International Journal of Mathematics And its Applications
Volume 3, Issue 2 (2015), 65–73.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

On $\tilde{g}(1,2)^{\star}$-closed Sets

Research Article

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Abstract: In this present paper, we introduce a new class of sets namely $\tilde{g}(1,2)^{\star}$-closed sets in bitopological spaces. The notion of $\tilde{g}(1,2)^{\star}$-interior is defined and some of its basic properties are studied. Also we introduce the concept of $\tilde{g}(1,2)^{\star}$-closure in bitopological spaces using the notion of $\tilde{g}(1,2)^{\star}$-closed sets.

MSC: 54E55.

Keywords: $\tilde{g}(1,2)^{\star}$-closed set, $(1,2)^{\star}$-$\hat{g}$-closed set, $(1,2)^{\star}$-sg-closed set, $(1,2)^{\star}$-gsp-closed set, $(1,2)^{\star}$-$\ddot{g}$-closed set, $(1,2)^{\star}$-ψ-closed set.
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1. Introduction

Levine [4] introduced generalized closed sets in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. Veerakumar [16] introduced $\hat{g}$-closed sets in topological spaces. Sheik John [15] introduced ω-closed sets in topological spaces. After advent of these notions, many topologists introduced various types of generalized closed sets and studied their fundamental properties. Quite Recently, Ravi and Ganesan [6] introduced and studied $\ddot{g}$-closed sets in topological spaces as another generalization of closed sets and proved that the class of $\ddot{g}$-closed sets properly lies between the class of closed sets and the class of ω-closed sets. Ravi et al [12, 14], Ravi and Thivagar [8] and Duszynski et al [1] introduced $(1,2)^{\star}$-αg-closed sets, $(1,2)^{\star}$-g-closed sets, $(1,2)^{\star}$-sg-closed sets and $(1,2)^{\star}$-$\hat{g}$-closed sets respectively. Ravi et al [7] introduced $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed sets in bitopological spaces. In this paper, we introduce a new class of sets namely $\tilde{g}(1,2)^{\star}$-closed sets in bitopological spaces. This class lies between the class of $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed sets and the class of $(1,2)^{\star}$-αg-closed sets. The notion of $\tilde{g}(1,2)^{\star}$-interior is defined and some of its basic properties are studied. Also we introduce the concept of $\tilde{g}(1,2)^{\star}$-closure in bitopological spaces using the notion of $\tilde{g}(1,2)^{\star}$-closed sets, and we obtain some related results.

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2. Preliminaries

Throughout this paper, (X, τ_1, τ_2) (briefly, X) will denote bitopological space.

Definition 2.1. *Let S be a subset of X. Then S is said to be $\tau_{1,2}$-open [9] if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$. The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed. Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.*

Definition 2.2 ([9]). *Let S be a subset of a bitopological space X. Then*

(1) the $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined as $\cap \{F : S \subseteq F$ and F is $\tau_{1,2}$-closed$\}$.

(2) the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as $\cup \{F : F \subseteq S$ and F is $\tau_{1,2}$-open$\}$.

Definition 2.3. *A subset A of a bitopological space X is called*

(1) $(1,2)^{\star}$-semi-open set [8] if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(A));

(2) $(1,2)^{\star}$-α-open set [3] if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A)));

(3) $(1,2)^{\star}$-β-open set [10] if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A))).

The complements of the above mentioned open sets are called their respective closed sets. The $(1,2)^{\star}$-semi-closure [5] (resp. $(1,2)^{\star}$-α-closure [5], $(1,2)^{\star}$-β-closure [10]) of a subset A of X, denoted by $(1,2)^{\star}$-scl(A) (resp. $(1,2)^{\star}$-αcl(A), $(1,2)^{\star}$-βcl(A)), is defined to be the intersection of all $(1,2)^{\star}$-semi-closed (resp. $(1,2)^{\star}$-α-closed, $(1,2)^{\star}$-β-closed) sets of (X, τ_1, τ_2) containing A. It is known that $(1,2)^{\star}$-scl(A) (resp. $(1,2)^{\star}$-αcl(A), $(1,2)^{\star}$-βcl(A)) is a $(1,2)^{\star}$-semiclosed (resp. $(1,2)^{\star}$-α-closed, $(1,2)^{\star}$-β-closed) set.

