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The First and Second Zagreb Indices of Generalized Complementary Prisms

Research Article

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Abstract: The first and second zagreb indices of a graph G are defined as $M_1(G) = \sum\limits_{uv \in E(G)} [deg(u) + deg(v)]$ (or equivalently $\sum\limits_{u \in V(G)} [deg(u)^2]$ and $M_2(G) = \sum\limits_{uv \in E(G)} [deg(u)deg(v)]$ respectively. In this paper, we have obtained the first and second zagreb indices of the generalized complementary prisms $G_{m+n}, \mathcal{G}_{m,n}, \mathcal{G}^c_{m,m}$ and $G^P_{m,m}$.

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Keywords: First Zagreb Index, Second Zagreb index, Complementary Prism and Generalized Complementary Prism

1. Introduction

Throughout this paper, all graphs we considered are simple and connected. For a vertex $v \in V(G)$, $deg(v)$ denotes the degree of v. The Zagreb indices have been introduced by Gutman and Trinajstic [3, 4] and are defined as:

$$M_1(G) = \sum_{e=uv \in E(G)} [deg(u) + deg(v)] = \sum_{v \in V(G)} [deg(v)]^2, M_2(G) = \sum_{e=uv \in E(G)} [deg(u)deg(v)]$$

Further, so many authors have studied about Zagreb indices in graphs [1, 2, 6, 7]. Kathiresan and Arockiaraj introduced some generalization of complementary prisms and studied the Wiener index of those generalized complementary prisms [5]. Let G and H be any two graphs on p_1 and p_2 vertices, respectively and let R and S be subsets of $V(G) = \{u_1, u_2, \dots, u_{p_1}\}$ and $V(H) = \{v_1, v_2, \dots, v_{p_2}\}$ respectively. The *complementary product* $G(R)\square H(S)$ has the vertex set $\{(u_i, v_j) : 1 \leq i \leq p_1, 1 \leq j \leq p_2\}$ and (u_i, v_j) and (u_h, v_k) are adjacent in $G(R)\square H(S)$

(1) if $i = h, u_i \in R$ and $v_jv_k \in E(H)$, or if $i = h, u_i \notin R$ and $v_jv_k \notin E(H)$ or

(2) if $j = k, v_j \in S$ and $u_iu_h \in E(G)$, or if $j = k$, $v_j \notin S$ and $u_iu_h \notin E(G)$.

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In other words, $G(R)\square H(S)$ is the graph formed by replacing each vertex $u_i \in R$ of G by a copy of H, each vertex $u_i \notin R$ of G by a copy of $\overline{H}$, each vertex $v_j \in S$ of H by a copy of G and each vertex $v_j \notin S$ of H by a copy of $\overline{G}$. If $R = V(G)$ (respectively, $S = V(H)$), the complementary product can be written as $G\square H(S)$ (respectively, $G(R)\square H$). The *complementary prism* $G\overline{G}$ obtained from G is $G\square K_2(S)$ with $|S| = 1$. That is, $G\overline{G}$ has a copy of G and a copy of $\overline{G}$ with a matching between the corresponding vertices. In $G\overline{G}$, we have an edge $v\overline{v}$ for each vertex v in G. The authors consider this edge as K_2 or $K_{1,1}$ or P_2. By taking m copies of G and n copies of $\overline{G}$, they generalize the complementary prism as a graph $G\square H(S)$, where $H = K_{m+n}$ (or $K_{m,n}$) and S is a subset of $V(H)$ having m vertices and $H = C_{2m}$ (or P_{2m}) whose vertex set is $\{v_1, v_2, \ldots, v_{2m}\}$ and $S = \{v_1, v_3, \ldots, v_{2m-1}\}$.

In this paper, we have obtained the first and second zagreb indices of the generalized complementary prisms $G_{m+n}, G_{m,n}, G^c_{m,m}$ and $G^P_{m,m}$.

