \otimes [s_\beta, s_\gamma]$ respectively, then*

(i) $[s_l, s_u] \oplus [s_\alpha, s_\beta]=(a^1_{lu}+a^1_{\beta\gamma}, a^2_{lu}+a^2_{\beta\gamma}, a^3_{lu}+a^3_{\beta\gamma}, a^4_{lu}+a^4_{\beta\gamma}, a^5_{lu}+a^5_{\beta\gamma}, a^6_{lu}+a^6_{\beta\gamma}, a^7_{lu}+a^7_{\beta\gamma}, a^8_{lu}+a^8_{\beta\gamma}, a^9_{lu}+a^9_{\beta\gamma}, a^{10}_{lu}+a^{10}_{\beta\gamma})$. *Here, notation $\oplus$ denotes addition operation of uncertain linguistic terms.*

(ii) $[s_l, s_u] \ominus [s_\beta, s_\gamma]=(a^1_{lu}-a^{10}_{\beta\gamma}, a^2_{lu}-a^9_{\beta\gamma}, a^3_{lu}-a^8_{\beta\gamma}, a^4_{lu}-a^7_{\beta\gamma}, a^5_{lu}-a^6_{\beta\gamma}, a^6_{lu}-a^5_{\beta\gamma}, a^7_{lu}-a^4_{\beta\gamma}, a^8_{lu}-a^3_{\beta\gamma}), a^9_{lu}-a^2_{\beta\gamma}, a^{10}_{lu}-a^1_{\beta\gamma}),$. *Here, notation $\ominus$ denotes addition operation of uncertain linguistic terms.*

(iii) $[s_l, s_u] \otimes [s_\beta, s_\gamma]=(a^1_{lu}*a^1_{\beta\gamma}, a^2_{lu}*a^2_{\beta\gamma}, a^3_{lu}*a^3_{\beta\gamma}, a^4_{lu}*a^4_{\beta\gamma}, a^5_{lu}*a^5_{\beta\gamma}, a^6_{lu}*a^6_{\beta\gamma}, a^7_{lu}*a^7_{\beta\gamma}, a^8_{lu}*a^8_{\beta\gamma}, a^9_{lu}*a^9_{\beta\gamma}, a^{10}_{lu}*a^{10}_{\beta\gamma})$.

(iv) $\lambda \otimes [s_l, s_u] = (\lambda a^1_{lu}, \lambda a^2_{lu}, \lambda a^3_{lu}, \lambda a^4_{lu}, \lambda a^5_{lu}, \lambda a^6_{lu}, \lambda a^7_{lu}, \lambda a^8_{lu}, \lambda a^9_{lu}, \lambda a^{10}_{lu})$, *$\lambda$ be a crisp number ($\lambda > 0$). Here, notation $\otimes$ denotes addition operation of uncertain linguistic terms.*

Example 3.3. *Two experts judgment on the effect of climate change on Indian agriculture is medium and High respectively. This linguistic variable transformed in to decagonal fuzzy number* $[s_3, s_3]$ *=(3.3, 3.6, 3.9, 4.2, 4.5, 4.8, 5.1, 5.4, 5.7, 6.0),* $[s_3, s_4]$ *= (4.8, 5.1, 5.4, 5.7, 6.0, 6.3, 6.6, 6.9, 7.2, 7.5). Then sum, difference and multiplication of their opinion is as follows,*

1. $[s_l, s_u] \oplus [s_\beta, s_\gamma] =$ *(8.1, 8.7, 9.3, 9.9, 10.5, 11.1, 11.7, 12.3, 12.9, 13.5)*

2. $[s_l, s_u] \ominus [s_\beta, s_\gamma] =$ *(-1.5, -1.5, -1.5, -1.5, -1.5, -1.5, -1.5, -1.5, -1.5, -1.5,)*

3. $[s_l, s_u] \otimes [s_\beta, s_\gamma] =$ *(15.84, 18.36, 21.06, 23.94, 27, 30.24, 33.66, 37.26, 41.04, 45)*