Definition 2.4. *A subset A of a bitopological space (X, τ_1, τ_2) is called*

(1) $(1,2)^{\star}$-g-closed set [14] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^{\star}$-g-closed set is called $(1,2)^{\star}$-g-open set;

(2) $(1,2)^{\star}$-sg-closed set [8] if $(1,2)^{\star}$-scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-semi-open in X. The complement of $(1,2)^{\star}$-sg-closed set is called $(1,2)^{\star}$-sg-open set;

(3) $(1,2)^{\star}$-gs-closed set [8] if $(1,2)^{\star}$-scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^{\star}$-gs-closed set is called $(1,2)^{\star}$-gs-open set;

(4) $(1,2)^{\star}$-αg-closed set [12] if $(1,2)^{\star}$-αcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^{\star}$-αg-closed set is called $(1,2)^{\star}$- αg-open set;

(5) $(1,2)^{\star}$-$\hat{g}$-closed set [1] or $(1,2)^{\star}$-ω-closed set [2] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-semi-open in X. The complement of $(1,2)^{\star}$-$\hat{g}$-closed (or $(1,2)^{\star}$-ω-closed) set is called $(1,2)^{\star}$-$\hat{g}$-open (or $(1,2)^{\star}$-ω-open) set;

(6) $(1,2)^{\star}$-ψ-closed set [7] if $(1,2)^{\star}$-scl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-sg-open in X. The complement of $(1,2)^{\star}$-ψ-closed set is called $(1,2)^{\star}$-ψ-open set;

(7) $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed set [7] if $(1,2)^{\star}$-αcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-sg-open in X. The complement of $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed set is called $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-open set;

(8) $(1,2)^\star$*-gsp-closed set [10] if* $(1,2)^\star$*-$\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of* $(1,2)^\star$*-gsp-closed set is called* $(1,2)^\star$*-gsp-open set.*

Remark 2.5. *The collection of all $(1,2)^\star$-$\ddot{g}_\alpha$-closed (resp. $(1,2)^\star$-$\hat{g}$-closed, $(1,2)^\star$-g-closed, $(1,2)^\star$-gs-closed, $(1,2)^\star$- αg-closed, $(1,2)^\star$-sg-closed, $(1,2)^\star$-ψ-closed, $(1,2)^\star$-α-closed, $(1,2)^\star$-semi-closed, $(1,2)^\star$-gsp-closed) sets of X is denoted by $(1,2)^\star$-$\ddot{G}_\alpha C(X)$ (resp. $(1,2)^\star$-$\hat{G}C(X)$, $(1,2)^\star$-$GC(X)$, $(1,2)^\star$-$GSC(X)$, $(1,2)^\star$-$\alpha GC(X)$, $(1,2)^\star$- $SGC(X)$, $(1,2)^\star$-$\psi C(X)$, $(1,2)^\star$-$\alpha C(X)$, $(1,2)^\star$-$SC(X)$, $(1,2)^\star$-$GSPC(X)$).*

We denote the power set of X by P(X).

Remark 2.6.

(1) Every $\tau_{1,2}$-closed set is $(1,2)^\star$-semi-closed but not conversely [8].

(2) Every $\tau_{1,2}$-closed set is $(1,2)^\star$-α-closed but not conversely [5].

(3) Every $(1,2)^\star$-semi-closed set is $(1,2)^\star$-ψ-closed but not conversely [7].

(4) Every $(1,2)^\star$-semi-closed set is $(1,2)^\star$-sg-closed but not conversely [8].

(5) Every $(1,2)^\star$-$\hat{g}$-closed set is $(1,2)^\star$-g-closed but not conversely [1].

(6) Every $(1,2)^\star$-sg-closed set is $(1,2)^\star$-gs-closed but not conversely [11].

(7) Every $(1,2)^\star$-g-closed set is $(1,2)^\star$- αg-closed but not conversely [12].

