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Drifting Effect of Electrons on Ion Acoustic Waves in an Unmagnetized Quantum Plasmas

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Abstract: Both supersonic (mach number $M > 1$) and subsonic ($M < 1$) compressive solitary waves of high amplitudes are shown to exist in this plasma for quantum mechanical effects of electrons. Consideration of electron inertia facilitating its drift motion and quantum effects are responsible to the growth of high amplitude solitary waves. Highly energetic plasma particles are trapped in the pseudopotential of great depths under the quantum effects to result much higher amplitude compressive solitons.

Keywords: KdV, Ion acoustic, Solitary.

1. Introduction

The microscopic behaviour of plasma particles demand the use of quantum mechanical effects in the formation of nonlinear waves subject to de-Broglie condition. The quantum mechanical effects when the de-Broglie wave length λ_{Be} of the charge carriers is comparable to the dimensions of the system are expected to be employed frequently in coming years. With the help of Schrodinger-Poisson and the Winger-Poisson equations, quantum plasmas can be modeled consisting of electrons of extremely high particle number densities and low temperatures unlike classical plasma. S. Ali, W.M. Moslem, P.K.Shukla, I. Kourakis [12] studied fully nonlinear ion-sound waves in a dense Fermi magneto plasma. The non-linear effects in quantum dusty plasma was first attempted by S. Ali and P. K. Shukla [13]. Abdi kian [1] studied non linear Propagation of Acoustic Soliton Waves in dense quantum Electron-Positron Magneto plasma. O subsonic ion-acoustic solitary waves are shown to exist based on mach number and propagation direction. C.Bhowmik and A. P. Misra [2] investigated oblique modulation of electron-acoustic waves in a Fermi electron-ion plasma modulational instability (MI) of electron acoustic wave (EAW) in a quantum plasma consisting of inertialess hot electrons, inertial cold electrons and one component of immobile ions using non linear Schrodinger equation (NLSE). P. Bertrand, V. T. Nguyen, M. Gros, B. Izrar, M. R. Feix and J. Gutierrez [11] studied classical Vlasov Plasma description through quantum numerical methods through computer simulations. S. Chandra, S. N. Paul, B. Ghosh [14] studied effect of electron-inertia and ion-streaming on Ion-Acoustic Waves in Quantum Plasma. The hydrodynamic formalism of the plasma model with quantum effects is more convenient for direct use of macroscopic plasma quantities like density and average velocity. F. Hass, L. G. Garcia. Geodert and G. Manfredi [5] establishes quantum ion-acoustic waves. Recently, quantum effects are being used even to establish ion-acoustic solitary waves (IASW), dust ion-acoustic waves which attract immense interests from the researchers of the world. F. Hass [4] studied magnetohydrodynamic

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model for quantum plasmas. Y. Jung [15] investigated quantum-mechanical effects on electron-electron scattering in dense high- temperature plasmas. M. Marklund and G. Brodin [7] studied short-wavelength soliton in a fully degenerate quantum plasma.

The properties of non-linear stability of plasma oscillations were first studied by D.Pines, [3] for high density and low temperature plasmons. P. K. Shukla and S. Ali [10] establishes dust acoustic waves in quantum plasmas. P. K. Shukla [9] investigated a new dust mode in quantum plasmas. L. Stenflo, P. K. Shukla and M. Marklund [6] studied new low frequency oscillations in quantum dusty plasmas. N. Sadiq and M. Ahmed [15] investigated kinetic Alfven waves in dense quantum plasmas with effect of spin magnetization. For the higher values of equilibrium number density, the plasma frequency $w_p^2 = \dfrac{4\pi n_{\alpha 0}e^2}{m_\alpha}$, $(\alpha = e, p)$ of course is sufficiently high. Hence strong density corrections significantly change the states of plasma when the de-Broglie wavelength λ_{Be} is larger than the average inter-particle distance such that $n_{e0}\lambda_{Be}^3 \geq 1$.

