



## Decompositions of $\mathcal{I}$ - $\pi g$ -continuity

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**Abstract :** In this paper, we introduce the notions of  $\mathcal{I}$ - $\pi$ -open sets,  $\mathcal{I}$ - $\pi g$ -open sets,  $\mathcal{I}$ - $\pi g\alpha$ -open sets,  $\mathcal{I}$ - $\pi gp$ -open sets,  $\mathcal{I}$ - $E_r$ -sets and  $\mathcal{I}$ - $E_r^*$ -sets in ideal topological spaces and investigate some of their properties and using these notions we obtain three decompositions of  $\mathcal{I}$ - $\pi g$ -continuity.

**Keywords :**  $\mathcal{I}$ - $\pi g\alpha$ -continuity,  $\mathcal{I}$ - $\pi gp$ -continuity,  $\mathcal{I}$ - $E_r$ -continuity,  $\mathcal{I}$ - $E_r^*$ -continuity and  $\mathcal{I}$ - $\pi g$ -continuity.

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## 1 Introduction and Preliminaries

In 1968, Zaitsev [12] introduced the concept of  $\pi$ -closed sets and in 1970, Levine [5] initiated the study of so called  $g$ -closed sets in topological spaces. The concept of  $g$ -continuity was introduced and studied by Balachandran et. al. in 1991 [1]. Dontchev and Noiri [2] defined the notions of  $\pi g$ -closed sets and  $\pi g$ -continuity in topological spaces. Quite Recently Ravi et. al. [9] obtained three different decompositions of  $\pi g$ -continuity in topological spaces by providing two types of weaker forms of continuity, namely  $E_r$ -continuity and  $E_r^*$ -continuity and in [10], they also obtained three different decompositions of  $\pi g$ -continuity via idealization.

Recently, Rajamani et. al. [7] introduced  $\mathcal{I}$ - $g$ -open sets,  $\mathcal{I}$ - $gp$ -open sets,  $\mathcal{I}$ - $gs$ -open sets and obtained three different decompositions of  $\mathcal{I}$ - $g$ -continuity and in [8], they also introduced  $\mathcal{I}$ - $rg$ -open sets,  $\mathcal{I}$ - $g\alpha^{**}$ -open sets,  $\mathcal{I}$ - $gpr$ -open sets and obtained three different decompositions of  $\mathcal{I}$ - $rg$ -continuity. In this paper, we introduce the notions of  $\mathcal{I}$ - $\pi$ -open sets,  $\mathcal{I}$ - $\pi g$ -open sets,  $\mathcal{I}$ - $\pi g\alpha$ -open sets,  $\mathcal{I}$ - $\pi gp$ -open sets,  $\mathcal{I}$ - $E_r$ -sets and  $\mathcal{I}$ - $E_r^*$ -sets to obtain three decompositions of  $\mathcal{I}$ - $\pi g$ -continuity.

Let  $(X, \tau)$  be a topological space. An ideal is defined as a nonempty collection  $\mathcal{I}$  of subsets of  $X$  satisfying the following two conditions:

- (i) If  $A \in \mathcal{I}$  and  $B \subseteq A$ , then  $B \in \mathcal{I}$
- (ii) If  $A \in \mathcal{I}$  and  $B \in \mathcal{I}$ , then  $A \cup B \in \mathcal{I}$ .

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For a subset  $A \subseteq X$ ,  $A^*(\mathcal{I}) = \{x \in X / U \cap A \notin \mathcal{I} \text{ for each neighborhood } U \text{ of } x\}$  is called the local function of  $A$  with respect to  $\mathcal{I}$  and  $\tau$  [4]. We simply write  $A^*$  instead of  $A^*(\mathcal{I})$  in case there is no chance for confusion.  $X^*$  is often a proper subset of  $X$ .

For every ideal topological space  $(X, \tau, \mathcal{I})$  there exists a topology  $\tau^*(\mathcal{I})$ , finer than  $\tau$ , generated by  $\beta(\mathcal{I}, \tau) = \{U \setminus I : U \in \tau \text{ and } I \in \mathcal{I}\}$ , but in general  $\beta(\mathcal{I}, \tau)$  is not always a topology [11]. Also,  $\text{cl}^*(A) = A \cup A^*$  defines a Kuratowski closure operator for  $\tau^*(\mathcal{I})$  [11].