2. Main Results

Proposition 2.1. *For any* $m, n \geq 1, M_1(G_{m+n}) = (m+n)M_1(G) + 4mq(m+n-1) + mp(m+n-1)^2 + np(m+n+p-2)^2 - 4nq(m+n+p-2)$.

Proof. In G_{m+n},

$$deg(u) = \begin{cases} deg_G(u) + m + n - 1, & \text{if } u \text{ is in a copy of } G \\ m + n + p - 2 - deg_G(u), & \text{if } u \text{ is in a copy of } \overline{G}. \end{cases}$$

Therefore,

$$\begin{aligned}
M_1(G_{m+n}) &= \sum_{u \in V(G_{m+n})} [deg(u)]^2 \\
&= \sum_{i=1}^{m} \sum_{u \in i^{th}\ copy\ of\ G} [deg(u)]^2 + \sum_{i=1}^{n} \sum_{u \in i^{th}\ copy\ of \overline{G}} [deg(u)]^2 \\
&= m \sum_{u \in a\ copy\ of\ G} [deg(u)]^2 + n \sum_{u \in a\ copy\ of\ \overline{G}} [deg(u)]^2 \\
&= m \sum_{u \in V(G)} [deg_G(u) + m + n - 1]^2 + n \sum_{u \in V(G)} [m + n + p - deg_G(u) - 2]^2 \\
&= m \left[\sum_{u \in V(G)} (deg_G(u))^2 + \sum_{u \in V(G)} [2(m+n-1)deg_G(u)] + \sum_{u \in V(G)} (m+n-1)^2 \right] + n \left[\sum_{u \in V(G)} (m+n+p-2)^2 \right. \\
&\quad \left. - \sum_{u \in V(G)} [2(m+n+p-2)deg_G(u)] + \sum_{u \in V(G)} [deg_G(u)]^2 \right] \\
&= mM_1(G) + 2m(m+n-1) \sum_{u \in V(G)} deg_G(u) + mp(m+n-1)^2 \\
&\quad + np(m+n+p-2)^2 - 2n(m+n+p-2) \sum_{u \in V(G)} deg_G(u) + nM_1(G) \\
&= (m+n)M_1(G) + 4mq(m+n-1) + mp(m+n-1)^2 + np(m+n+p-2)^2 - 4nq(m+n+p-2).
\end{aligned}$$

□

Proposition 2.2. *For any* $m, n \geq 1, M_1(G_{m,n}) = (m+n)M_1(G) + 4mnq + mn^2p + np(m+p-1)^2 - 4nq(m+p-1)$.

Proof. In $G_{m,n}$,

$$deg(u) = \begin{cases} deg_G(u) + n, & \text{when } u \text{ is in a copy of } G \\ m + p - 1 - deg_G(u), & \text{when } u \text{ is in a copy of } \overline{G}. \end{cases}$$

Therefore,

$$\begin{aligned} M_1(G_{m,n}) &= \sum_{u \in V(G_{m,n})} [deg(u)]^2 = \sum_{i=1}^{m} \left[\sum_{u \in i^{th}\ copy\ of\ G} (deg(u))^2 \right] + \sum_{i=1}^{n} \left[\sum_{u \in i^{th}\ copy\ of\ \overline{G}} (deg(u))^2 \right] \\ &= m \sum_{u \in V(G)} [deg_G(u) + n]^2 + n \sum_{u \in V(G)} [m + p - 1 - deg_G(u)]^2 \\ &= m \left[\sum_{u \in V(G)} (deg_G(u))^2 + \sum_{u \in V(G)} (2n deg_G(u)) + \sum_{u \in V(G)} n^2 \right] \\ &\quad + n \left[\sum_{u \in V(G)} (m + p - 1)^2 - \sum_{u \in V(G)} [2(m + p - 1) deg_G(u)] + \sum_{u \in V(G)} [deg_G(u)]^2 \right] \\ &= mM_1(G) + 4mnq + mn^2p + np(m + p - 1)^2 - 4nq(m + p - 1) + nM_1(G) \\ &= (m + n)M_1(G) + 4mnq + mn^2p + np(m + p - 1)^2 - 4nq(m + p - 1). \end{aligned}$$