The parameters-quantum diffraction (H) and the equilibrium density ratio of the cold to the hot electrons are shown to significantly effect the MI of the system. Further Ali et.al have studied in an electron-positron-ion quantum unmagnetized plasma, the existence of linear and non-linear ion-acoustic waves with the help of the Korteweg-de-vries (KdV) equation and the energy integral for $H_e = 0$ and only with quantum statistical effects. The quantum corrections are shown to result significantly in the formation of IAW. Further quantum transport models in order to study the dispersion properties and non-linear dynamics of unmagnetized and magnetized quantum plasmas are investigated by Hass [7] and many other authors. The cold quantum dusty plasmas supporting new dust modes have been investigated by many authors [12, 13, 14, 15]. At extremely low temperatures, the plasma behaves like a Fermi gas and quantum mechanical effects are expected to play a significant role in the behavior of charged particles.

In this paper, the quantum effects are introduced in electron populations in the plasma without Fermi pressures. The paper is organized in the following manner- the basic equations governing the plasma is formulated in section 2, the energy integral to investigate solitary waves is derived in section 3, section 4 contains the conditions of existence for solitary waves, and results for the existence of solitary waves are analyzed in section 5.

2. Basic Equations

We consider the two component plasmas having ions and electrons under the influence of quantum effects and electron inertia. The normalized fluid equations of motion with quantum parameter H_e (for electrons) are given by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{1}$$

$$\frac{\partial v_{ix}}{\partial t} + v_{ix}\frac{\partial v_{ix}}{\partial x} = -\left[\frac{\partial \varphi}{\partial x} + \frac{\alpha}{n_i}\frac{\partial n_i}{\partial x}\right], \tag{2}$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0, \tag{3}$$

$$\frac{\partial v_{ex}}{\partial t} + v_{ex}\frac{\partial v_{ex}}{\partial x} = \frac{1}{Q}\left[\frac{\partial \varphi}{\partial x} - \frac{1}{n_e}\frac{\partial n_e}{\partial x}\right] + \frac{{H_e}^2}{2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2}{\partial x^2}\left(\sqrt{n_e}\right)\right] \tag{4}$$

for electrons, where

$$Q = \frac{m_e}{m_i}, \quad {H_e}^2 = \frac{\hbar \omega_{pe}}{2k_B T_e}, \quad \alpha = \frac{T_e}{T_i} = \frac{\text{electrontemperature}}{\text{iontemperature}}.$$

We have normalized densities n_i, n_e by the equilibrium density n velocities v_{ix}, v_{ex} by C_s time t by $\frac{\lambda_D}{C_s}$ and potential φ by $\frac{T_e}{e}$.

3. Derivation of Energy Integral

Let us consider the transformation $\eta = k_x x - Mt$, where $M = \frac{\text{wave velocity}}{C_s}$. Therefore

$$\frac{\partial}{\partial x} = k_x \frac{\partial}{\partial \eta}, \quad \frac{\partial}{\partial t} = -M \frac{\partial}{\partial \eta}.$$

Using the above transformation in (1)-(4) and applying $v_{ix} = 0$, $v_{ex} = v_e'(initial\ drift)$, $\varphi = 0$ at $n_e = n_i = 1$ we get,

$$k_x v_{ix} = M\left(1 - \frac{1}{n_i}\right) \tag{5}$$

$$k_x {v_{ix}}^2 - 2Mv_{ix} + 2k_x\varphi + 2\alpha k_x \log n_i = 0 \tag{6}$$

$$k_x v_{ex} = M\left(1 - \frac{1}{n_e}\right) + \frac{k_x v_e'}{n_e} \tag{7}$$

$$Qk_x\left({v_{ex}}^2 - {v_e'}^2\right) - 2QM(v_{ex} - v_e') - 2k_x\varphi + 2k_x log n_e = \frac{Q{k_x}^3{H_e}^2}{\sqrt{n_e}}\frac{\partial}{\partial\eta}\left\{\frac{1}{2\sqrt{n_e}}\frac{\partial n_e}{\partial \eta}\right\} \tag{8}$$

Using the charge neutrality condition $n_i = n_e = n$ and eliminating v_{ix} from (5) and (6), v_{ex} from (7) and (8) we get,

$$\frac{M^2}{k_x}\left(1 - \frac{1}{n}\right)^2 - \frac{2M^2}{k_x}\left(1 - \frac{1}{n}\right) + 2k_x\varphi + 2\alpha k_x \log n = 0 \tag{9}$$

$$\frac{M^2 Q}{k_x}\left(1 - \frac{1}{n}\right)^2 - \frac{2M^2 Q}{k_x}\left(1 - \frac{1}{n}\right) + Qk_x{v_e'}^2\left(\frac{1}{n^2} - 1\right) + 2QMv_e'\left(1 - \frac{1}{n^2}\right) - 2k_x\varphi + 2k_x log n = \frac{{k_x}^3}{\sqrt{n}}Q{H_e}^2\frac{\partial}{\partial\eta}\left(\frac{1}{2\sqrt{n}}\frac{\partial n}{\partial\eta}\right) \tag{10}$$