Additionally,  $\text{cl}^*(A) \subseteq \text{cl}(A)$  for any subset  $A$  of  $X$  [3]. Throughout this paper,  $X$  denotes the ideal topological space  $(X, \tau, \mathcal{I})$  and also  $\text{cl}(A)$  and  $\text{int}(A)$  denote the closure of  $A$  and the interior of  $A$  in  $(X, \tau)$ , respectively.  $\text{int}^*(A)$  will denote the interior of  $A$  in  $(X, \tau^*, \mathcal{I})$ .

**Definition 1.1.** A subset  $A$  of an ideal topological space  $(X, \tau, \mathcal{I})$  is said to be

- (i)  $\mathcal{I}$ -pre-open [6] if  $A \subseteq \text{int}^*(\text{cl}^*(A))$ ,
- (ii)  $\mathcal{I}$ - $\alpha$ -open [6] if  $A \subseteq \text{int}^*(\text{cl}^*(\text{int}^*(A)))$ ,
- (iii) a  $\mathcal{I}$ -t-set [7] if  $\text{int}^*(\text{cl}^*(A)) = \text{int}^*(A)$ ,
- (iv) an  $\mathcal{I}$ - $\alpha^*$ -set [7] if  $\text{int}^*(\text{cl}^*(\text{int}^*(A))) = \text{int}^*(A)$ ,
- (v)  $\mathcal{I}$ -regular closed [8] if  $A = \text{cl}^*(\text{int}^*(A))$ .

The complement of  $\mathcal{I}$ -regular closed set is  $\mathcal{I}$ -regular open [8].

Also, we have  $\mathcal{I}$ - $\alpha\text{int}(A) = A \cap \text{int}^*(\text{cl}^*(\text{int}^*(A)))$  [6] and  $\mathcal{I}$ - $\text{pint}(A) = A \cap \text{int}^*(\text{cl}^*(A))$  [6], where  $\mathcal{I}$ - $\alpha\text{int}(A)$  denotes the  $\mathcal{I}$ - $\alpha$ -interior of  $A$  in  $(X, \tau, \mathcal{I})$  which is the union of all  $\mathcal{I}$ - $\alpha$ -open sets of  $(X, \tau, \mathcal{I})$  contained in  $A$ .  $\mathcal{I}$ - $\text{pint}(A)$  has similar meaning.

## 2 $\mathcal{I}$ - $\pi$ g-open sets, $\mathcal{I}$ - $\pi$ g $\alpha$ -open sets and $\mathcal{I}$ - $\pi$ gp-open sets

**Definition 2.1.** A subset  $A$  of an ideal topological space  $(X, \tau, \mathcal{I})$  is called

- (i)  $\mathcal{I}$ - $\pi$ -open if the finite union of  $\mathcal{I}$ -regular open sets,  
The complement of  $\mathcal{I}$ - $\pi$ -open set is  $\mathcal{I}$ - $\pi$ -closed,
- (ii)  $\mathcal{I}$ - $\pi$ g-open if  $F \subseteq \text{int}^*(A)$  whenever  $F \subseteq A$  and  $F$  is  $\mathcal{I}$ - $\pi$ -closed in  $X$ ,
- (iii)  $\mathcal{I}$ - $\pi$ g $\alpha$ -open if  $F \subseteq \mathcal{I}$ - $\alpha\text{int}(A)$  whenever  $F \subseteq A$  and  $F$  is  $\mathcal{I}$ - $\pi$ -closed in  $X$ ,
- (iv)  $\mathcal{I}$ - $\pi$ gp-open if  $F \subseteq \mathcal{I}$ - $\text{pint}(A)$  whenever  $F \subseteq A$  and  $F$  is  $\mathcal{I}$ - $\pi$ -closed in  $X$ .

**Proposition 2.2.** For a subset of an ideal topological space, the following hold:

- (i) Every  $\mathcal{I}$ - $\pi$ g-open set is  $\mathcal{I}$ - $\pi$ g $\alpha$ -open.
- (ii) Every  $\mathcal{I}$ - $\pi$ g $\alpha$ -open set is  $\mathcal{I}$ - $\pi$ gp-open.
- (iii) Every  $\mathcal{I}$ - $\pi$ g-open set is  $\mathcal{I}$ - $\pi$ gp-open.

