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Some Decompositions of πg-Continuity

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Abstract : In this paper, we introduce the notions of E_r-sets and E_r^*-sets in topological spaces and investigate some of their properties and using these notions we obtain three decompositions of πg-continuity.

Keywords : $\pi g\alpha$-continuity, πgp-continuity, E_r-continuity, E_r^*-continuity and πg-continuity.

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1 Introduction and Preliminaries

In 1968, Zaitsev [15] introduced the concept of π-closed sets and in 1970, Levine [7] initiated the study of so called g-closed sets in topological spaces. The concept of g-continuity was introduced and studied by Balachandran et. al. in 1991 [3]. Dontchev and Noiri [4] defined the notions of πg-closed sets and πg-continuity in topological spaces. In 1993, Palaniappan and Rao [10] introduced the notions of regular generalized closed (rg-closed) sets and rg-continuity in topological spaces. In 2000, Sundaram and Rajamani [13] obtained three different decompositions of rg-continuity by providing two types of weaker forms of continuity, namely C_r-continuity and C_r^*-continuity. In this paper, we introduce the notions of E_r-sets and E_r^*-sets to obtain three decompositions of πg-continuity by providing two types of weaker forms of continuity, namely E_r-continuity and E_r^*-continuity.

Let (X, τ) be a topological space and also cl(A) and int(A) denote the closure of A and the interior of A in (X, τ), respectively.

Definition 1.1. A subset A of *(X, τ)* is said to be

(i) *α-open [9] if $A \subseteq int(cl(int(A)))$,*

(ii) *preopen [8] if $A \subseteq int(cl(A))$,*

(iii) *regular open [12] if $A = int(cl(A))$,*

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(iv) π-open [15] if the finite union of regular open sets,

(v) πg-open [4] iff $F \subseteq int(A)$ whenever $F \subseteq A$ and F is π-closed in (X, τ),

(vi) πgp-open [11] iff $F \subseteq pint(A)$ whenever $F \subseteq A$ and F is π-closed in (X, τ),

(vii) $\pi g\alpha$-open [2] iff $F \subseteq \alpha int(A)$ whenever $F \subseteq A$ and F is π-closed in (X, τ),

(viii) a t-set [14] if $int(A) = int(cl(A))$,

(ix) an α^*-set [5] if $int(A) = int(cl(int(A)))$.

The complements of the above mentioned open sets are called their respective closed sets.

The preinterior $pint(A)$ *(resp.* α*-interior,* $\alpha int(A)$*) of* A *is the union of all preopen sets (resp.* α*-open sets) contained in* A*. The* α*-closure* $\alpha cl(A)$ *of* A *is the intersection of all* α*-closed sets containing* A*.*

Lemma 1.2 ([1]). *If* A *is a subset of* X*, then*

(i) $pint(A) = A \cap int(cl(A))$,

(ii) $\alpha int(A) = A \cap int(cl(int(A)))$ and $\alpha cl(A) = A \cup cl(int(cl(A)))$.

Remark 1.3. *The following hold in a topological space.*

(i) Every πg*-open set is* πgp*-open but not conversely [11].*

(ii) Every πg*-open set is* $\pi g\alpha$*-open but not conversely [2].*

2 $\pi g\alpha$-open Sets

Proposition 2.1. *For a subset of a topological space, the following hold: Every* $\pi g\alpha$*-open set is* πgp*-open.*

Proof. It follows from the definitions. □

Remark 2.2. *The converse of Proposition 2.1 is not true, in general.*

Example 2.3. *Let* $X = \{a, b, c, d, e\}$ *and* $\tau = \{\phi, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$*. Then* $\{b, c, e\}$ *is* πgp*-open set but not* $\pi g\alpha$*-open.*

Remark 2.4. *By Proposition 2.1 and Remark 1.3, we have the following diagram. In this diagram, there is no implication which is reversible as shown by examples above.*

πg-open

↙

$\pi g\alpha$-open ↓

↘

πgp-open

3 E_r-sets and E_r^*-sets

Definition 3.1. *A subset A of a topological space (X, τ) is called*

(i) a E_r-set if $A = U \cap V$, where U is πg-open and V is a t-set,

(ii) a E_r^*-set if $A = U \cap V$, where U is πg-open and V is an α^*-set.

We have the following proposition:

Proposition 3.2. *For a subset of a topological space, the following hold:*

(i) Every t-set is an α^*-set [5] and a E_r-set.

(ii) Every α^*-set is a E_r^*-set.

(iii) Every E_r-set is a E_r^*-set.

(iv) Every πg-open set is a E_r-set.

