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On $I_{\pi gp^\star}$-closed sets in ideal topological spaces

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Abstract : In this paper, a new class of sets called $I_{\pi gp^\star}$-closed sets is introduced and its properties are studied in ideal topological space. Moreover $I_{\pi gp^\star}$-continuity and the notion of quasi-p*-I-normal spaces are introduced.

Keywords : π-open set, $I_{\pi gp^\star}$-closed set, $I_{\pi gp^\star}$-continuity, quasi-p*-I-normal space.

AMS Subject Classification: 54A05, 54C08.

1 Introduction and preliminaries

An ideal topological space is a topological space (X, τ) with an ideal I on X, and is denoted by (X, τ, I). A$^*(I)$ = {x $\in$ X | U $\cap$ A $\notin I$ for each open neighborhood U of x} is called the local function of A with respect to I and τ [11]. When there is no chance for confusion A$^*(I)$ is denoted by A*. For every ideal topological space (X, τ, I), there exists a topology τ^* finer than τ, generated by the base $\beta(I, \tau)$ = {U$\backslash I$ | U$\in \tau$ and $I \in I$}. In general $\beta(I, \tau)$ is not always a topology [10]. Observe additionally that Cl*(A) = A* $\cup$ A [20] defines a Kuratowski closure operator for τ^*. Int*(A) will denote the interior of A in (X, $\tau^\star$).

In this paper, we define and study a new notion $I_{\pi gp^\star}$-closed set by using the notion of pre*-I-open set. Some new notions depending on $I_{\pi gp^\star}$-closed sets such as $I_{\pi gp^\star}$-open sets, $I_{\pi gp^\star}$-continuity and $I_{\pi gp^\star}$-irresoluteness are also introduced and a decomposition of pre*-I-continuity is given. Also by using $I_{\pi gp^\star}$-closed sets characterizations of quasi-p*-I-normal spaces are obtained. Several preservation theorems for quasi-p*-I-normal spaces are given.

Throughout this paper, space (X, τ) (or simply X) always means topological space on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and the interior of A are denoted by Cl(A) and Int(A), respectively. A subset A of a topological space (X, τ) is said to be regular open [19](resp. regular closed [19]) if A = Int(Cl(A)) (resp. A = Cl(Int(A))).

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The finite union of regular open sets is said to be π-open [21] in (X, τ). The complement of a π-open set is π-closed [21].

A subset A of a topological space (X, τ) is said to be pre-open [12] if A $\subseteq$ Int(Cl(A)) and the complement of a pre-open set is called pre-closed [12]. The intersection of all pre-closed sets containing A is called the pre-closure [12] of A and is denoted by pCl(A). Note that pCl(A) = A $\cup$ Cl(Int(A)) [2]. A subset A of a space (X, τ) is said to be πg-closed [4] (resp. πgp-closed [14]) if Cl(A) $\subseteq$ U (resp. pCl(A) $\subseteq$ U) whenever A $\subseteq$ U and U is π-open in X.

A function f : (X, τ) $\rightarrow$ (Y, σ) is said to be m-π-closed [6] if f(V) is π-closed in (Y, σ) for every π-closed in (X, τ). A function f : (X, τ) $\rightarrow$ (Y, σ) is said to be πg-continuous [4] (resp. πgp-continuous [13, 14]) if f^{-1}(V) is πg-closed (resp. πgp-closed) in (X, τ) for every closed set V of (Y, σ). A space (X, τ) is said to be quasi-p-normal [16] if for every pair of disjoint π-closed subsets A, B of X, there exist disjoint pre-open sets U, V of X such that A $\subseteq$ U and B $\subseteq$ V.