*Proof.*

- (i) Let  $A$  be an  $\mathcal{I}$ - $\pi g$ -open. Then, for any  $\mathcal{I}$ - $\pi$ -closed set  $F$  with  $F \subseteq A$ , we have  $F \subseteq \text{int}^*(A) \subseteq \text{int}^*((\text{int}^*(A))^*) \cup \text{int}^*(A) = \text{int}^*((\text{int}^*(A))^*) \cup \text{int}^*(\text{int}^*(A)) \subseteq \text{int}^*((\text{int}^*(A))^* \cup \text{int}^*(A)) = \text{int}^*(\text{cl}^*(\text{int}^*(A)))$ . That is,  $F \subseteq A \cap \text{int}^*(\text{cl}^*(\text{int}^*(A))) = \mathcal{I}\text{-}\alpha\text{int}(A)$  which implies that  $A$  is  $\mathcal{I}$ - $\pi g\alpha$ -open.
- (ii) Let  $A$  be  $\mathcal{I}$ - $\pi g\alpha$ -open. Then, for any  $\mathcal{I}$ - $\pi$ -closed set  $F$  with  $F \subseteq A$ , we have  $F \subseteq \mathcal{I}\text{-}\alpha\text{int}(A) = A \cap \text{int}^*(\text{cl}^*(\text{int}^*(A))) \subseteq A \cap \text{int}^*(\text{cl}^*(A)) = \mathcal{I}\text{-}\pi\text{int}(A)$  which implies that  $A$  is  $\mathcal{I}$ - $\pi g\pi$ -open.
- (iii) It is an immediate consequence of (i) and (ii).

□

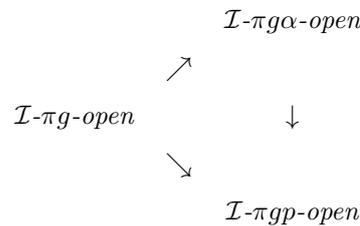
**Remark 2.3.** The converses of Proposition 2.2 are not true, in general.

**Example 2.4.** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$  and  $\mathcal{I} = \{\emptyset, \{d\}\}$ . Then  $\{a, b, d\}$  is  $\mathcal{I}$ - $\pi g\pi$ -open set but not  $\mathcal{I}$ - $\pi g\alpha$ -open.

**Example 2.5.** In Example 2.4,  $\{a, b, d\}$  is  $\mathcal{I}$ - $\pi g\pi$ -open set but not  $\mathcal{I}$ - $\pi g$ -open.

**Example 2.6.** Let  $X = \{a, b, c, d, e\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$  and  $\mathcal{I} = \{\emptyset\}$ . Clearly  $\{a, c, d, e\}$  is  $\mathcal{I}$ - $\pi g\alpha$ -open set but not  $\mathcal{I}$ - $\pi g$ -open.

**Remark 2.7.** By Proposition 2.2, we have the following diagram. In this diagram, there is no implication which is reversible as shown by examples above.



### 3 $\mathcal{I}$ - $E_r$ -sets and $\mathcal{I}$ - $E_r^*$ -sets

**Definition 3.1.** A subset  $A$  of an ideal topological space  $(X, \tau, \mathcal{I})$  is called

- (i) a  $\mathcal{I}$ - $E_r$ -set if  $A = U \cap V$ , where  $U$  is  $\mathcal{I}$ - $\pi g$ -open and  $V$  is a  $\mathcal{I}$ - $t$ -set,
- (ii) a  $\mathcal{I}$ - $E_r^*$ -set if  $A = U \cap V$ , where  $U$  is  $\mathcal{I}$ - $\pi g$ -open and  $V$  is an  $\mathcal{I}$ - $\alpha^*$ -set.

We have the following proposition:

**Proposition 3.2.** For a subset of an ideal topological space, the following hold:

- (i) Every  $\mathcal{I}$ - $t$ -set is an  $\mathcal{I}$ - $\alpha^*$ -set [7] and a  $\mathcal{I}$ - $E_r$ -set.
- (ii) Every  $\mathcal{I}$ - $\alpha^*$ -set is a  $\mathcal{I}$ - $E_r^*$ -set.
- (iii) Every  $\mathcal{I}$ - $E_r$ -set is a  $\mathcal{I}$ - $E_r^*$ -set.
- (iv) Every  $\mathcal{I}$ - $\pi g$ -open set is both  $\mathcal{I}$ - $E_r$ -set and  $\mathcal{I}$ - $E_r^*$ -set.