(8) Every $(1,2)^\star$-g-closed set is $(1,2)^\star$-sg-closed but not conversely [11].

(9) Every $\tau_{1,2}$-closed set is $(1,2)^\star$-$\hat{g}$-closed but not conversely [1].

(10) Every $(1,2)^\star$-$\hat{g}$-closed set is $(1,2)^\star$-sg-closed but not conversely [1].

3. $\tilde{g}(1,2)^\star$-closed Sets

We introduce the following definition.

Definition 3.1. *A subset A of a bitopological space X is called*

(1) $(1,2)^\star$-$\ddot{g}$-closed set [7] if $\tau_{1,2}$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-sg-open in X. The complement of $(1,2)^\star$-$\ddot{g}$-closed set is called $(1,2)^\star$-$\ddot{g}$-open set.

(2) $\tilde{g}(1,2)^\star$-closed if $(1,2)^\star$-$\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-$\hat{g}$-open in X. The complement of $\tilde{g}(1,2)^\star$-closed set is called $\tilde{g}(1,2)^\star$-open set. The collection of all $(1,2)^\star$-$\ddot{g}$-closed (resp. $\tilde{g}(1,2)^\star$-closed) sets in X is denoted by $(1,2)^\star$-$\ddot{G}C(X)$ (resp. $(1,2)^\star$-$\tilde{G}C(X)$).

Proposition 3.2 ([7])**.** *Every $\tau_{1,2}$-closed set is $(1,2)^\star$-$\ddot{g}$-closed but not conversely.*

Proposition 3.3 ([7])**.** *Every $(1,2)^\star$-$\ddot{g}$-closed set is $(1,2)^\star$-$\ddot{g}_\alpha$-closed but not conversely.*

Proposition 3.4 ([7]). *Every $(1,2)^{\star}$-$\ddot{g}$-closed set is $(1,2)^{\star}$-ψ-closed but not conversely.*

Proposition 3.5. *Every $(1,2)^{\star}$-$\ddot{g}$-closed set is $(1,2)^{\star}$-$\hat{g}$-closed.*

Proof. Suppose that A $\subseteq$ G and G is $(1,2)^{\star}$-semi-open in X. Since every $(1,2)^{\star}$-semi-open set is $(1,2)^{\star}$-sg-open and A is $(1,2)^{\star}$-$\ddot{g}$-closed, therefore $\tau_{1,2}$-cl(A) $\subseteq$ G. Hence A is $(1,2)^{\star}$-$\hat{g}$-closed in X. □

The converse of Proposition 3.5 need not be true as seen from the following example.

Example 3.6. *Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{d\}, \{b, c, d\}\}$ and $\tau_2 = \{\emptyset, X, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{d\}, \{b, c\}, \{b, c, d\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{a\}, \{a, d\}, \{a, b, c\}\}$ are called $\tau_{1,2}$-closed. Clearly, the set $\{a, c, d\}$ is a $(1,2)^{\star}$-$\hat{g}$-closed but not a $(1,2)^{\star}$-$\ddot{g}$-closed set in X.*

Proposition 3.7. *Every $(1,2)^{\star}$-α-closed set is $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed.*

Proof. If A is an $(1,2)^{\star}$-α-closed subset of X and G is any $(1,2)^{\star}$-sg-open set containing A, we have $(1,2)^{\star}$-αcl(A) = A $\subseteq$ G. Hence A is $(1,2)^{\star}$-$\ddot{g}_{\alpha}$- closed in X. □

The converse of Proposition 3.7 need not be true as seen from the following example.

Example 3.8. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then the sets in $\{\emptyset, X, \{a, b\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{c\}\}$ are called $\tau_{1,2}$-closed. Then $(1,2)^{\star}$-$\alpha C(X) = \{\emptyset, \{c\}, X\}$ and $(1,2)^{\star}$-$\ddot{G}_{\alpha}C(X) = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}$. Clearly, the set $\{a, c\}$ is an $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed but not an $(1,2)^{\star}$-α-closed set in X.*

Remark 3.9. *$(1,2)^{\star}$-$\hat{g}$-closed set is different from $\tilde{g}(1,2)^{\star}$-closed.*

Example 3.10.