A space (X, τ) is said to be quasi-normal [21] if for every pair of disjoint π-closed subsets A, B of X, there exist disjoint open sets U, V of X such that A $\subseteq$ U and B $\subseteq$ V. An ideal I is said to be codense [5] if $\tau \cap I = \emptyset$. A subset A of an ideal topological space X is said to be $\star$-perfect [9] (resp. $\star$-dense-in-itself [9], semi-I-open [8], pre-I-open [3], semi*-I-open [7], pre*-I-open [17], I-R-closed [1]) if $A^\star$ = A (resp. A $\subseteq A^\star$, A $\subseteq Cl^\star$(Int(A)), A $\subseteq$ Int($Cl^\star$(A)), A $\subseteq$ Cl($Int^\star$(A)), A $\subseteq Int^\star$(Cl(A)), A = $Cl^\star$(Int(A))).

The complement of semi-I-open (resp. pre-I-open, semi*-I-open, pre*-I-open) is semi-I-closed [8] (resp. pre-I-closed [3], semi*-I-closed [7], pre*-I-closed [17]). A subset A of an ideal topological space X is said to be $I_{\pi g}$-closed [15] if $A^\star \subseteq$ U whenever A $\subseteq$ U and U is π-open in X. A function f : (X, τ, I) $\rightarrow$ (Y, σ) is said to be $I_{\pi g}$-continuous [15] if f^{-1}(V) is $I_{\pi g}$-closed in (X, τ, I) for every closed set V of (Y, σ).

Lemma 1.1 ([18]). *Let (X, τ, I) be an ideal topological space and A $\subseteq$ X. If A $\subseteq A^\star$, then $A^\star$ = Cl($A^\star$) = Cl(A) = $Cl^\star$(A).*

Theorem 1.2 ([15]). *Every πg-closed set is $I_{\pi g}$-closed but not conversely.*

Theorem 1.3 ([15]). *For a function f : (X, τ, I) $\rightarrow$ (Y, σ), the following holds: Every πg-continuous function is $I_{\pi g}$-continuous but not conversely.*

Theorem 1.4 ([14]). *Every πg-closed set is πgp-closed but not conversely.*

Theorem 1.5 ([14]). *For a function f: (X, τ) $\rightarrow$ (Y, σ), the following holds: Every πg-continuous function is πgp-continuous but not conversely.*

Proposition 1.6 ([3, 8]). *Every pre-I-open set is pre-open but not conversely.*

2 $I_{\pi gp^\star}$-closed sets

Definition 2.1. *Let (X, τ, I) be an ideal topological space and let A be a subset of X. The union of all pre*-I-open sets contained in A is called the pre*-I-interior of A and is denoted by $p^\star_I Int(A)$.*

Definition 2.2. *Let (X, τ, I) be an ideal topological space and let A be a subset of X. The intersection of all pre*-I-closed sets containing A is called the pre*-I-closure of A and is denoted by $p^\star_I Cl(A)$.*

Lemma 2.3. *Let (X, τ, I) be an ideal topological space. For a subset A of X, the followings hold:*

(i) $p_I^\star Cl(A) = A \cup Cl^\star(Int(A))$, *[18]*

(ii) $p_I^\star Int(A) = A \cap Int^\star(Cl(A))$.

Definition 2.4. *A subset A of an ideal topological space (X, τ, I) is called $I_{\pi gp^\star}$-closed if $p_I^\star Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in X. The complement of $I_{\pi gp^\star}$-closed set is said to be $I_{\pi gp^\star}$-open.*

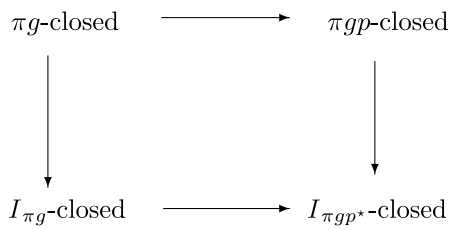
Proposition 2.5. *Let (X, τ, I) be an ideal topological space and $A \subseteq X$. Then the following properties hold:*

(i) If A is πgp-closed, then A is $I_{\pi gp^\star}$-closed,

(ii) If A is $I_{\pi g}$-closed, then A is $I_{\pi gp^\star}$-closed.

Proof. The proof is obvious. □

Remark 2.1. *From Proposition 2.5, we have the following diagram.*



where none of these implications is reversible as shown in the following examples.