From Proposition 3.2, We have the following diagram.

$$\begin{array}{ccccc}
 \mathcal{I}\text{-}\pi g\text{-open set} & \longrightarrow & \mathcal{I}\text{-}E_r\text{-set} & \longleftarrow & \mathcal{I}\text{-}t\text{-set} \\
 & & \downarrow & & \downarrow \\
 & & \mathcal{I}\text{-}E_r^*\text{-set} & \longleftarrow & \mathcal{I}\text{-}\alpha^*\text{-set}
 \end{array}$$

**Remark 3.3.** The converses of implications in Diagram II need not be true as the following examples show.

**Example 3.4.** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$  and  $\mathcal{I} = \{\emptyset, \{c\}\}$ . Then  $\{a, b\}$  is  $\mathcal{I}\text{-}E_r$ -set but not a  $\mathcal{I}\text{-}t$ -set.

**Example 3.5.** In Example 3.4,  $\{b, c, d\}$  is  $\mathcal{I}\text{-}E_r$ -set but not an  $\mathcal{I}\text{-}\pi g$ -open set.

**Example 3.6.** In Example 2.4,  $\{a, b, d\}$  is  $\mathcal{I}\text{-}E_r^*$ -set but not a  $\mathcal{I}\text{-}E_r$ -set.

**Example 3.7.** In Example 3.4,  $\{a, b\}$  is  $\mathcal{I}\text{-}E_r^*$ -set but not an  $\mathcal{I}\text{-}\alpha^*$ -set.

**Example 3.8.** In Example 2.4,  $\{b\}$  is  $\mathcal{I}\text{-}\alpha^*$ -set but not a  $\mathcal{I}\text{-}t$ -set.

**Proposition 3.9.** A subset  $A$  of  $X$  is  $\mathcal{I}\text{-}\pi g$ -open if and only if it is both  $\mathcal{I}\text{-}\pi g p$ -open and a  $\mathcal{I}\text{-}E_r$ -set in  $X$ .

*Proof.* Necessity is trivial. We prove the sufficiency. Assume that  $A$  is  $\mathcal{I}\text{-}\pi g p$ -open and a  $\mathcal{I}\text{-}E_r$ -set in  $X$ . Let  $F \subseteq A$  and  $F$  is  $\mathcal{I}\text{-}\pi$ -closed in  $X$ . Since  $A$  is a  $\mathcal{I}\text{-}E_r$ -set in  $X$ ,  $A = U \cap V$ , where  $U$  is  $\mathcal{I}\text{-}\pi g$ -open and  $V$  is a  $\mathcal{I}\text{-}t$ -set. Since  $A$  is  $\mathcal{I}\text{-}\pi g p$ -open,  $F \subseteq \mathcal{I}\text{-}pint(A) = A \cap int^*(cl^*(A)) = (U \cap V) \cap int^*(cl^*(U \cap V)) \subseteq (U \cap V) \cap int^*(cl^*(U) \cap cl^*(V)) = (U \cap V) \cap int^*(cl^*(U)) \cap int^*(cl^*(V))$ . This implies  $F \subseteq int^*(cl^*(V)) = int^*(V)$  since  $V$  is a  $\mathcal{I}\text{-}t$ -set. Since  $F$  is  $\mathcal{I}\text{-}\pi$ -closed,  $U$  is  $\mathcal{I}\text{-}\pi g$ -open and  $F \subseteq U$ , we have  $F \subseteq int^*(U)$ . Therefore,  $F \subseteq int^*(U) \cap int^*(V) = int^*(U \cap V) = int^*(A)$ . Hence  $A$  is  $\mathcal{I}\text{-}\pi g$ -open in  $X$ .  $\square$

**Corollary 3.10.** A subset  $A$  of  $X$  is  $\mathcal{I}\text{-}\pi g$ -open if and only if it is both  $\mathcal{I}\text{-}\pi g \alpha$ -open and a  $\mathcal{I}\text{-}E_r$ -set in  $X$ .

*Proof.* This is an immediate consequence of Proposition 3.9.  $\square$

**Proposition 3.11.** A subset  $A$  of  $X$  is  $\mathcal{I}\text{-}\pi g$ -open if and only if it is both  $\mathcal{I}\text{-}\pi g \alpha$ -open and a  $\mathcal{I}\text{-}E_r^*$ -set in  $X$ .