□

Proposition 2.3. *For any $m \geq 1$, $M_1(G^c_{m,m}) = 2mM_1(G) + mp^3 + 2mp^2 + 5mp + 4mq - 4mpq$.*

Proof. In $G^c_{m,m}$,

$$deg(u) = \begin{cases} deg_G(u) + 2, & \text{when } u \text{ is in a copy of } G \\ p + 1 - deg_G(u), & \text{when } u \text{ is in a copy of } \overline{G}. \end{cases}$$

Therefore,

$$\begin{aligned} M_1(G^c_{m,m}) &= \sum_{u \in V(G^c_{2m})} [deg(u)]^2 \\ &= \sum_{i=1}^{m} \sum_{u \in i^{th}\ copy\ of\ G} [deg(u)]^2 + \sum_{i=1}^{m} \sum_{u \in i^{th}\ copy\ of\ \overline{G}} [deg(u)]^2 \\ &= m \sum_{u \in i^{th}\ copy\ of\ G} [deg(u)]^2 + m \sum_{u \in i^{th}\ copy\ of\ \overline{G}} [deg(u)]^2 \\ &= m \sum_{u \in V(G)} [deg_G(u) + 2]^2 + m \sum_{u \in V(G)} [p + 1 - deg_G(u)]^2 \\ &= m \left[\sum_{u \in V(G)} [deg_G(u)]^2 + \sum_{u \in V(G)} [4 deg_G(u)] + \sum_{u \in V(G)} 4 \right] \\ &\quad + m \left[\sum_{u \in V(G)} (p + 1)^2 - \sum_{u \in V(G)} [2(p + 1) deg_G(u)] + \sum_{u \in V(G)} [deg_G(u)]^2 \right] \\ &= mM_1(G) + 8mq + 4mp + mp(p + 1)^2 + 4mq(p + 1) + mM_1(G) \\ &= 2mM_1(G) + mp^3 + 2mp^2 + 5mp + 4mq - 4mpq. \end{aligned}$$

□

Proposition 2.4. *For any integer $m \geq 1$, $M_1(G^P_{m,m}) = 2mM_1(G) + (m-1)[p^3 + 2p^2 - 4pq + 5p - 4q] + p^3 - 4pq + p + 4q$.*

Proof. For any vertex u in $G^P_{m,m}$,

$$deg(u) = \begin{cases} deg_G(u) + 1, & \text{if } u \text{ is in the first copy of } G \\ deg_G(u) + 2, & \text{if } u \text{ is in the remaining copies of } G \\ p - deg_G(u), & \text{if } u \text{ is in the } m^{th} \text{ copy of } \overline{G} \\ p + 1 - deg_G(u), & \text{if } u \text{ is in the remaining copies of } \overline{G}. \end{cases}$$