Example 2.6.

(i) Let $X = \{a, b, c, d, e\}$, $\tau = \{X, \emptyset, \{b, c\}, \{a, d\}, \{a, b, c, d\}\}$ and $I = \{\emptyset, \{a\}\}$. Then $A = \{b\}$ is $I_{\pi gp^\star}$-closed set but it is not $I_{\pi g}$-closed.

(ii) Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{b\}, \{d\}, \{b, d\}, \{b, c, d\}\}$ and $I = \{\emptyset, \{c\}, \{d\}, \{c, d\}\}$. Then B= $\{d\}$ is $I_{\pi gp^\star}$-closed set but it is not πgp-closed.

Theorem 2.7. *Every $\star$-dense-in-itself and $I_{\pi gp^\star}$-closed set is a πgp-closed set.*

Proof. Let A⊆ U, and U is π-open in X. Since A is $I_{\pi gp^\star}$-closed, $p_I^\star$Cl(A) ⊆ U. By Lemmas 1.1 and 2.3, $p_I^\star$Cl(A) = A ∪ Cl*(Int(A)) = A ∪ Cl(Int(A)) = pCl(A). Then, pCl(A) ⊆ U. So A is πgp-closed. □

Theorem 2.8. *Every π-open and $I_{\pi gp^\star}$-closed set is I-R-closed.*

Proof. $p_I^\star$Cl(A) ⊆ A, since A is π-open and $I_{\pi gp^\star}$-closed. We have Cl*(Int(A)) ⊆ A. Since A is open, then A is clearly semi-I-open and thus A ⊆ Cl*(Int(A)). Therefore A = Cl*(Int(A)), which shows that A is I-R-closed. □

Remark 2.2. *The union of two $I_{\pi gp^\star}$-closed sets need not be $I_{\pi gp^\star}$-closed.*

Example 2.9. *Consider the Example 2.7(1). Let $A = \{a, b\}$ and $B = \{c, d\}$. Then A and B are $I_{\pi gp^\star}$-closed sets but $A \cup B = \{a, b, c, d\}$ is not $I_{\pi gp^\star}$-closed.*

Remark 2.3. *The intersection of two $I_{\pi gp^\star}$-closed sets need not be $I_{\pi gp^\star}$-closed.*

Example 2.10. *Consider the Example 2.7(2). Let $A = \{b, c\}$ and $B = \{a, b, d\}$. Then A and B are $I_{\pi gp^\star}$-closed sets but $A \cap B = \{b\}$ is not $I_{\pi gp^\star}$-closed.*

Theorem 2.11. *Let A be $I_{\pi gp^\star}$-closed in (X, τ, I). Then $p^\star_I Cl(A) \setminus A$ does not contain any non-empty π-closed set.*

Proof. Let F be a π-closed set such that F $\subseteq$ $p^\star_I$Cl(A) $\setminus$ A. Then F $\subseteq$ X $\setminus$ A implies A $\subseteq$ X $\setminus$ F. Therefore $p^\star_I$Cl(A) $\subseteq$ X $\setminus$ F. That is F $\subseteq$ X $\setminus$ $p^\star_I$Cl(A). Hence F $\subseteq$ $p^\star_I$Cl(A) $\cap$ (X $\setminus$ $p^\star_I$Cl(A))= $\emptyset$. This shows F = $\emptyset$. □

Theorem 2.12. *If A is $I_{\pi gp^\star}$-closed and $A \subseteq B \subseteq p^\star_I Cl(A)$, then B is $I_{\pi gp^\star}$-closed.*

Proof. Let A be $I_{\pi gp^\star}$-closed and B $\subseteq$ U, where U is π-open. Then A $\subseteq$ B implies A $\subseteq$ U. Since A is $I_{\pi gp^\star}$-closed, $p^\star_I$Cl(A) $\subseteq$ U. B $\subseteq$ $p^\star_I$Cl(A) implies $p^\star_I$Cl(B) $\subseteq$ $p^\star_I$Cl(A). Therefore $p^\star_I$Cl(B) $\subseteq$ U and hence B is $I_{\pi gp^\star}$-closed. □