Definition 3.4. *A Decagonal fuzzy number* $\widetilde{D}$ *can also be defined as* $\widetilde{D} = P_l(t),\ Q_l(u),\ R_l(v),\ S_l(w),\ P_u(t),\ Q_u(u),\ R_u(v),\ S_u(w),\ t \in [0, 0.25],\ u \in [0.25, 0.5],\ v \in [0.5, 0.75]$ *and* $w \in [0, 0.1]$, *where*

$$P_l(t) = \frac{1}{4}\frac{(x - a_{lu}^1)}{(a_{lu}^2 - a_{lu}^1)},\ Q_l(u) = \frac{1}{4} + \frac{1}{4}\frac{(x - a_{lu}^2)}{(a_{lu}^3 - a_{lu}^2)},\ R_l(v) = \frac{1}{2} + \frac{1}{4}\frac{(x - a_{lu}^3)}{(a_{lu}^4 - a_{lu}^3)},\ S_l(w) = \frac{3}{4} + \frac{1}{4}\frac{(x - a_{lu}^4)}{(a_{lu}^5 - a_{lu}^4)},$$

$$S_u(w) = 1 - \frac{1}{4}\frac{(x - a_{lu}^6)}{(a_{lu}^7 - a_{lu}^6)},\ R_u(v) = \frac{3}{4} - \frac{1}{4}\frac{(x - a_{lu}^7)}{(a_{lu}^8 - a_{lu}^7)},\ Q_u(u) = \frac{1}{2} - \frac{1}{4}\frac{(x - a_{lu}^8)}{(a_{lu}^9 - a_{lu}^8)},\ P_u(t) = \frac{1}{4}\frac{(a_{lu}^{10} - x)}{(a_{lu}^{10} - a_{lu}^9)}$$

Here,

- $P_l(t), Q_l(u), R_l(v), S_l(w)$, *is bounded and continuous incresing function over* $[0, 0.25]$, $[0.25, 0.5]$, $[0.5, 0.75]$ *and* $[0, 0.1]$ *respectively.*

- $P_u(t), Q_u(u), R_u(v), S_u(w)$, *is bounded and continuous decresing function over* $[0, 0.25]$, $[0.25, 0.5]$, $[0.5, 0.75]$ *and* $[0, 0.1]$ *respectively.*

Definition 3.5. *The* $\alpha-$ *cut of the fuzzy set of the universe of discourse* X *is defined as* $\widetilde{D}_\alpha = \{x \in X/\mu_{\widetilde{A}}(x) \geq \alpha\}$, *where* $\alpha \in [0, 1]$.

$$\widetilde{D}_\alpha = \begin{cases} [P_l(\alpha), P_u(\alpha)], & \text{for } \alpha \in [0, 0.25) \\ [Q_l(\alpha), Q_u(\alpha)], & \text{for } \alpha \in [0.25, 0.5) \\ [R_l(\alpha), R_u(\alpha)], & \text{for } \alpha \in [0.5, 0.75) \\ [S_l(\alpha), S_u(\alpha)], & \text{for } \alpha \in [0.75, 1] \end{cases}$$

Definition 3.6. *If* $P_l(x) = \alpha$ *and* $P_u(x) = \alpha$, *then* α *cut operations interval* $\widetilde{D}_\alpha$ *is obtained as*

1. $[P_l(\alpha), P_u(\alpha)] = [4\alpha(a_{lu}^2 - a_{lu}^1) + a_{lu}^1, -4\alpha(a_{lu}^{10} - a_{lu}^9) + a_{lu}^{10})]$
   Similarly, we can obtain $\alpha-$ *cut operations interval* $\widetilde{D}_\alpha$ *for* $[Q_l(\alpha), Q_u(\alpha)]$, $[R_l(\alpha), R_u(\alpha)]$ *and* $[S_l(\alpha), S_u(\alpha)]$ *as follows;*

2. $[Q_l(\alpha), Q_u(\alpha)] = [4\alpha(a_{lu}^3 - a_{lu}^2) + 2a_{lu}^2 - a_{lu}^3, -4\alpha(a_{lu}^9 - a_{lu}^8) + 2a_{lu}^9 - a_{lu}^8]$

3. $[R_l(\alpha), R_u(\alpha)] = [4\alpha(a_{lu}^4 - a_{lu}^3) + 3a_{lu}^3 - 2a_{lu}^4, -4\alpha(a_{lu}^8 - a_{lu}^7) + 3a_{lu}^8 - 2a_{lu}^7]$