(1) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, \{a, b\}, X\}$. Then $\{b\}$ is $\tilde{g}(1,2)^{\star}$-closed set but not $(1,2)^{\star}$-$\hat{g}$-closed.

(2) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, \{b, c\}, X\}$. Then $\{b\}$ is $(1,2)^{\star}$-$\hat{g}$-closed set but not $\tilde{g}(1,2)^{\star}$-closed.

Proposition 3.11. *Every $(1,2)^{\star}$-$\ddot{g}$-closed set is $(1,2)^{\star}$-g-closed.*

Proof. If A is a $(1,2)^{\star}$-$\ddot{g}$-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is $(1,2)^{\star}$-sg-open, we have G $\supseteq$ $\tau_{1,2}$-cl(A). Hence A is $(1,2)^{\star}$-g-closed in X. □

The converse of Proposition 3.11 need not be true as seen from the following example.

Example 3.12. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b, c\}\}$ are called both $\tau_{1,2}$-open and $\tau_{1,2}$-closed. Then $(1,2)^{\star}$-$\ddot{G}C(X) = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $(1,2)^{\star}$-$GC(X) = P(X)$. Clearly, the set $\{a, b\}$ is a $(1,2)^{\star}$-g-closed but not a $(1,2)^{\star}$-$\ddot{g}$-closed set in X.*

Proposition 3.13. *Every $\tilde{g}(1,2)^{\star}$-closed set is $(1,2)^{\star}$-αg-closed.*

Proof. If A is a $\tilde{g}(1,2)^{\star}$-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is $(1,2)^{\star}$-$\hat{g}$-open, we have $(1,2)^{\star}$-αcl(A) $\subseteq$U. Hence A is $(1,2)^{\star}$-αg-closed in X. □

The converse of Proposition 3.13 need not be true as seen from the following example.

Example 3.14. *In Example 3.12, $\{a,\ c\}$ is $(1,2)^{\star}$-αg-closed set but not $\tilde{g}(1,2)^{\star}$-closed.*

Proposition 3.15. *Every $(1,2)^{\star}$-$\ddot{g}$-closed set is $(1,2)^{\star}$-αg-closed.*

Proof. If A is a $(1,2)^{\star}$-$\ddot{g}$-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is $(1,2)^{\star}$-sg-open, we have G $\supseteq \tau_{1,2}$-cl(A) $\supseteq (1,2)^{\star}$-αcl(A). Hence A is $(1,2)^{\star}$-αg-closed in X. □

The converse of Proposition 3.15 need not be true as seen from the following example.

Example 3.16. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\ \emptyset,\ X,\ \{a,\ b\}\}$ and $\tau_2 = \{\emptyset,\ X,\ \{c\}\}$. Then the sets in $\{\emptyset,\ X,\ \{c\},\ \{a,\ b\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset,\ X,\ \{c\},\ \{a,\ b\}\}$ are called $\tau_{1,2}$-closed. Then $(1,2)^{\star}$-$\ddot{G}C(X) = \{\emptyset,\ \{c\},\ \{a,\ b\},\ X\}$ and $(1,2)^{\star}$- $\alpha GC(X) = P(X)$. Clearly, the set $\{a,\ c\}$ is an $(1,2)^{\star}$- αg-closed but not a $(1,2)^{\star}$-$\ddot{g}$-closed set in X.*

Proposition 3.17. *Every $(1,2)^{\star}$-$\ddot{g}$-closed set is $(1,2)^{\star}$-gs-closed.*

Proof. If A is a $(1,2)^{\star}$-$\ddot{g}$-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is $(1,2)^{\star}$-sg-open, we have G $\supseteq \tau_{1,2}$-cl(A) $\supseteq (1,2)^{\star}$-scl(A). Hence A is $(1,2)^{\star}$-gs-closed in X. □

The converse of Proposition 3.17 need not be true as seen from the following example.