Addition of (9) and (10) can be expressed as

$$\frac{M^2(1+Q)}{K_x}\left(1 - \frac{1}{n}\right)^2 - \frac{2M^2(1+Q)}{K_x}\left(1 - \frac{1}{n}\right) + 2K_x(1+\alpha)\log n + QK_x{v_e'}^2\left(\frac{1}{n^2} - 1\right) + 2QMv_e'\left(1 - \frac{1}{n^2}\right) \tag{11}$$

Multiplying both sides of (11) by $\left(\frac{1}{2\sqrt{n}}\frac{\partial n}{\partial \eta}\right)$, it can be simplified as

$$\left(\frac{1}{2\sqrt{n}}\frac{\partial n}{\partial\eta}\right)\frac{\partial}{\partial\eta}\left(\frac{1}{2\sqrt{n}}\frac{\partial n}{\partial\eta}\right) = \frac{1}{2{k_x}^3 Q{H_e}^2}\left[\begin{array}{c}\frac{M^2(1+Q)}{k_x}\left(1 - \frac{2}{n} + \frac{1}{n^2}\right) - \frac{2M^2(1+Q)}{k_x}\left(1 - \frac{1}{n}\right) + \\ 2k_x(1+\alpha)\log n + Qk_x{v_e'}^2\left(\frac{1}{n^2} - 1\right) + 2QMv_e'\left(1 - \frac{1}{n^2}\right)\end{array}\right]\frac{dn}{d\eta} \tag{12}$$

Integrating (12) and using the condition $\frac{dn}{d\eta} = 0$ at $n = 1$, we can have the energy integral

$$\frac{1}{2}\left(\frac{dn}{d\eta}\right)^2 + \psi(n, M, \alpha, k_x, H_i, H_e) = 0, \tag{13}$$

where

$$(n, M, \alpha, k_x, H_e) = \frac{-2n}{{k_x}^3 Q{H_e}^2}\left[\begin{array}{c}\frac{M^2(1+Q)}{k_x}\left(\frac{1}{n} - n + 2\log n\right) + \frac{2M^2(1+Q)}{k_x}(n - 1 - \log n) + \\ 2k_x(1+\alpha)(n - 1 - n\log n) + Qk_x{v_e'}^2\left(n + \frac{1}{n} - 2\right) + \\ 2QMv_e'\left(2 - n - \frac{1}{n}\right)\end{array}\right] \tag{14}$$

which is called the Sagdeev Potential.

4. Conditions for the Existence of Solitary Waves

From (14), $\Psi'(n)$ can be found as follows:

$$\psi'(n) = \frac{-2}{{k_x}^3 Q{H_e}^2}\left[\begin{array}{c}\frac{M^2(1+Q)}{k_x}(\frac{1}{n} - n + 2\log n) + \frac{2M^2(1+Q)}{k_x}(n-1-\log n) + \\ 2k_x(1+\alpha)(n-1-n\log n) + Qk_x {v'_e}^2\left(n+\frac{1}{n}-2\right) + 2QMv'_e\left(2-n-\frac{1}{n}\right)\end{array}\right] - \frac{-2n}{{k_x}^3 Q{H_e}^2}\left[\begin{array}{c}\frac{M^2(1+Q)}{k_x}\left(\frac{2}{n}-1-\frac{1}{n^2}\right) + \frac{2M^2(1+Q)}{k_x}\left(1-\frac{1}{n}\right) - 2k_x(1+\alpha)\log n + \\ Qk_x{v'_e}^2\left(1-\frac{1}{n^2}\right) + 2QMv'_e\left(\frac{1}{n^2}-1\right)\end{array}\right] \tag{15}$$