Therefore, $M_1(G^P_{m,m})$

$$\begin{aligned}
&= \sum_{u \in V(G^P_{m,m})} [deg(u)]^2 \\
&= \sum_{u \in 1^{st} \text{ copy of } G} [deg(u)]^2 + \sum_{i=2}^{m} \left[\sum_{u \in i^{th} \text{ copy of } G} [deg(u)]^2 \right] + \sum_{i=1}^{m-1} \left[\sum_{u \in i^{th} \text{ copy of } \overline{G}} [deg(u)]^2 \right] + \sum_{u \in m^{th} \text{ copy of } \overline{G}} [deg(u)]^2 \\
&= \sum_{u \in V(G)} [deg_G(u) + 1]^2 + (m-1) \sum_{u \in V(G)} [deg_G(u) + 2]^2 + (m-1) \sum_{u \in V(\overline{G})} [deg_{\overline{G}}(u) + 2]^2 + \sum_{u \in V(\overline{G})} [deg_{\overline{G}}(u) + 1]^2 \\
&= \sum_{u \in V(G)} [deg_G(u) + 1]^2 + (m-1) \sum_{u \in V(G)} [deg_G(u) + 2]^2 + (m-1) \sum_{u \in V(G)} [(p+1) - deg_G(u)]^2 + \sum_{u \in V(G)} [p - deg_G(u)]^2 \\
&= \sum_{u \in V(G)} [deg_G(u)]^2 + \sum_{u \in V(G)} 1 + 2 \sum_{u \in V(G)} deg_G(u) + (m-1) \left[\sum_{u \in V(G)} [deg_G(u)]^2 + \sum_{u \in V(G)} 4 + 4 \sum_{u \in V(G)} deg_G(u) \right] \\
&\quad + (m-1) \left[\sum_{u \in V(G)} (p+1)^2 + \sum_{u \in V(G)} [deg_G(u)]^2 - \sum_{u \in V(G)} [2(p+1) deg_G(u)] \right] \\
&\quad + \sum_{u \in V(G)} p^2 + \sum_{u \in V(G)} [deg_G(u)]^2 - \sum_{u \in V(G)} [2p\ deg_G(u)] \\
&= M_1(G) + p + 4q + (m-1)[M_1(G) + 4p + 8q] + (m-1)[p(p+1)^2 + M_1(G) - 4(p+1)q] + p^3 + M_1(G) - 4pq \\
&= 2mM_1(G) + (m-1)[p^3 + 2p^2 - 4pq + 5p - 4q] + p^3 - 4pq + p + 4q.
\end{aligned}$$

□

Theorem 2.5. *For any $m, n \geq 1$,*

$$\begin{aligned}
M_2(G_{m+n}) &= mM_2(G) + nM_2(\overline{G}) + (m+n-1)[mM_1(G) + nM_1(\overline{G})] \\
&\quad + (m+n-1)^2 \left[mq + n \left(\frac{p(p-1)}{2} - q \right) \right] \\
&\quad + \frac{m(m-1)}{2} [M_1(G) + (m+n-1)^2 p + 4(m+n-1)q] \\
&\quad + \frac{n(n-1)}{2} \left[M_(\overline{G}) + (m+n-1)^2 p + 4(m+n-1) \left(\frac{p(p-1)}{2} - q \right) \right] \\
&\quad + mn[2(m+n+p-2)q + (m+n-1)(m+n+p-2)p - M_1(G) - 2(m+n-1)q].
\end{aligned}$$

Proof. In $G_{m+n}, deg(u) = \begin{cases} deg_G(u) + m + n - 1, & \text{where } u \text{ is in a copy of } G \\ m + n + p - deg_G(u) - 2, & \text{where } u \text{ is in a copy of } \overline{G}. \end{cases}$ Therefore,