Example 3.18. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\emptyset,\ X,\ \{a\}\}$ and $\tau_2 = \{\emptyset,\ X\}$. Then the sets in $\{\emptyset,\ X,\ \{a\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset,\ X,\ \{b,\ c\}\}$ are called $\tau_{1,2}$-closed. We have $(1,2)^{\star}$-$\ddot{G}C(X) = \{\emptyset,\ \{b,\ c\},\ X\}$ and $(1,2)^{\star}$-$GSC(X) = \{\emptyset,\ \{b\},\ \{c\},\ \{a,\ b\},\ \{a,\ c\},\ \{b,\ c\},\ X\}$. Clearly, the set $\{c\}$ is a $(1,2)^{\star}$-gs-closed but not a $(1,2)^{\star}$-$\ddot{g}$-closed set in X.*

Proposition 3.19. *Every $(1,2)^{\star}$-$\ddot{g}$-closed set is $(1,2)^{\star}$-sg-closed.*

Proof. If A is a $(1,2)^{\star}$-$\ddot{g}$-closed subset of X and G is any $(1,2)^{\star}$-semi-open set containing A, since every $(1,2)^{\star}$-semi-open set is $(1,2)^{\star}$-sg-open, we have G $\supseteq \tau_{1,2}$-cl(A) $\supseteq (1,2)^{\star}$-scl(A). Hence A is $(1,2)^{\star}$-sg-closed in X. □

The converse of Proposition 3.19 need not be true as seen from the following example.

Example 3.20. *In Example 3.18, we have $(1,2)^{\star}$-$\ddot{G}C(X) = \{\emptyset,\ \{b,\ c\},\ X\}$ and $(1,2)^{\star}$-$SGC(X) = \{\emptyset,\ \{b\},\ \{c\},\ \{b,\ c\},\ X\}$. Clearly, the set $\{b\}$ is a $(1,2)^{\star}$-sg-closed but not a $(1,2)^{\star}$-$\ddot{g}$-closed set in X.*

Proposition 3.21. *Every $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed set is $\tilde{g}(1,2)^{\star}$-closed.*

Proof. If A is an $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed subset of X and G is any $(1,2)^{\star}$-$\hat{g}$-open set containing A, since every $(1,2)^{\star}$-$\hat{g}$-open set is $(1,2)^{\star}$-sg-open, we have $(1,2)^{\star}$-αcl(A) $\subseteq$G. Hence A is $\tilde{g}(1,2)^{\star}$-closed in X. □

The converse of Proposition 3.21 need not be true as seen from the following example.

Example 3.22. *In Example 3.10(1), $\{a,\ c\}$ is $\tilde{g}(1,2)^{\star}$-closed set but not $(1,2)^{\star}$-$\ddot{g}_{\alpha}$-closed.*

Proposition 3.23. *Every $(1,2)^{\star}$-α-closed set is $\tilde{g}(1,2)^{\star}$-closed.*

Proof. If A is an $(1,2)^{\star}$-α-closed subset of X and G is any $(1,2)^{\star}$-$\hat{g}$-open set containing A, we have $(1,2)^{\star}$-αcl(A) = A $\subseteq$G. Hence A is $\tilde{g}(1,2)^{\star}$-closed in X. □

The converse of Proposition 3.23 need not be true as seen from the following example.

Example 3.24. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\emptyset,\ X,\ \{a,\ b\}\}$ and $\tau_2 = \{\emptyset,\ \{a,\ c\},\ X\}$. Then $\{b,\ c\}$ is $\tilde{g}(1,2)^{\star}$-closed set but not $(1,2)^{\star}$-α-closed.*

Proposition 3.25. *Every $(1,2)^{\star}$-ψ-closed set is $(1,2)^{\star}$-sg-closed.*

Proof. Suppose that A $\subseteq$ G and G is $(1,2)^{\star}$-semi-open in X. Since every $(1,2)^{\star}$-semi-open set is $(1,2)^{\star}$-sg-open and A is $(1,2)^{\star}$-ψ-closed, therefore $(1,2)^{\star}$- scl(A) $\subseteq$G. Hence A is $(1,2)^{\star}$-sg-closed in X. □

The converse of Proposition 3.25 need not be true as seen from the following example.