4. $[S_l(\alpha), S_u(\alpha)] = [4\alpha(a_{lu}^5 - a_{lu}^4) + 4a_{lu}^4 - 3a_{lu}^5, -4\alpha(a_{lu}^7 - a_{lu}^6) + 4a_{lu}^7 - 3a_{lu}^6]$

Hence, α*-cut of Decagonal Fuzzy Number*

$$\widetilde{D}_\alpha = \begin{cases} [4\alpha(a_{lu}^2 - a_{lu}^1) + a_{lu}^1, -4\alpha(a_{lu}^{10} - a_{lu}^9) + a_{lu}^{10})], & \text{for } \alpha \in [0, 0.25) \\ [4\alpha(a_{lu}^3 - a_{lu}^2) + 2a_{lu}^2 - a_{lu}^3, -4\alpha(a_{lu}^9 - a_{lu}^8) + 2a_{lu}^9 - a_{lu}^8], & \text{for } \alpha \in [0.25, 0.5) \\ [4\alpha(a_{lu}^4 - a_{lu}^3) + 3a_{lu}^3 - 2a_{lu}^4, -4\alpha(a_{lu}^8 - a_{lu}^7) + 3a_{lu}^8 - 2a_{lu}^7], & \text{for } \alpha \in [0.5, 0.75) \\ [4\alpha(a_{lu}^5 - a_{lu}^4) + 4a_{lu}^4 - 3a_{lu}^5, -4\alpha(a_{lu}^7 - a_{lu}^6) + 4a_{lu}^7 - 3a_{lu}^6], & \text{for } \alpha \in [0.75, 1] \end{cases}$$

4 A proposed operation for Addition, Subtraction, Multiplication on Decagonal Fuzzy Number

4.1 Addition of two Decagonal Fuzzy Numbers

Definition 4.1. *Let $[s_l, s_u]$ and $[s_\beta, s_\gamma]$ be two arbitrary uncertain linguistic terms , (a_{lu}^1, a_{lu}^2, a_{lu}^3, a_{lu}^4, a_{lu}^5, a_{lu}^6, a_{lu}^7, a_{lu}^8, a_{lu}^9, a_{lu}^{10}) and ($a_{\beta\gamma}^1$, $a_{\beta\gamma}^2$, $a_{\beta\gamma}^3$, $a_{\beta\gamma}^4$, $a_{\beta\gamma}^5$, $a_{\beta\gamma}^6$, $a_{\beta\gamma}^7$, $a_{\beta\gamma}^8$, $a_{\beta\gamma}^9$, $a_{\beta\gamma}^{10}$) be their corresponding decagonal fuzzy numbers ; then the addition operation of α-cuts $[s_l, s_u]$ and $[s_\beta, s_\gamma]$ using interval arithmetic defined as, $[s_l, s_u] \oplus_\alpha [s_\beta, s_\gamma]$*

$$= \begin{cases} \big([4\alpha(a_{lu}^2 - a_{lu}^1) + a_{lu}^1, -4\alpha(a_{lu}^{10} - a_{lu}^9) + a_{lu}^{10})] + \\ [4\alpha(a_{\beta\gamma}^2 - a_{\beta\gamma}^1) + a_{\beta\gamma}^1, -4\alpha(a_{\beta\gamma}^{10} - a_{\beta\gamma}^9) + a_{\beta\gamma}^{10})]\big), & \text{for } \alpha \in [0, 0.25) \\ \big([4\alpha(a_{lu}^3 - a_{lu}^2) + 2a_{lu}^2 - a_{lu}^3, -4\alpha(a_{lu}^9 - a_{lu}^8) + 2a_{lu}^9 - a_{lu}^8] + \\ [4\alpha(a_{\beta\gamma}^3 - a_{\beta\gamma}^2) + 2a_{\beta\gamma}^2 - a_{\beta\gamma}^3, -4\alpha(a_{\beta\gamma}^9 - a_{\beta\gamma}^8) + 2a_{\beta\gamma}^9 - a_{\beta\gamma}^8]\big), & \text{for } \alpha \in [0.25, 0.5) \\ \big([4\alpha(a_{lu}^4 - a_{lu}^3) + 3a_{lu}^3 - 2a_{lu}^4, -4\alpha(a_{lu}^8 - a_{lu}^7) + 3a_{lu}^8 - 2a_{lu}^7] + \\ [4\alpha(a_{\beta\gamma}^4 - a_{\beta\gamma}^3) + 3a_{\beta\gamma}^3 - 2a_{\beta\gamma}^4, -4\alpha(a_{\beta\gamma}^8 - a_{\beta\gamma}^7) + 3a_{\beta\gamma}^8 - 2a_{\beta\gamma}^7]\big) & \text{for } \alpha \in [0.5, 0.75) \\ \big([4\alpha(a_{lu}^5 - a_{lu}^4) + 4a_{lu}^4 - 3a_{lu}^5, -4\alpha(a_{lu}^7 - a_{lu}^6) + 4a_{lu}^7 - 3a_{lu}^6] + \\ [4\alpha(a_{\beta\gamma}^5 - a_{\beta\gamma}^4) + 4a_{\beta\gamma}^4 - 3a_{\beta\gamma}^5, -4\alpha(a_{\beta\gamma}^7 - a_{\beta\gamma}^6) + 4a_{\beta\gamma}^7 - 3a_{\beta\gamma}^6]\big), & \text{for } \alpha \in [0.75, 1] \end{cases}$$