Theorem 2.13. *Let (X, τ, I) be an ideal topological space. Then every subset of X is $I_{\pi gp^\star}$-closed if and only if every π-open set is I-R-closed.*

Proof. Necessity: It is obvious from Theorem 2.9.
Sufficiency: Suppose that every π-open set is I-R-closed. Let A be a subset of X and U be π-open such that A $\subseteq$ U. By hypothesis Cl*(Int(A)) $\subseteq$ Cl*(Int(U)) = U. Then $p^\star_I$Cl(A)$\subseteq$ U. So A is $I_{\pi gp^\star}$-closed. □

Theorem 2.14. *Let (X, τ, I) be an ideal topological space. $A \subseteq X$ is $I_{\pi gp^\star}$-open if and only if $F \subseteq p^\star_I Int(A)$ whenever F is π-closed and $F \subseteq A$.*

Proof. Necessity: Let A be $I_{\pi gp^\star}$-open and F be π-closed such that F $\subseteq$ A. Then X\A $\subseteq$ X\F where X\F is π-open. $I_{\pi gp}$-closedness of X\A implies $p^\star_I$Cl(X\A) $\subseteq$ X\F. Then F $\subseteq$ $p^\star_I$Int(A).
Sufficiency: Suppose F is π-closed and F $\subseteq$ A implies F $\subseteq$ $p^\star_I$Int(A). Let X\A $\subseteq$ U where U is π-open. Then X\U $\subseteq$ A where X\U is π-closed. By hypothesis X\U $\subseteq$ $p^\star_I$Int(A). That is $p^\star_I$Cl(X\A) $\subseteq$ U. So, A is $I_{\pi gp^\star}$-open. □

Definition 2.15. *A subset A of an ideal topological space (X, τ, I) is called P_I-set if $A = U \cup V$ where U is π-closed and V is pre*-I-open.*

Proposition 2.16. *Every π-closed set is P_I-set but not conversely.*

Example 2.17. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $I = \{\emptyset, \{b\}\}$. Then $\{a, c\}$ is P_I-set but not π-closed set.*

Proposition 2.18. *Every pre*-I-open set is P_I-set but not conversely.*

Example 2.19. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $I = \{\emptyset, \{a\}\}$. Then $\{a, b\}$ is P_I-set but not pre*-I-open set.*

Proposition 2.20. *Every pre*-I-open set is $I_{\pi gp^\star}$-open but not conversely.*

Proof. Let A be pre*-I-open set. Then A $\subseteq$ Int*(Cl(A)). Assume that F is π-closed and F $\subseteq$ A. Then F $\subseteq$ Int*(Cl(A)) which implies F $\subseteq$ A $\cap$ Int*(Cl(A)) = p$^\star_I$Int(A) by Lemma 2.3. Hence, by Theorem 2.17, A is $I_{\pi gp^\star}$-open.

Example 2.21. *Consider the Example 2.20. Let $A = \{b\}$. Then A is $I_{\pi gp^\star}$-open set but not pre*-I-open set.*

Theorem 2.22. *For a subset A of (X, τ, I) the following conditions are equivalent:*

(i) A is pre*-I-open,

(ii) A is $I_{\pi gp^\star}$-open and a P_I-set.

Proof. (i) $\Rightarrow$ (ii) It is obvious.
(ii) $\Rightarrow$ (i) Let A be $I_{\pi gp^\star}$-open and a P$_I$-set. Then there exist a π-closed set U and pre*-I-open set V such that A = U $\cup$ V. Since U $\subseteq$ A and A is $I_{\pi gp^\star}$-open, by Theorem 2.17, U $\subseteq$ p$^\star_I$Int(A) and U $\subseteq$ Int*(Cl(A)). Also, V $\subseteq$ Int*(Cl(V)) $\subseteq$ Int*(Cl(A)). Then A $\subseteq$ Int*(Cl(A)). So A is pre*-I-open. □

The following examples show that concepts of $I_{\pi gp^\star}$-open set and P$_I$-set are independent.