$$\begin{aligned}
M_2(G_{m+n}) &= \sum_{uv\in E(G_{m+n})} [deg(u)\ deg(v)] \\
&= \sum_{uv\in \text{an edge of a copy of } G} [deg(u)deg(v)] + \sum_{uv\in \text{an edge of a copy of } \overline{G}} [deg(u)deg(v)] \\
&\quad + \sum_{uv\in K_{m+n} \text{ with } u,v\in V(G)} [deg(u)deg(v)] + \sum_{uv\in K_{m+n} \text{ with } u,v\in V(\overline{G})} [deg(u)deg(v)] \\
&\quad \sum_{uv\in K_{m+n} \text{ with } u\in V(G) \text{ and } v\in V(\overline{G})} [deg(u)deg(v)] \\
&= m \sum_{uv\in E(G)} [(deg_G(u) + m + n - 1)(deg_G(v) + m + n - 1)] \\
&\quad + n \sum_{uv\in E(\overline{G})} [(deg_{\overline{G}}(u) + m + n - 1)(deg_{\overline{G}}(v) + m + n - 1)] \\
&\quad + \binom{m}{2} \sum_{u\in V(G)} [deg_G(u) + m + n - 1)]^2 + \binom{n}{2} \sum_{u\in V(\overline{G})} [deg_{\overline{G}}(u) + m + n - 1]^2 \\
&\quad + mn \sum_{u\in V(G)} [(deg_G(u) + m + n - 1)(m + n + p - 2 - deg_G(u))] \\
&= m \left[\sum_{uv\in E(G)} [deg_G(u)deg_G(v)] + \sum_{uv\in E(G)} [(m+n-1)[deg_G(u) + deg_G(v)]] + \sum_{uv\in E(G)} (m+n-1)^2 \right] \\
&\quad + n \left[\sum_{uv\in E(\overline{G})} [deg_{\overline{G}}(u)deg_{\overline{G}}(v)] + \sum_{uv\in E(\overline{G})} [(m+n-1)[deg_{\overline{G}}(u) + deg_{\overline{G}}(v)] + \sum_{uv\in E(\overline{G})} (m+n-1)^2 \right] \\
&\quad + \frac{m(m-1)}{2} \left[\sum_{u\in V(G)} [deg_G(u)]^2 + \sum_{u\in V(G)} (m+n-1)^2 + \sum_{u\in V(G)} [2(m+n-1)deg_G(u)] \right] \\
&\quad + \frac{n(n-1)}{2} \left[\sum_{u\in V(\overline{G})} [deg_{\overline{G}}(u)]^2 + \sum_{u\in V(\overline{G}} (m+n-1)^2 + \sum_{u\in V(\overline{G})} [2(m+n-1)deg_{\overline{G}}(u)] \right] \\
&\quad + mn \left[\sum_{u\in V(G)} [(m+n+p-2)deg_G(u)] + \sum_{u\in V(G)} [(m+n-1)(m+n+p-2)] \right. \\
&\quad \left. - \sum_{u\in V(G)} [deg_G(u)]^2 - \sum_{u\in V(G)} [(m+n-1)deg_G(u)] \right] \\
&= m[M_2(G) + (m+n-1)M_1(G) + (m+n-1)^2 q] \\
&\quad + n \left[M_2(\overline{G}) + (m+n-1)M_1(\overline{G}) + (m+n-1)^2 \left(\binom{p}{2} - q \right) \right] \\
&\quad + \frac{m(m-1)}{2} [M_1(G) + (m+n-1)^2 p + (m+n-1)(4q)] \\
&\quad + \frac{n(n-1)}{2} \left[M_1(\overline{G}) + (m+n-1)^2 p + 4(m+n-1) \left(\binom{p}{2} - q \right) \right] \\
&\quad + mn[2(m+n+p-2)q + (m+n-1)(m+n+p-2)p - M_1(G) - 2(m+n-1)q]
\end{aligned}$$

$$= mM_2(G) + nM_2(\overline{G}) + (m+n-1)[mM_1(G) + nM_1(\overline{G})] + (m+n-1)^2\left[mq + n\left(\frac{p(p-1)}{2} - q\right)\right] + \frac{m(m-1)}{2}$$
$$[M_1(G) + (m+n-1)^2 p + 4(m+n-1)q] + \frac{n(n-1)}{2}\left[M_(\overline{G}) + (m+n-1)^2 p + 4(m+n-1)\left(\frac{p(p-1)}{2} - q\right)\right]$$
$$+ mn[2(m+n+p-2)q + (m+n-1)(m+n+p-2)p - M_1(G) - 2(m+n-1)q].$$

□

Theorem 2.6. *For any integer $m, n \geq 1$,*

$$M_2(G_{m,n}) = m[M_2(G) + nM_1(G) + n^2 q]$$
$$+ n\left[M_2(\overline{G}) + mM_1(\overline{G}) + m^2\left(\frac{p(p-1)}{2} - q\right)\right] + mn[2(m+p-1)q + (m+p-1)np - M_1(G) - 2nq]$$