Example 3.26. *In Example 3.12, we have $(1,2)^{\star}$-$\psi C(X) = \{\emptyset,\ X,\ \{a\},\ \{b,\ c\}\}$ and $(1,2)^{\star}$-$SGC(X) = P(X)$. Clearly, the set $\{a,\ b\}$ is a $(1,2)^{\star}$-sg-closed but not a $(1,2)^{\star}$-ψ-closed set in X.*

Proposition 3.27. *Every $(1,2)^{\star}$-$\ddot{g}$-closed set is $(1,2)^{\star}$-gsp-closed.*

Proof. If A is a $(1,2)^{\star}$-$\ddot{g}$-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is $(1,2)^{\star}$-sg-open, we have G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ $(1,2)^{\star}$-βcl(A). Hence A is $(1,2)^{\star}$-gsp-closed in X. □

The converse of Proposition 3.27 need not be true as seen from the following example.

Example 3.28. *Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\emptyset,\ X,\ \{b\}\}$ and $\tau_2 = \{\emptyset,\ X\}$. We have $(1,2)^{\star}$-$\ddot{G}$ $C(X) = \{\emptyset,\ \{a,\ c\},\ X\}$ and $(1,2)^{\star}$-$GSPC(X) = \{\emptyset,\ \{a\},\ \{c\},\ \{a,\ b\},\ \{a,\ c\},\ \{b,\ c\},\ X\}$. Clearly, the set $\{c\}$ is a $(1,2)^{\star}$-gsp-closed but not a $(1,2)^{\star}$-$\ddot{g}$-closed set in X.*

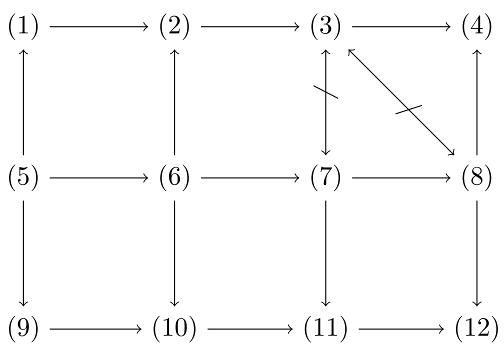
Remark 3.29. *The concepts of $\tilde{g}(1,2)^{\star}$-closed sets and $(1,2)^{\star}$-g-closed sets are independent.*

Example 3.30.

(1) In Example 3.10 (2), $\{b\}$ is $(1,2)^{\star}$-g-closed set but it is not $\tilde{g}(1,2)^{\star}$-closed set.

(2) Let $X = \{a,\ b,\ c\}$, $\tau_1 = \{\emptyset,\ X,\ \{a\}\}$ and $\tau_2 = \{\emptyset,\ X,\ \{a,\ b\}\}$. Then $\{b\}$ is $\tilde{g}(1,2)^{\star}$-closed set but it is not $(1,2)^{\star}$-g-closed set.

Remark 3.31. *From the above Propositions, Examples and Remarks, we obtain the following diagram, where $A \longrightarrow B$ (resp. $A \nleftrightarrow B$) represents A implies B but not conversely (resp. A and B are independent of each other).*



(1) $(1,2)^\star$-*α-closed*	*(2)* $(1,2)^\star$-*$\ddot{g}_\alpha$-closed*	*(3)* $\tilde{g}(1,2)^\star$-*closed*
(4) $(1,2)^\star$-*αg-closed*	*(5)* $\tau_{1,2}$-*closed*	*(6)* $(1,2)^\star$-*$\ddot{g}$-closed*
(7) $(1,2)^\star$-*$\hat{g}$-closed*	*(8)* $(1,2)^\star$-*g-closed*	*(9)* $(1,2)^\star$-*semi-closed*
(10) $(1,2)^\star$-*ψ-closed*	*(11)* $(1,2)^\star$-*sg-closed*	*(12)* $(1,2)^\star$-*gs-closed.*