Example 2.23. *Let (X, τ, I) be the same ideal topological space as in Example 2.7(i). Then $\{a, b, d\}$ is a P_I-set but not $I_{\pi gp^\star}$-open set.*

Example 2.24. *Let (X, τ, I) be the same ideal topological space as in Example 2.7(i). Then $\{b, d\}$ is $I_{\pi gp^\star}$-open set but not a P_I-set.*

Proposition 2.25. *Every pre-open set is pre*-I-open but not conversely.*

Proof. Let A be pre-open set. Then A $\subseteq$ Int(Cl(A)) which implies A $\subseteq$ Int*(Cl(A)). Hence A is pre*-I-open set. □

Example 2.26. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \emptyset, \{a\}\}$ and $I = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Then $\{b\}$ is pre*-I-open set but not pre-open set.*

3 $I_{\pi gp^\star}$-continuity and $I_{\pi gp^\star}$-irresoluteness

Definition 3.1. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be $I_{\pi gp^\star}$-continuous (resp. pre*-I-continuous) if $f^{-1}(V)$ is $I_{\pi gp^\star}$-closed (resp. pre*-I-closed) in X for every closed set V of Y.*

Definition 3.2. *A function $f : (X, \tau, I) \to (Y, \sigma, J)$ is said to be $I_{\pi gp^\star}$-irresolute if $f^{-1}(V)$ is $I_{\pi gp^\star}$-closed in X for every $J_{\pi gp^\star}$-closed set V of Y.*

Definition 3.3. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be P_I-continuous if $f^{-1}(V)$ is P_I-set in (X, τ, I) for every closed set V of Y.*

Theorem 3.4. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is pre*-I-continuous if and only if it is P_I-continuous and $I_{\pi gp^\star}$-continuous.*

Proof. This is an immediate consequence of Theorem 2.25. □

Remark 3.1. *The following Examples show that:*

(i) *every $I_{\pi gp^\star}$-continuous function is not πgp-continuous,*

(ii) *every $I_{\pi gp^\star}$-continuous function is not $I_{\pi g}$-continuous.*

Example 3.5. *Let (X, τ, I) be the same ideal topological space as in Example 2.7(2). Let $Y = \{x, y, z\}$ and $\sigma = \{Y, \emptyset, \{y, z\}\}$. Define a function $f : (X, \tau, I) \to (Y, \sigma)$ as follows: $f(a) = f(b) = z$, $f(c) = y$ and $f(d) = x$. Then f is a $I_{\pi gp^\star}$-continuous function but it is not πgp-continuous.*

Example 3.6. *Let (X, τ, I) be the same ideal topological space as in Example 2.7(1). Let $Y = \{x, y, z\}$ and $\sigma = \{Y, \emptyset, \{x, y\}\}$. Define a function $f : (X, \tau, I) \to (Y, \sigma)$ as follows: $f(a) = f(d) = f(e) = x$, $f(b) = z$ and $f(c) = y$. Then f is a $I_{\pi gp^\star}$-continuous function but it is not $I_{\pi g}$-continuous.*

Theorem 3.7. *For a function $f : (X, \tau, I) \to (Y, \sigma)$, the following properties hold:*

$$\begin{array}{ccc} \pi g\text{-continuous} & \longrightarrow & \pi gp\text{-continuous} \\ \downarrow & & \downarrow \\ I_{\pi g}\text{-continuous} & \longrightarrow & I_{\pi gp^\star}\text{-continuous} \end{array}$$

Proof. The proof is obvious by Remark 2.6. □

The composition of two $I_{\pi gp^\star}$-continuous functions need not be $I_{\pi gp^\star}$-continuous. Consider the following Example:

Example 3.8. *Let (X, τ, I) be the same ideal topological space as in Example 2.7(2), $Y = \{x, y, z\}$, $\sigma = \{Y, \emptyset, \{y, z\}\}$, $J = \{\emptyset, \{x\}\}$, $Z = \{1, 2\}$ and $\eta = \{Z, \emptyset, \{1\}\}$. Define $f : (X, \tau, I) \to (Y, \sigma, J)$ by $f(a) = f(c) = x$, $f(b) = y$ and $f(d) = z$ and $g : (Y, \sigma, J) \to (Z, \eta)$ by $g(x) = 1$ and $g(y) = g(z) = 2$. Then f and g are $I_{\pi gp^\star}$-continuous. $\{2\}$ is closed in (Z, η), $(g \circ f)^{-1}(\{2\}) = f^{-1}(g^{-1}(\{2\})) = f^{-1}(\{y, z\}) = \{b, d\}$ which is not $I_{\pi gp^\star}$-closed in (X, τ, I). Hence $g \circ f$ is not $I_{\pi gp^\star}$-continuous.*

Theorem 3.9. *Let $f : (X, \tau, I) \to (Y, \sigma, J)$ and $g : (Y, \sigma, J) \to (Z, \eta, K)$ be any two functions. Then*

(i) *$g \circ f$ is $I_{\pi gp^\star}$-continuous, if g is continuous and f is $I_{\pi gp^\star}$-continuous,*

(ii) *$g \circ f$ is $I_{\pi gp^\star}$-continuous, if g is $J_{\pi gp^\star}$-continuous and f is $I_{\pi gp^\star}$-irresolute,*

(iii) *$g \circ f$ is $I_{\pi gp^\star}$-irresolute, if g is $J_{\pi gp^\star}$-irresolute and f is $I_{\pi gp^\star}$-irresolute.*

Proof.

(i) Let V be closed in Z. Then $g^{-1}(V)$ is closed in Y, since g is continuous. $I_{\pi gp^\star}$-continuity of f implies that $f^{-1}(g^{-1}(V))$ is $I_{\pi gp^\star}$-closed in X. Hence $g \circ f$ is $I_{\pi gp^\star}$-continuous.

(ii) Let V be closed in Z. Since g is $J_{\pi gp^\star}$-continuous, $g^{-1}(V)$ is $J_{\pi gp^\star}$-closed in Y. As f is $I_{\pi gp^\star}$-irresolute, $f^{-1}(g^{-1}(V))$ is $I_{\pi gp^\star}$-closed in X. Hence $g \circ f$ is $I_{\pi gp^\star}$-continuous.

(iii) Let V be $K_{\pi gp^\star}$-closed in Z. Then $g^{-1}(V)$ is $J_{\pi gp^\star}$-closed in Y, since g is $J_{\pi gp^\star}$-irresolute. Because f is $I_{\pi gp^\star}$-irresolute, $f^{-1}(g^{-1}(V))$ is $I_{\pi gp^\star}$-closed in X. Hence $g \circ f$ is $I_{\pi gp^\star}$-irresolute.

□

4 Quasi-p*-I-normal spaces

Definition 4.1. *An ideal topological space (X, τ, I) is said to be quasi-p*-I-normal if for every pair of disjoint π-closed subsets A, B of X, there exist disjoint pre*-I-open sets U, V of X such that $A \subseteq U$ and $B \subseteq V$.*

Proposition 4.2. *If X is a quasi-p-normal space, then X is quasi-p*-I-normal.*

Proof. It is obtained from Proposition 2.28. □

Theorem 4.3. *The following properties are equivalent for a space X:*

(i) X is quasi-p*-I-normal,

(ii) for any disjoint π-closed sets A and B, there exist disjoint $I_{\pi gp^\star}$-open sets U, V of X such that $A \subseteq U$ and $B \subseteq V$,

(iii) for any π-closed set A and any π-open set B containing A, there exists an $I_{\pi gp^\star}$-open set U such that $A \subseteq U \subseteq p^\star_I Cl(U) \subseteq B$.