Proof. In $G_{m,n}, deg(u) = \begin{cases} deg_G(u) + n, & \text{where } u \text{ is in a copy of } G \\ m + p - 1 - deg_G(u), & \text{where } u \text{ is in a copy of } \overline{G}. \end{cases}$ Therefore,

$$M_2(G_{m,n}) = \sum_{uv \in E(G_{m,n})} [deg(u)deg(v)]$$
$$= \sum_{uv \in \text{an edge of a copy of } G} [deg(u)deg(v)] + \sum_{uv \in \text{an edge of a copy of } \overline{G}} [deg(u)deg(v)]$$
$$+ \sum_{uv \in K_{m+n} \text{ with } u \in V(G), v \in V(\overline{G})} [deg(u)deg(v)]$$
$$= m \sum_{uv \in E(G)} [(deg_G(u) + n)(deg_G(v) + n)]$$
$$+ n \sum_{uv \in E(\overline{G})} [(deg_{\overline{G}}(u) + m)(deg_{\overline{G}}(v) + m)] + mn \sum_{uv \in V(G)} [(deg_G(u) + n)(m + p - 1 - deg_G(u))]$$
$$= m\left[\sum_{uv \in E(G)} [deg_G(u)deg_G(v)] + \sum_{uv \in E(G)} [n(deg_G(u) + deg_G(v))] + \sum_{uv \in E(G)} n^2\right]$$
$$+ n\left[\sum_{uv \in E(\overline{G})} (deg_{\overline{G}}(u)deg_{\overline{G}}(v)) + \sum_{uv \in E(\overline{G})} [m(deg_{\overline{G}}(u) + deg_{\overline{G}}(v))] + \sum_{uv \in E(\overline{G})} m^2\right]$$
$$+ mn\left[\sum_{u \in V(G)} [(m+p-1)deg_G(u)] + \sum_{u \in V(G)} [n(m+p-1)]\right.$$
$$\left. - \sum_{u \in V(G)} [deg_G(u)]^2 - \sum_{u \in V(G)} [ndeg_G(u)]\right]$$
$$= m[M_2(G) + nM_1(G) + n^2 q]$$
$$+ n\left[M_2(\overline{G}) + mM_1(\overline{G}) + m^2\left(\binom{p}{2} - q\right)\right] + mn[(m+p-1)(2q) + n(m+p-1)p - M_1(G) - n(2q)]$$
$$= m[M_2(G) + nM_1(G) + n^2 q]$$
$$+ n\left[M_2(\overline{G}) + mM_1(\overline{G}) + m^2\left(\frac{p(p-1)}{2} - q\right)\right] + mn[2(m+p-1)q + n(m+p-1)p - M_1(G) - 2nq].$$

□

Theorem 2.7. *For any integer* $m \geq 1$, $M_2(G^c_{m,m}) = m[M_2(G) + M_2(\overline{G}) + 2M_1(\overline{G}) + 4\binom{p}{2}] + 4m[(p-1)q + p(p+1)]$.

Proof. In $G^c_{m,m}, deg(u) = \begin{cases} deg_G(u) + 2, & \text{where } u \text{ is in a copy of } G \\ p + 1 - deg_G(u), & \text{where } u \text{ is in a copy of } \overline{G} \end{cases}$