To verify this new addition operation with ordinary addition operation, we take the example 3.3, $[s_3, s_3] = $ (3.3,3.6, 3.9,4.2,4.5 4.8, 5.1,5.4, 5.7,6.0), $[s_3, s_4] = $ (4.8,5.1, 5.4,5.7,6.0,6.3,6.6, 6.9,7.2,7.5)

For $\alpha \in [0, 0.25)$ $[s_3, s_3]_\alpha = [1.2\alpha + 3.3, -1.2\alpha + 6], [s_3, s_4]_\alpha = [1.2\alpha + 4.8, -1.2\alpha + 7.5]$

$[s_3, s_3]_\alpha + [s_3, s_4]_\alpha = [2.4\alpha + 8.1, -2.4\alpha + 13.5]$

For $\alpha \in [0.25, 0.5)$ $[s_3, s_3]_\alpha = [1.2\alpha + 3.3, -1.2\alpha + 6], [s_3, s_4]_\alpha = [1.2\alpha + 4.8, -1.2\alpha + 7.5]$

$[s_3, s_3]_\alpha + [s_3, s_4]_\alpha = [2.4\alpha + 8.1, -2.4\alpha + 13.5]$

For $\alpha \in [0.5, 0.75)$ $[s_3, s_3]_\alpha = [1.2\alpha + 3.3, -1.2\alpha + 6], [s_3, s_4]_\alpha = [1.2\alpha + 4.8, -1.2\alpha + 7.5]$

$[s_3, s_3]_\alpha + [s_3, s_4]_\alpha = [2.4\alpha + 8.1, -2.4\alpha + 13.5]$

For $\alpha \in [0.75, 1]$ $[s_3, s_3]_\alpha = [1.2\alpha + 3.3, -1.2\alpha + 6], [s_3, s_4]_\alpha = [1.2\alpha + 4.8, -1.2\alpha + 7.5]$

$[s_3, s_3]_\alpha + [s_3, s_4]_\alpha = [2.4\alpha + 8.1, -2.4\alpha + 13.5]$

As for $\alpha \in [0, 0.25)$, $\alpha \in [0.5, 0.5)$, $\alpha \in [0.5, 0.75)$ and $\alpha \in [0.75, 1]$ arithmetic interval same, $[s_3, s_3]_\alpha + [s_3, s_4]_\alpha = [2.4\alpha + 8.1, -2.4\alpha + 13.5]$ for $\alpha \in [0.75, 1]$

$$\begin{aligned}\text{when}\quad &\alpha = 0, \Rightarrow [s_3, s_3]_0 + [s_3, s_4]_0 = [8.1, 13.5]\\ &\alpha = 0.25, \Rightarrow [s_3, s_3]_{0.25} + [s_3, s_4]_{0.25} = [8.7, 12.9]\\ &\alpha = 0.5, \Rightarrow [s_3, s_3]_{0.5} + [s_3, s_4]_{0.5} = [9.3, 12.3]\\ &\alpha = 0.75, \Rightarrow [s_3, s_3]_{0.75} + [s_3, s_4]_{0.75} = [9.9, 11.7]\\ &\alpha = 1, \Rightarrow [s_3, s_3]_1 + [s_3, s_4]_1 = [10.5, 11.1]\end{aligned}$$