Proof. (i) ⇒ (ii) The proof is obvious.
(ii) ⇒ (iii) Let A be any π-closed set of X and B any π-open set of X such that $A \subseteq B$. Then A and $X\backslash B$ are disjoint π-closed subsets of X. Therefore, there exist disjoint $I_{\pi gp^\star}$-open sets U and V such that $A \subseteq U$ and $X\backslash B \subseteq V$. By the definition of $I_{\pi gp^\star}$-open set, We have that $X\backslash B \subseteq p^\star_I\mathrm{Int}(V)$ and $U \cap p^\star_I\mathrm{Int}(V) = \emptyset$. Therefore, we obtain $p^\star_I\mathrm{Cl}(U) \subseteq p^\star_I\mathrm{Cl}(X\backslash V)$ and hence $A \subseteq U \subset p^\star_I\mathrm{Cl}(U) \subseteq B$.
(iii) ⇒ (i) Let A and B be any disjoint π-closed sets of X. Then $A \subseteq X\backslash B$ and $X\backslash B$ is π-open and hence there exists an $I_{\pi gp^\star}$-open set G of X such that $A \subseteq G \subseteq p^\star_I\mathrm{Cl}(G) \subseteq X\backslash B$. Put $U = p^\star_I\mathrm{Int}(G)$ and $V = X\backslash\ p^\star_I\mathrm{Cl}(G)$. Then U and V are disjoint pre*-I-open sets of X such that $A \subseteq U$ and $B \subseteq V$. Therefore, X is quasi-p*-I-normal. □

Theorem 4.4. *Let $f : X \to Y$ be an $I_{\pi gp^\star}$-continuous m-π-closed injection. If Y is quasi-normal, then X is quasi-p*-I-normal.*

Proof. Let A and B be disjoint π-closed sets of Y. Since f is m-π-closed injection, f(A) and f(B) are disjoint π-closed sets of Y. By the quasi-normality of X, there exist disjoint open sets U and V such that $f(A) \subseteq U$ and $f(B) \subseteq V$. Since f is $I_{\pi gp^\star}$-continuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint $I_{\pi gp^\star}$-open sets such that $A \subseteq f^{-1}(U)$ and $B \subseteq f^{-1}(V)$. Therefore X is quasi-p*-I-normal by Theorem 4.3. □

Theorem 4.5. *Let $f : X \to Y$ be an $I_{\pi gp^\star}$-irresolute m-π-closed injection. If Y is quasi-p*-I-normal, then X is quasi-p*-I-normal.*

Proof. Let A and B be disjoint π-closed sets of Y. Since f is m-π-closed injection, f(A) and f(B) are disjoint π-closed sets of Y. By quasi-p*-I-normality of Y, there exist disjoint $I_{\pi gp^\star}$ open sets U and V such that f(A) $\subseteq$ U and f(B) $\subseteq$ V. Since f is $I_{\pi gp^\star}$-irresolute, then f^{-1}(U) and f^{-1}(V) are disjoint $I_{\pi gp^\star}$-open sets such that A $\subseteq$ f^{-1}(U) and B $\subseteq$ f^{-1}(V). Therefore X is quasi-p*-I-normal. □

Theorem 4.6. *Let (X, τ, I) be an ideal topological space where I is codense. Then X is quasi-p*-I-normal if and only if it is quasi-p-normal.*

5 Conclusion

General topology is important in many fields of applied sciences as well as branches of mathematics. In reality it is used in data mining, computational topology for geometric design and molecular design, computer-aided design, computer-aided geometric design, digital topology, information systems, particle physics and quantum physics etc. The notions of sets and functions in topological spaces, ideal topological spaces, minimal spaces and ideal minimal spaces are extensively developed and used in many engineering problems, information systems, particle physics, computational topology and mathematical sciences.

By researching generalizations of closed sets in various fields in general topology, some new separation axioms have been founded and they turn out to be useful in the study of digital topology. Therefore, all functions defined in this paper will have many possibilities of applications in digital topology and computer graphics.

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