Therefore,

$$\begin{aligned}
M_2(G^c_{m,m}) &= \sum_{uv \in E(G^c_{m,m})} [deg(u)deg(v)] \\
&= \sum_{uv \in \text{an edge of a copy of } G} [deg(u)deg(v)] + \sum_{uv \in \text{an edge of a copy of } \overline{G}} [deg(u)deg(v)] \\
&\quad + \sum_{uv \in \text{an edge of a copy of } C_{2m}} [deg(u)deg(v)] \\
&= \sum_{uv \in \text{an edge of a copy of } G} [(deg_G(u) + 2)(deg_G(v) + 2)] \\
&\quad + \sum_{uv \in \text{an edge of a copy of } \overline{G}} [(deg_{\overline{G}}(u) + 2)(deg_{\overline{G}}(v) + 2)] + 2m \sum_{u \in V(G)} [deg(u)deg(\overline{u})] \\
&= \sum_{uv \in \text{an edge of a copy of } G} [(deg_G(u) + 2)(deg_G(v) + 2)] \\
&\quad + \sum_{uv \in \text{an edge of a copy of } \overline{G}} [(deg_{\overline{G}}(u) + 2)(deg_{\overline{G}}(v) + 2)] + 2m \sum_{u \in V(G)} [(deg_G(u) + 2)(p + 1 - deg_G(u))] \\
&= m\left[\sum_{uv \in E(G)} [deg_G(u)deg_G(v)] + 2\sum_{uv \in E(G)} [deg_G(u) + deg_G(v)] + \sum_{uv \in E(G)} 4\right] \\
&\quad + m\left[\sum_{uv \in E(\overline{G})} [deg_{\overline{G}}(u)deg_{\overline{G}}(v)] + 2\sum_{uv \in E(\overline{G})} [deg_{\overline{G}}(u) + deg_{\overline{G}}(v)] + \sum_{uv \in E(\overline{G})} 4\right] \\
&\quad + 2m\left[\sum_{u \in V(G)} [(p+1)deg(u)] + 2\sum_{u \in V(G)} (p+1) - \sum_{u \in V(G)} [deg(u)]^2 - 2\sum_{u \in V(G)} deg(u)\right] \\
&= m[M_2(G) + 2M_1(G) + 4q] + m[M_2(\overline{G}) + 2M_1(\overline{G}) + 4(\binom{p}{2} - q)] + 2m[(p+1)(2q) + 2(p+1)p - M_1(G) - 4q] \\
&= m[M_2(G) + M_2(\overline{G}) + 2M_1(\overline{G}) + 4\binom{p}{2}] + 2m[2(p-1)q + 2p(p+1)].
\end{aligned}$$

□

Proposition 2.8. *For any integer* $m \geq 1$, $M_2(G^P_{m,m}) = m[M_2(G) + M_2(\overline{G})] + (2m-1)M_1(\overline{G}) + (4m-3)\binom{p}{2} + 4q(m-1)(p+1) + (4m-5)p(p+1) + 2pq + 2p^2 - 8mq + 6q$.

Proof. In $(G^p_{m,m})$,

$$deg(u) = \begin{cases} deg_G(u) + 1, & \text{where } u \text{ is in the first copy of } G \\ deg_G(u) + 2, & \text{where } u \text{ is in the remaining copies of } G \\ deg_{\overline{G}}(u) + 1, & \text{where } u \text{ is in the } m^{th} \text{ copy of } \overline{G} \\ deg_{\overline{G}}(u) + 2, & \text{where } u \text{ is in the remaining copies of } \overline{G}. \end{cases}$$