Therefore, sum of two experts opinion is,

$[s_3, s_3]_\alpha + [s_3, s_4]_\alpha = [8.1, 8.7, 9.3, 9.9, 10.5, 11.1, 11.7, 12.3, 12.9, 13.5]$. Since all the points coincide with the sum of two decagonal fuzzy numbers, addition of two alpha cuts lies with in interval. Hence, it is verified.

4.2 Subtraction of two Decagonal Fuzzy Numbers

Definition 4.2. *Let $[s_l, s_u]$ and $[s_\beta, s_\gamma]$ be two arbitrary uncertain linguistic terms , $(a^1_{lu}, a^2_{lu}, a^3_{lu}, a^4_{lu}, a^5_{lu}, a^6_{lu}, a^7_{lu}, a^8_{lu}, a^9_{lu}, a^{10}_{lu})$ and $(a^1_{\beta\gamma}, a^2_{\beta\gamma}, a^3_{\beta\gamma}, a^4_{\beta\gamma}, a^5_{\beta\gamma}, a^6_{\beta\gamma}, a^7_{\beta\gamma}, a^8_{\beta\gamma}, a^9_{\beta\gamma}, a^{10}_{\beta\gamma})$ be their corresponding decagonal fuzzy numbers ; then the subtraction operation of α-cuts $[s_l, s_u]$ and $[s_\beta, s_\gamma]$ using interval arithmetic defined as, $[s_l, s_u] \ominus_\alpha [s_\beta, s_\gamma]$*

$$= \begin{cases} \big([4\alpha(a^2_{lu} - a^1_{lu}) + a^1_{lu}, -4\alpha(a^{10}_{lu} - a^9_{lu}) + a^{10}_{lu})] - \\ [4\alpha(a^2_{\beta\gamma} - a^1_{\beta\gamma}) + a^1_{\beta\gamma}, -4\alpha(a^{10}_{\beta\gamma} - a^9_{\beta\gamma}) + a^{10}_{\beta\gamma})]\big), & \textit{for } \alpha \in [0, 0.25) \\ \big([4\alpha(a^3_{lu} - a^2_{lu}) + 2a^2_{lu} - a^3_{lu}, -4\alpha(a^9_{lu} - a^8_{lu}) + 2a^9_{lu} - a^8_{lu}] - \\ [4\alpha(a^3_{\beta\gamma} - a^2_{\beta\gamma}) + 2a^2_{\beta\gamma} - a^3_{\beta\gamma}, -4\alpha(a^9_{\beta\gamma} - a^8_{\beta\gamma}) + 2a^9_{\beta\gamma} - a^8_{\beta\gamma}]\big), & \textit{for } \alpha \in [0.25, 0.5) \\ \big([4\alpha(a^4_{lu} - a^3_{lu}) + 3a^3_{lu} - 2a^4_{lu}, -4\alpha(a^8_{lu} - a^7_{lu}) + 3a^8_{lu} - 2a^7_{lu}] - \\ [4\alpha(a^4_{\beta\gamma} - a^3_{\beta\gamma}) + 3a^3_{\beta\gamma} - 2a^4_{\beta\gamma}, -4\alpha(a^8_{\beta\gamma} - a^7_{\beta\gamma}) + 3a^8_{\beta\gamma} - 2a^7_{\beta\gamma}]\big) & \textit{for } \alpha \in [0.5, 0.75) \\ \big([4\alpha(a^5_{lu} - a^4_{lu}) + 4a^4_{lu} - 3a^5_{lu}, -4\alpha(a^7_{lu} - a^6_{lu}) + 4a^7_{lu} - 3a^6_{lu}] - \\ [4\alpha(a^5_{\beta\gamma} - a^4_{\beta\gamma}) + 4a^4_{\beta\gamma} - 3a^5_{\beta\gamma}, -4\alpha(a^7_{\beta\gamma} - a^6_{\beta\gamma}) + 4a^7_{\beta\gamma} - 3a^6_{\beta\gamma}]\big), & \textit{for } \alpha \in [0.75, 1] \end{cases}$$

To verify this new subtraction operation with ordinary subtraction operation, we take the previous arguments in the addition.