*Proof.* Necessity is trivial. We prove the sufficiency. Assume that  $A$  is  $\mathcal{I}\text{-}\pi g \alpha$ -open and a  $\mathcal{I}\text{-}E_r^*$ -set in  $X$ . Let  $F \subseteq A$  and  $F$  is  $\mathcal{I}\text{-}\pi$ -closed in  $X$ . Since  $A$  is a  $\mathcal{I}\text{-}E_r^*$ -set in  $X$ ,  $A = U \cap V$ , where  $U$  is  $\mathcal{I}\text{-}\pi g$ -open and  $V$  is an  $\mathcal{I}\text{-}\alpha^*$ -set. Now since  $F$  is  $\mathcal{I}\text{-}\pi$ -closed,  $F \subseteq U$  and  $U$  is  $\mathcal{I}\text{-}\pi g$ -open,  $F \subseteq int^*(U)$ . Since  $A$  is  $\mathcal{I}\text{-}\pi g \alpha$ -open,  $F \subseteq \mathcal{I}\text{-}\alpha int(A) = A \cap int^*(cl^*(int^*(A))) = (U \cap V) \cap int^*(cl^*(int^*(U \cap V))) = (U \cap V) \cap int^*(cl^*(int^*(U) \cap int^*(V))) \subseteq (U \cap V) \cap int^*(cl^*(int^*(U)) \cap cl^*(int^*(V))) = (U \cap V) \cap int^*(cl^*(int^*(U))) \cap int^*(cl^*(int^*(V))) = (U \cap V) \cap int^*(cl^*(int^*(U))) \cap int^*(V)$ , since  $V$  is an  $\mathcal{I}\text{-}\alpha^*$ -set. This implies  $F \subseteq int^*(V)$ . Therefore,  $F \subseteq int^*(U) \cap int^*(V) = int^*(U \cap V) = int^*(A)$ . Hence  $A$  is  $\mathcal{I}\text{-}\pi g$ -open in  $X$ .  $\square$

**Remark 3.12.**

(i) The concepts of  $\mathcal{I}\text{-}\pi g p$ -open sets and  $\mathcal{I}\text{-}E_r$ -sets are independent of each other.

(ii) The concepts of  $\mathcal{I}$ - $\pi g\alpha$ -open sets and  $\mathcal{I}$ - $E_r$ -sets are independent of each other.

(iii) The concepts of  $\mathcal{I}$ - $\pi g\alpha$ -open sets and  $\mathcal{I}$ - $E_r^*$ -sets are independent of each other.

**Example 3.13.** Consider the Example 2.4. Then

(i)  $\{d\}$  is  $\mathcal{I}$ - $E_r$ -set but not a  $\mathcal{I}$ - $\pi gp$ -open.

(ii)  $\{a, b, d\}$  is  $\mathcal{I}$ - $\pi gp$ -open but not a  $\mathcal{I}$ - $E_r$ -set.

**Example 3.14.**

(i) In Example 2.4,  $\{b, c, d\}$  is  $\mathcal{I}$ - $E_r$ -set but not an  $\mathcal{I}$ - $\pi g\alpha$ -open set.

(ii) In Example 2.6,  $\{a, c, d, e\}$  is  $\mathcal{I}$ - $\pi g\alpha$ -open set but not a  $\mathcal{I}$ - $E_r$ -set.

**Example 3.15.**

(i) In Example 2.5,  $\{b, c, d\}$  is  $\mathcal{I}$ - $E_r^*$ -set but not an  $\mathcal{I}$ - $\pi g\alpha$ -open set.

(ii) In Example 2.6,  $\{a, c, d, e\}$  is  $\mathcal{I}$ - $\pi g\alpha$ -open set but not a  $\mathcal{I}$ - $E_r^*$ -set.

## 4 Decompositions of $\mathcal{I}$ - $\pi g$ -continuity

**Definition 4.1.** A mapping  $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$  is said to be  $\mathcal{I}$ - $\pi g\alpha$ -continuous (resp.  $\mathcal{I}$ - $\pi g$ -continuous,  $\mathcal{I}$ - $\pi gp$ -continuous,  $\mathcal{I}$ - $E_r$ -continuous and  $\mathcal{I}$ - $E_r^*$ -continuous) if for every  $V \in \sigma$ ,  $f^{-1}(V)$  is  $\mathcal{I}$ - $\pi g\alpha$ -open (resp.  $\mathcal{I}$ - $\pi g$ -open,  $\mathcal{I}$ - $\pi gp$ -open, a  $\mathcal{I}$ - $E_r$ -set and a  $\mathcal{I}$ - $E_r^*$ -set) in  $(X, \tau, \mathcal{I})$ . From Propositions 3.9 and 3.11 and Corollary 3.10 we have the following decompositions of  $\mathcal{I}$ - $\pi g$ -continuity.