4. Properties of $\tilde{g}(1,2)^\star$-closed Sets

Remark 4.1. *Union of any two $\tilde{g}(1,2)^\star$-closed sets in X need not be a $\tilde{g}(1,2)^\star$-closed set as seen from the following example.*

Example 4.2. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ are called $\tau_{1,2}$- closed. Then $(1,2)^\star$-$\tilde{G}C(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$. Clearly, the sets $\{a\}$ and $\{b\}$ are $\tilde{g}(1,2)^\star$-closed but their union $\{a, b\}$ is not a $\tilde{g}(1,2)^\star$-closed set in X.*

Proposition 4.3. *If a set A is $\tilde{g}(1,2)^\star$-closed in X then $(1,2)^\star$-$\alpha cl(A) - A$ contains no nonempty $\tau_{1,2}$-closed set in X.*

Proof. Suppose that A is $\tilde{g}(1,2)^\star$-closed. Let F be a $\tau_{1,2}$-closed subset of $(1,2)^\star$-αcl(A) − A. Then A $\subseteq$ F^c. But A is $\tilde{g}(1,2)^\star$-closed, therefore $(1,2)^\star$-αcl(A) $\subseteq$ F^c. Consequently, F $\subseteq$ $((1,2)^\star$-αcl(A))^c. We already have F $\subseteq$ $(1,2)^\star$-αcl(A). Thus F $\subseteq$ $(1,2)^\star$-αcl(A) $\cap$ $((1,2)^\star$-αcl(A))^c and F is empty. □

The converse of Proposition 4.3 need not be true as seen from the following example.

Example 4.4. *In Example 3.12, if $A = \{b\}$, then $(1,2)^\star$-$\alpha cl(A) - A$ does not contain any nonempty $\tau_{1,2}$-closed set. But A is not a $\tilde{g}(1,2)^\star$-closed set in X.*

Theorem 4.5. *If a set A is $\tilde{g}(1,2)^\star$-closed in X then $(1,2)^\star$-$\alpha cl(A) - A$ contains no nonempty $(1,2)^\star$-$\hat{g}$-closed set.*

Proof. Suppose that A is $\tilde{g}(1,2)^\star$-closed. Let S be a $(1,2)^\star$-$\hat{g}$-closed subset of $(1,2)^\star$-αcl(A) − A. Then A $\subseteq$ S^c. Since A is $\tilde{g}(1,2)^\star$-closed, we have $(1,2)^\star$-αcl(A) $\subseteq$ S^c. Consequently, S $\subseteq$ $((1,2)^\star$-αcl(A))^c. Hence, S $\subseteq$ $(1,2)^\star$-αcl(A) $\cap$ $((1,2)^\star$-αcl(A))$^c = \emptyset$. Therefore S is empty. □

Theorem 4.6. *If A is $\tilde{g}(1,2)^\star$-closed in X and $A \subseteq B \subseteq (1,2)^\star$-$\alpha cl(A)$, then B is $\tilde{g}(1,2)^\star$-closed in X.*

Proof. Let B $\subseteq$ U where U is $(1,2)^\star$-$\hat{g}$-open set in X. Then A $\subseteq$ U. Since A is $\tilde{g}(1,2)^\star$-closed, $(1,2)^\star$-αcl(A) $\subseteq$ U. Since B $\subseteq$ $(1,2)^\star$-αcl(A), $(1,2)^\star$-αcl(B) $\subseteq$ $(1,2)^\star$-αcl(A). Therefore $(1,2)^\star$-αcl(B) $\subseteq$ U and B is $\tilde{g}(1,2)^\star$-closed in X. □

Proposition 4.7. *If A is a $(1,2)^\star$-$\hat{g}$-open and $\tilde{g}(1,2)^\star$-closed in X, then A is $(1,2)^\star$-α-closed in X.*

Proof. Since A is $(1,2)^\star$-$\hat{g}$-open and $\tilde{g}(1,2)^\star$-closed, $(1,2)^\star$-αcl(A) $\subseteq$ A and hence A is $(1,2)^\star$-α-closed in X. □

5. $\tilde{g}(1,2)^\star$-interior

We introduce the following definition.