$[s_3, s_3]$ = (3.3,3.6, 3.9,4.2,4.5 4.8, 5.1,5.4, 5.7,6.0), $[s_3, s_4]$ = (4.8,5.1, 5.4,5.7,6.0,6.3,6.6, 6.9,7.2,7.5)

$$\begin{aligned}\text{For } \alpha \in [0, 0.25),\quad &[s_3, s_3]_\alpha = [1.2\alpha + 3.3, -1.2\alpha + 6], [s_3, s_4]_\alpha = [1.2\alpha + 4.8, -1.2\alpha + 7.5]\\ &[s_3, s_3]_\alpha - [s_3, s_4]_\alpha = [-1.5, -1.5]\end{aligned}$$

By doing the above process, the same arithmetic interval was obtained for $\alpha \in [0, 0.25)$, $\alpha \in [0.5, 0.5)$, $\alpha \in [0.5, 0.75)$ and $\alpha \in [0.75, 1]$. Therefore, $[s_3, s_3]_\alpha - [s_3, s_4]_\alpha = [-1.5, -1.5]$ for $\alpha \in [0, 1]$. Since we have got the constant term, the difference between two experts opinions is $[s_3, s_3] \ominus_\alpha [s_3, s_4] = (-1.5, -1.5, -1.5, -1.5, -1.5, -1.5, -1.5, -1.5, -1.5, -1.5)$. Hence, it is confirmed with the ordinary subtraction operation.

4.3 Multiplication of two Decagonal Fuzzy Numbers

Definition 4.3. *Let $[s_l, s_u]$ and $[s_\beta, s_\gamma]$ be two arbitrary uncertain linguistic terms , $(a^1_{lu}, a^2_{lu}, a^3_{lu}, a^4_{lu}, a^5_{lu}, a^6_{lu}, a^7_{lu}, a^8_{lu}, a^9_{lu}, a^{10}_{lu})$ and $(a^1_{\beta\gamma}, a^2_{\beta\gamma}, a^3_{\beta\gamma}, a^4_{\beta\gamma}, a^5_{\beta\gamma}, a^6_{\beta\gamma}, a^7_{\beta\gamma}, a^8_{\beta\gamma}, a^9_{\beta\gamma}, a^{10}_{\beta\gamma})$ be their corresponding decagonal fuzzy numbers ; then the multiplication operation of α-cuts $[s_l, s_u]$ and $[s_\beta, s_\gamma]$ using interval* *arithmetic defined as,* $[s_l, s_u] \otimes_\alpha [s_\beta, s_\gamma]$

$$= \begin{cases} \big([4\alpha(a^2_{lu} - a^1_{lu}) + a^1_{lu}, -4\alpha(a^{10}_{lu} - a^9_{lu}) + a^{10}_{lu})]* & \\ [4\alpha(a^2_{\beta\gamma} - a^1_{\beta\gamma}) + a^1_{\beta\gamma}, -4\alpha(a^{10}_{\beta\gamma} - a^9_{\beta\gamma}) + a^{10}_{\beta\gamma})]\big), & \textit{for } \alpha \in [0, 0.25) \\ \big([4\alpha(a^3_{lu} - a^2_{lu}) + 2a^2_{lu} - a^3_{lu}, -4\alpha(a^9_{lu} - a^8_{lu}) + 2a^9_{lu} - a^8_{lu}]* & \\ [4\alpha(a^3_{\beta\gamma} - a^2_{\beta\gamma}) + 2a^2_{\beta\gamma} - a^3_{\beta\gamma}, -4\alpha(a^9_{\beta\gamma} - a^8_{\beta\gamma}) + 2a^9_{\beta\gamma} - a^8_{\beta\gamma}]\big), & \textit{for } \alpha \in [0.25, 0.5) \\ \big([4\alpha(a^4_{lu} - a^3_{lu}) + 3a^3_{lu} - 2a^4_{lu}, -4\alpha(a^8_{lu} - a^7_{lu}) + 3a^8_{lu} - 2a^7_{lu}]* & \\ [4\alpha(a^4_{\beta\gamma} - a^3_{\beta\gamma}) + 3a^3_{\beta\gamma} - 2a^4_{\beta\gamma}, -4\alpha(a^8_{\beta\gamma} - a^7_{\beta\gamma}) + 3a^8_{\beta\gamma} - 2a^7_{\beta\gamma}]\big) & \textit{for } \alpha \in [0.5, 0.75) \\ \big([4\alpha(a^5_{lu} - a^4_{lu}) + 4a^4_{lu} - 3a^5_{lu}, -4\alpha(a^7_{lu} - a^6_{lu}) + 4a^7_{lu} - 3a^6_{lu}]* & \\ [4\alpha(a^5_{\beta\gamma} - a^4_{\beta\gamma}) + 4a^4_{\beta\gamma} - 3a^5_{\beta\gamma}, -4\alpha(a^7_{\beta\gamma} - a^6_{\beta\gamma}) + 4a^7_{\beta\gamma} - 3a^6_{\beta\gamma}]\big), & \textit{for } \alpha \in [0.75, 1] \end{cases}$$