Therefore,

$$
\begin{aligned}
M_2(G^P_{m,m}) &= \sum_{uv\in E(G^p_{m,m})}[deg(u)deg(v)] \\
&= \sum_{uv\in \text{an edge of a copy of } G}[deg(u)deg(v)] + \sum_{uv\in \text{an edge of a copy of } \overline{G}}[deg(u)deg(v)] \\
&\quad + \sum_{uv\in \text{an edge of a copy of } P_{m,m}}[deg(u)deg(v)] \\
&= \sum_{uv\in E(G)}[(deg_G(u)+1)(deg_G(v)+1)] \\
&\quad + (m-1)\sum_{uv\in E(G)}[(deg_G(u)+2)(deg_G(v)+2)] + \sum_{uv\in E(\overline{G})}[(deg_{\overline{G}}(u)+1)(deg_{\overline{G}}(v)+1)] \\
&\quad + (m-1)\sum_{uv\in E(\overline{G})}[(deg_{\overline{G}}(u)+2)(deg_{\overline{G}}(v)+2)] + \sum_{u\in V(G)}[(deg_G(u)+1)(p+1-deg_G(u))] \\
&\quad + (2m-3)\sum_{uv\in V(G)}[(deg_G(u)+2)(p+1-deg_G(u))] + \sum_{u\in V(G)}[(deg_G(u)+2)(p-deg_G(u))] \\
&= \sum_{uv\in E(G)}[deg_G(u)deg_G(v)] + \sum_{uv\in E(G)}(deg_G(u)+deg_G(v)) + \sum_{uv\in E(G)}1 \\
&\quad + (m-1)\left[\sum_{uv\in E(G)}[deg_G(u)deg_G(v)] + 2\sum_{uv\in E(G)}[deg_G(u)+deg_G(v)] + \sum_{uv\in E(G)}4\right] \\
&\quad + \sum_{uv\in E(\overline{G})}[deg_{\overline{G}}(u)deg_{\overline{G}}(v)] + \sum_{uv\in E(\overline{G})}[deg_{\overline{G}}(u)+deg_{\overline{G}}(v)] + \sum_{uv\in E(\overline{G})}1 \\
&\quad + (m-1)\left[\sum_{uv\in E(\overline{G})}[deg_{\overline{G}}(u)deg_{\overline{G}}(v)] + 2\sum_{uv\in E(\overline{G})}[deg_{\overline{G}}(u)+deg_{\overline{G}}(v)] + \sum_{uv\in E(\overline{G})}4\right] \\
&\quad + \sum_{u\in V(G)}[(p+1)deg_G(u)] + \sum_{u\in V(G)}(p+1) - \sum_{u\in V(G)}[deg_G(u)]^2 - \sum_{u\in V(G)}[deg_G(u)] \\
&\quad + (2m-3)\left[\sum_{u\in V(G)}[(p+1)deg_G(u)] + \sum_{u\in V(G)}[2(p+1)] - \sum_{u\in V(G)}[deg_G(u)]^2 - 2\sum_{u\in V(G)}deg_G(u)\right] \\
&\quad + \sum_{u\in V(G)}[p\ deg_G(u)] + \sum_{u\in V(G)}[2p] - \sum_{u\in V(G)}[deg_G(u)]^2 - 2\sum_{u\in V(G)}deg_G(u) \\
&= M_2(G) + M_1(G) + q + (m-1)[M_2(G)+2M_1(G)+4q] + M_2(\overline{G}) + M_1(\overline{G}) + (\binom{p}{2} - q) \\
&\quad + (m-1)[M_2(\overline{G}) + 2M_1(G) + 4(\binom{p}{2} - q)] + 2(p+1)q + p(p+1) - M_1(G) - 2q \\
&\quad + (2m-3)[2(p+1)q + 2p(p+1) - M_1(G) - 4q] + 2pq + 2p^2 - M_1(G) - 4q \\
&= m[M_2(G) + M_2(\overline{G})] + (2m-1)M_1(\overline{G}) + (4m-3)\binom{p}{2} \\
&\quad + 4q(m-1)(p+1) + (4m-5)p(p+1) + 2pq + 2p^2 - 8mq + 6q.
\end{aligned}
$$

□

Corollary 2.9. *For any graph* $G, M_1(G\overline{G}) = 2M_1(G) + p^3 - 4pq + p + 4q$ *and* $M_2(G\overline{G}) = M_1(\overline{G}) + M_2(G) + M_2(\overline{G}) + (p-1)\left[\frac{3p}{2} + 2q\right]$.

Proof. By taking $m = 1$ in Propositions 2.4 and 2.8, the result follows. □

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