**Theorem 4.2.** Let  $(X, \tau, \mathcal{I})$  be an ideal topological space. For a mapping  $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$ , the following properties are equivalent:

(i)  $f$  is  $\mathcal{I}$ - $\pi g$ -continuous;

(ii)  $f$  is  $\mathcal{I}$ - $\pi gp$ -continuous and  $\mathcal{I}$ - $E_r$ -continuous;

(iii)  $f$  is  $\mathcal{I}$ - $\pi g\alpha$ -continuous and  $\mathcal{I}$ - $E_r$ -continuous;

(iv)  $f$  is  $\mathcal{I}$ - $\pi g\alpha$ -continuous and  $\mathcal{I}$ - $E_r^*$ -continuous.

**Remark 4.3.**

(i) The concepts of  $\mathcal{I}$ - $\pi gp$ -continuity and  $\mathcal{I}$ - $E_r$ -continuity are independent of each other.

(ii) The concepts of  $\mathcal{I}$ - $\pi g\alpha$ -continuity and  $\mathcal{I}$ - $E_r$ -continuity are independent of each other.

(iii) The concepts of  $\mathcal{I}$ - $\pi g\alpha$ -continuity and  $\mathcal{I}$ - $E_r^*$ -continuity are independent of each other.

**Example 4.4.**

(i) Let  $X = Y = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$ ,  $\mathcal{I} = \{\emptyset, \{d\}\}$  and  $\sigma = \{\emptyset, Y, \{b, c, d\}\}$ . Let  $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$  be the identity function. Then  $f$  is  $\mathcal{I}$ - $E_r$ -continuous but not  $\mathcal{I}$ - $\pi gp$ -continuous.

- (ii) In Example 4.4(1), if  $\sigma$  is replaced by  $\sigma = \{\emptyset, Y, \{a, b, d\}\}$ , then  $f$  is  $\mathcal{I}$ - $\pi$ gp-continuous but not  $\mathcal{I}$ - $E_r$ -continuous.

**Example 4.5.**

- (i) In Example 4.4(1),  $f$  is  $\mathcal{I}$ - $E_r$ -continuous but not  $\mathcal{I}$ - $\pi$ g $\alpha$ -continuous.
- (ii) Let  $X = Y = \{a, b, c, d, e\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$ ,  $\mathcal{I} = \{\emptyset\}$  and  $\sigma = \{\emptyset, Y, \{a, c, d, e\}\}$ . Let  $f : (X, \tau, \mathcal{I}) \rightarrow (Y, \sigma)$  be the identity function. Then  $f$  is  $\mathcal{I}$ - $\pi$ g $\alpha$ -continuous but not  $\mathcal{I}$ - $E_r$ -continuous.

**Example 4.6.**

- (i) In Example 4.4(1),  $f$  is  $\mathcal{I}$ - $E_r^*$ -continuous but not  $\mathcal{I}$ - $\pi$ g $\alpha$ -continuous.
- (ii) In Example 4.5(2),  $f$  is  $\mathcal{I}$ - $\pi$ g $\alpha$ -continuous but not  $\mathcal{I}$ - $E_r^*$ -continuous.

## 5 Conclusion

Recently, Rajamani et. al. [7] introduced  $\mathcal{I}$ -g-open sets,  $\mathcal{I}$ -gp-open sets,  $\mathcal{I}$ -gs-open sets and obtained three different decompositions of  $\mathcal{I}$ -g-continuity and in [8], they also introduced  $\mathcal{I}$ -rg-open sets,  $\mathcal{I}$ -g $\alpha^{**}$ -open sets,  $\mathcal{I}$ -gpr-open sets and obtained three different decompositions of  $\mathcal{I}$ -rg-continuity. In this paper, we introduced the notions of  $\mathcal{I}$ - $\pi$ -open sets,  $\mathcal{I}$ - $\pi$ g-open sets,  $\mathcal{I}$ - $\pi$ g $\alpha$ -open sets,  $\mathcal{I}$ - $\pi$ gp-open sets,  $\mathcal{I}$ - $E_r$ -sets and  $\mathcal{I}$ - $E_r^*$ -sets to obtain three decompositions of  $\mathcal{I}$ - $\pi$ g-continuity.

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