From Proposition 3.2, We have the following diagram.

$$\begin{array}{ccccc} \pi g\text{-open set} & \longrightarrow & E_r\text{-set} & \longleftarrow & t\text{-set} \\ & & \downarrow & & \downarrow \\ & & E_r^*\text{-set} & \longleftarrow & \alpha^*\text{-set} \end{array}$$

Remark 3.3. *The converses of implications in Diagram II need not be true as the following examples show.*

Example 3.4. *Let $X = \{a, b, c, d, e\}$ and $\tau = \{\phi, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$. Then $\{b\}$ is E_r-set but not πg-open.*

Example 3.5. *In Example 3.4, $\{c\}$ is E_r-set but not t-set.*

Example 3.6. *In Example 3.4, $\{b, c\}$ is E_r^*-set but not E_r-set.*

Example 3.7. *In Example 3.4, $\{c\}$ is α^*-set but not t-set.*

Example 3.8. *In Example 3.4, $\{c, d, e\}$ is E_r^*-set but not α^*-set.*

Proposition 3.9. *A subset A of X is πg-open if and only if it is both πgp-open and a E_r-set in X.*

Proof. Necessity is trivial. We prove the sufficiency. Assume that A is πgp-open and a E_r-set in X. Let $F \subseteq A$ and F is π-closed in X. Since A is a E_r-set in X, $A = U \cap V$, where U is πg-open and V is a t-set. Since A is πgp-open, $F \subseteq p\mathrm{int}(A) = A \cap \mathrm{int}(\mathrm{cl}(A)) = (U \cap V) \cap \mathrm{int}(\mathrm{cl}(U \cap V)) \subseteq (U \cap V) \cap \mathrm{int}(\mathrm{cl}(U) \cap \mathrm{cl}(V)) = (U \cap V) \cap \mathrm{int}(\mathrm{cl}(U)) \cap \mathrm{int}(\mathrm{cl}(V))$. This implies $F \subseteq \mathrm{int}(\mathrm{cl}(V)) = \mathrm{int}(V)$ since V is a t-set. Since F is π-closed, U is πg-open and $F \subseteq U$, we have $F \subseteq \mathrm{int}(U)$. Therefore, $F \subseteq \mathrm{int}(U) \cap \mathrm{int}(V) = \mathrm{int}(U \cap V) = \mathrm{int}(A)$. Hence A is πg-open in X. □

Corollary 3.10. *A subset A of X is πg-open if and only if it is both $\pi g\alpha$-open and a E_r-set in X.*

Proof. This is an immediate consequence of Proposition 3.9. □

Proposition 3.11. *A subset A of X is πg-open if and only if it is both $\pi g\alpha$-open and a E_r^*-set in X.*

Proof. Necessity is trivial. We prove the sufficiency. Assume that A is $\pi g\alpha$-open and a E_r^*-set in X. Let $F \subseteq A$ and F is π-closed in X. Since A is a E_r^*-set in X, $A = U \cap V$, where U is πg-open and V is an α^*-set. Now since F is π-closed, $F \subseteq U$ and U is πg-open, $F \subseteq$ int(U). Since A is $\pi g\alpha$-open, $F \subseteq \alpha\text{int}(A)$ $= A \cap \text{int}(\text{cl}(\text{int}(A))) = (U \cap V) \cap \text{int}(\text{cl}(\text{int}(U \cap V))) = (U \cap V) \cap \text{int}(\text{cl}(\text{int}(U) \cap \text{int}(V))) \subseteq (U \cap V)$ $\cap\, \text{int}(\text{cl}(\text{int}(U)) \cap \text{cl}(\text{int}(V))) = (U \cap V) \cap \text{int}(\text{cl}(\text{int}(U))) \cap \text{int}(\text{cl}(\text{int}(V))) = (U \cap V) \cap \text{int}(\text{cl}(\text{int}(U)))$ $\cap\, \text{int}(V)$, since V is an α^*-set. This implies $F \subseteq \text{int}(V)$. Therefore, $F \subseteq \text{int}(U) \cap \text{int}(V) = \text{int}(U \cap V)$ $= \text{int}(A)$. Hence A is πg-open in X. □

Remark 3.12.

(i) The concepts of πgp-open sets and E_r-sets are independent of each other.

(ii) The concepts of $\pi g\alpha$-open sets and E_r-sets are independent of each other.

(iii) The concepts of $\pi g\alpha$-open sets and E_r^*-sets are independent of each other.