To verify this new multiplication operation, we take the same argument as we did in addition. $[s_3, s_3]$ = (3.3,3.6, 3.9,4.2,4.5 4.8, 5.1,5.4, 5.7,6.0), $[s_3, s_4]$ = (4.8,5.1, 5.4,5.7,6.0,6.3,6.6, 6.9,7.2,7.5)

$$\text{For } \alpha \in [0, 0.25), \quad [s_3, s_3]_\alpha = [1.2\alpha + 3.3, -1.2\alpha + 6], [s_3, s_4]_\alpha = [1.2\alpha + 4.8, -1.2\alpha + 7.5]$$
$$[s_3, s_3]_\alpha * [s_3, s_4]_\alpha = [(1.2\alpha + 3.3)(1.2\alpha + 4.8), (-1.2\alpha + 6)(-1.2\alpha + 7.5)]$$

By doing the above process, the same arithmetic interval was obtained for for $\alpha \in [0, 0.25)$, $\alpha \in [0.5, 0.5)$, $\alpha \in [0.5, 0.75)$ and $\alpha \in [0.75, 1]$.

Therefore, $[s_3, s_3]_\alpha * [s_3, s_4]_\alpha = [(1.2\alpha + 3.3)(1.2\alpha + 4.8), (-1.2\alpha + 6)(-1.2\alpha + 7.5)]$ for $\alpha \in [0, 1]$.

$$\begin{aligned} \text{when} \quad & \alpha = 0, \Rightarrow [s_3, s_3]_0 * [s_3, s_4]_0 = [15.84, 45] \\ & \alpha = 0.25, \Rightarrow [s_3, s_3]_{0.25} * [s_3, s_4]_{0.25} = [18.36, 41.04] \\ & \alpha = 0.5, \Rightarrow [s_3, s_3]_{0.5} * [s_3, s_4]_{0.5} = [21.06, 37.26] \\ & \alpha = 0.75, \Rightarrow [s_3, s_3]_{0.75} * [s_3, s_4]_{0.75} = [23.94, 33.66] \\ & \alpha = 1, \Rightarrow [s_3, s_3]_1 * [s_3, s_4]_1 = [27, 30.4] \end{aligned}$$

Therefore, product of the two experts opinion is,
$[s_3, s_3] \otimes_\alpha [s_3, s_4] = (15.84, 18.36, 21.06, 23.94, 27, 30.24, 33.66, 37.26, 41.04, 45)$.

Since all the points coincide with the product of two decagonal fuzzy numbers, product of two alpha cuts lies with in interval. Hence, it is verified

5 Conclusion

In this paper, decagonal fuzzy number has been introduced and its arithmetic operations are defined with an uncertain linguistic environment. In many real life cases, the decision data of human judgments with preferences are often vague so that the traditional ways of using crisp values are inadequate also using fuzzy numbers such as triangular, trapezoidal are not suitable in few case where the uncertainties arises in ten different points in such cases DFN can be used to solve the problems. DEMATEL method can be extended by representing linguistic variable in to decagonal fuzzy number will be the further research.

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