Definition 5.1. *For any $A \subseteq X$, $\tilde{g}(1,2)^\star$-int(A) is defined as the union of all $\tilde{g}(1,2)^\star$-open sets contained in A. That is $\tilde{g}(1,2)^\star$-int(A) $= \cup\{ G : G \subseteq A$ and G is $\tilde{g}(1,2)^\star$-open$\}$.*

Lemma 5.2. *For any* $A \subseteq X$, $\tau_{1,2}$*-int(A)* $\subseteq \tilde{g}(1,2)^{\star}$*-int(A)* $\subseteq A$.

The following two Propositions are easy consequences from definitions.

Proposition 5.3. *For any* $A \subseteq X$, *we have A is* $\tilde{g}(1,2)^{\star}$*-open if and only if* $\tilde{g}(1,2)^{\star}$*-int(A)* $= A$.

Proposition 5.4. *For any subsets A and B of X, we have*

(1) $\tilde{g}(1,2)^{\star}$*-int(A* $\cap$ *B)* $= \tilde{g}(1,2)^{\star}$*-int(A)* $\cap\ \tilde{g}(1,2)^{\star}$*-int(B).*

(2) $\tilde{g}(1,2)^{\star}$*-int(A* $\cup$ *B)* $\supseteq \tilde{g}(1,2)^{\star}$*-int(A)* $\cup\ \tilde{g}(1,2)^{\star}$*-int(B).*

(3) If $A \subseteq B$, *then* $\tilde{g}(1,2)^{\star}$*-int(A)* $\subseteq \tilde{g}(1,2)^{\star}$*-int(B).*

(4) $\tilde{g}(1,2)^{\star}$*-int(X)* $= X$ *and* $\tilde{g}(1,2)^{\star}$*-int(* $\emptyset$ *)* $= \emptyset$.

6. $\tilde{g}(1,2)^{\star}$-closure

Definition 6.1. *For every set* $A \subseteq X$, *we define the* $\tilde{g}(1,2)^{\star}$*-closure of A to be the intersection of all* $\tilde{g}(1,2)^{\star}$*-closed sets containing A. In symbols,* $\tilde{g}(1,2)^{\star}$*-cl(A)* $= \cap \{F : A \subseteq F \in (1,2)^{\star}\text{-}\tilde{G}C(X)\}$.

Lemma 6.2. *For any* $A \subseteq X$, $A \subseteq \tilde{g}(1,2)^{\star}$*-cl(A)* $\subseteq \tau_{1,2}$*-cl(A).*

Proof. It follows from the fact that every $\tau_{1,2}$-closed set is $\tilde{g}(1,2)^{\star}$-closed. □

Remark 6.3. *Both containment relations in Lemma 6.2 may be proper as seen from the following example.*

Example 6.4. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X\}$ *and* $\tau_2 = \{\emptyset, X, \{a, b\}\}$. *Then the sets in* $\{\emptyset, X, \{a, b\}\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\emptyset, X, \{c\}\}$ *are called* $\tau_{1,2}$*-closed. Let* $A = \{a\}$. *Then* $\tilde{g}(1,2)^{\star}$*-cl(A)* $= \{a, c\}$ *and so* $A \subset \tilde{g}(1,2)^{\star}$*-cl(A)* $\subset \tau_{1,2}$*-cl(A).*

Proposition 6.5. *For any* $A \subseteq X$, *we have A is* $\tilde{g}(1,2)^{\star}$*-closed if and only if* $\tilde{g}(1,2)^{\star}$*-cl(A)* $= A$.

Proposition 6.6. *For any two subsets A and B of X, we have*

(1) If $A \subseteq B$, *then* $\tilde{g}(1,2)^{\star}$*-cl(A)* $\subseteq \tilde{g}(1,2)^{\star}$*-cl(B).*

(2) $\tilde{g}(1,2)^{\star}$*-cl(A* $\cap$ *B)* $\subseteq \tilde{g}(1,2)^{\star}$*-cl(A)* $\cap\ \tilde{g}(1,2)^{\star}$*-cl(B).*

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