Example 3.13. *Let X= {a, b, c, d, e} and τ = {ϕ, {a}, {e}, {a, e}, {c, d}, {a, c, d}, {c, d, e}, {a, c, d, e}, {b, c, d, e}, X}. Then {b, c, e} is πgp-open but not E_r-set and {b, e} is E_r-set but not πgp-open.*

Example 3.14. *Let X = {a, b, c, d} and τ = {ϕ, X, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Then {a, d} is E_r-set but not $\pi g\alpha$-open set. Also {a, b, d} is an $\pi g\alpha$-open set but not E_r-set.*

Example 3.15. *In Example 3.14, {a, d} is E_r^*-set but not $\pi g\alpha$-open set and {a, b, d} is an $\pi g\alpha$-open set but not E_r^*-set.*

4 Decompositions of πg-continuity

Definition 4.1. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be πg-continuous [4] (resp. πgp-continuous [11], $\pi g\alpha$-continuous [2], E_r-continuous and E_r^*-continuous) if $f^{-1}(V)$ is πg-open (resp. πgp-open, $\pi g\alpha$-open, E_r-set and E_r^*-set) in (X, τ) for every open set V in (Y, σ).*

From Propositions 3.9 and 3.11 and Corollary 3.10 we have the following decompositions of πg-continuity.

Theorem 4.2. *For a mapping $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(i) f is πg-continuous;

(ii) f is πgp-continuous and E_r-continuous;

(iii) f is $\pi g\alpha$-continuous and E_r-continuous;

(iv) f is $\pi g\alpha$-continuous and E_r^*-continuous.

Remark 4.3.

(i) The concepts of πgp-continuity and E_r-continuity are independent of each other.

(ii) The concepts of $\pi g\alpha$-continuity and E_r-continuity are independent of each other.

(iii) The concepts of $\pi g\alpha$-continuity and E_r^*-continuity are independent of each other.

Example 4.4.

(i) Let $X = Y = \{a, b, c, d, e\}$, $\tau = \{\phi, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$ and $\sigma = \{\phi, \{b, c, e\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is πgp-continuous but not E_r-continuous.

(ii) Let $X = Y = \{a, b, c, d, e\}$, $\tau = \{\phi, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is E_r-continuous but not πgp-continuous.

Example 4.5.

(i) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\sigma = \{\phi, \{a, d\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is E_r-continuous but not $\pi g\alpha$-continuous.

(ii) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\sigma = \{\phi, \{a, b, d\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\pi g\alpha$-continuous but not E_r-continuous.

Example 4.6.

(i) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\sigma = \{\phi, \{a, d\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is E_r^*-continuous but not $\pi g\alpha$-continuous.

(ii) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\sigma = \{\phi, \{a, b, d\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $\pi g\alpha$-continuous but not E_r^*-continuous.

References

[1] D.Andrijevic, *Semi-preopen sets*, Mat. Vesnik, 38(1)(1986), 24-32.

[2] I.Arockiarani, K.Balachandran and C.Janaki, *On contra-$\pi g\alpha$-continuous functions*, Kochi J. Math., 3(2008), 201-209.

[3] K.Balachandran, P.Sundaram and H.Maki, *On generalized continuous maps in topological spaces*, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 12(1991), 5-13.

[4] J.Dontchev and T.Noiri, *Quasi-normal spaces and πg-closed sets*, Acta Math. Hungar., 89(3)(2000), 211-219.

[5] E.Hatir, T.Noiri and S.Yuksel, *A decomposition of continuity*, Acta Math. Hungar., 70(1996), 145-150.

[6] N.Levine, *A decomposition of continuity in topological spaces*, Amer. Math. Monthly, 68(1961), 44-46.

[7] N.Levine, *Generalized closed sets in topology*, Rend. Circ. Mat. Palermo, 19(2)(1970), 89-96.

[8] A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb, *On precontinuous and weak precontinuous mappings*, Proc. Math. Phys. Soc. Egypt, 53(1982), 47-53.

[9] O.Njastad, *On some classes of nearly open sets*, Pacific J. Math., 15(1965), 961-970.

[10] N.Palaniappan and K.C.Rao, *Regular generalized closed sets*, Kyungpook Math. J., 33(1993), 211-219.

[11] J.H.Park, *On πgp-closed sets in topological spaces*, Indian J. Pure Appl. Math., to appear.

[12] M.H.Stone, *Applications of the theory of Boolean rings to general topology*, Trans. Amer. Math. Soc., 41(1937), 375-481.

[13] P.Sundaram and M.Rajamani, *Some decompositions of regular generalized continuous maps in topological spaces*, Far East J. Math. Sci., II(2000), 179-188.

[14] J.Tong, *On decomposition of continuity in topological spaces*, Acta Math. Hungar., 54(1989), 51-55.

[15] V.Zaitsev, *On certain classes of topological spaces and their bicompactifications*, Dokl. Akad. Nauk. SSSR, 178(1968), 778-779.