From (14) and (15), it is clear that

$$\psi(1) = 0 = \psi'(1)$$

Again $\Psi''(n)$ can be found as,

$$\psi''(n) = \frac{-4}{{k_x}^3 Q{H_e}^2}\left[\begin{array}{c}\frac{M^2(1+Q)}{k_x}\left(\frac{2}{n}-\frac{1}{n^2}-1\right) + \frac{2M^2(1+Q)}{k_x}\left(1-\frac{1}{n}\right) - 2k_x(1+\alpha)\log n + \\ Qk_x{v'_e}^2\left(1-\frac{1}{n^2}\right) + 2QMv'_e\left(\frac{1}{n^2}-1\right)\end{array}\right] - \frac{2M}{{k_x}^3 Q{H_e}^2}\left[\frac{M^2(1+Q)}{k_x}\left(\frac{2}{n^3}-\frac{2}{n^2}\right) + \frac{2M^2(1+Q)}{k_x}\frac{1}{n^2} - \frac{2k_x(1+\alpha)}{n}\frac{2Qk_x{v'_e}^2}{n^3} - \frac{4QMv'_e}{n^3}\right]. \tag{16}$$

For solitary wave solution, it is essential to investigate the behavior of $\psi(n)$ near $n = 1$ and $n = N$, the maximum value N of n called the solitary wave amplitude. For this purpose we expand $\psi(n)$ as follows:

By Taylor series near $n = 1$ and $n = N$

$$\psi(n \approx 1) = \psi(1) + (n-1)\psi'(1) + \frac{(n-1)^2}{2}\psi''(1) + ... \tag{17}$$

$$\psi(n \approx 1) = \frac{-2(n-1)^2}{{k_x}^4 Q{H_e}^2}\left[M^2(1+Q) + Q{k_x}^2{v'_e}^2 - {k_x}^2(1+\alpha) - 2QMk_x v'_e\right] \tag{18}$$

and

$$\psi(n \approx N) = \frac{-4(n-N)}{{k_x}^4 Q{H_e}^2}\left[(N-1)\left\{M^2(1+Q) + Q{k_x}^2{v'_e}^2 - {k_x}^2(1+\alpha) - 2QMk_x v'_e\right\} - 2{k_x}^2(1+\alpha)N\log N\right] \tag{19}$$

Near $n \approx 1$, $\psi(n \approx 1) < 0$ if

$$A < M^2(1+Q) + Qk_x^2{v'}_e^2 - k_x^2(1+\alpha) - 2QMk_x v'_e > 0 \tag{20}$$

And was near $n \approx N$, $\psi(n \approx N) < 0$ if

$$A < \frac{2k_x^2(1+\alpha)N\log N}{(N-1)} \text{ for } N > 1 \tag{21}$$

$$A > \frac{2k_x^2(1+\alpha)N\log N}{(N-1)} \text{ for } N < 1 \tag{22}$$

5. Discussion

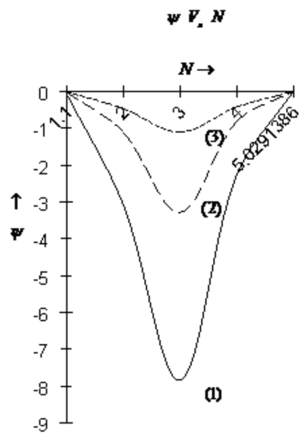

Figure 1: Plot of pseudopotential to characterize compressive solitary waves in a plasma with quantum effects $H_e = 1.5(1)$; $H_e = 2.5(2)$; $H_e = 3.5(3)$ and for fixed values of $M = 0.6(< 1)$, $\alpha = 0.1$, $k_x = 0.5$ and v'_e (electron's drift) $= 1$.

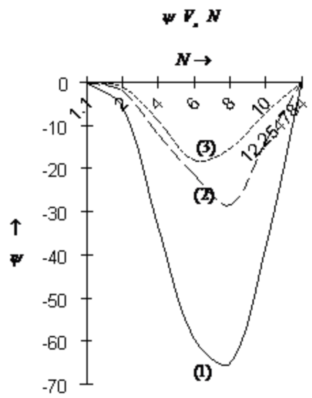

Figure 2: Plot of pseudopotential to characterize compressive solitary wave in quantum plasma for $H_e = 1.5(1)$; $H_e = 2.5(2)$ and $H_e = 3.5(3)$ when mach number is $M = 1.5(> 1)$, $\alpha = 0.1$, $k_x = 0.5$ and $v'_e = 1$

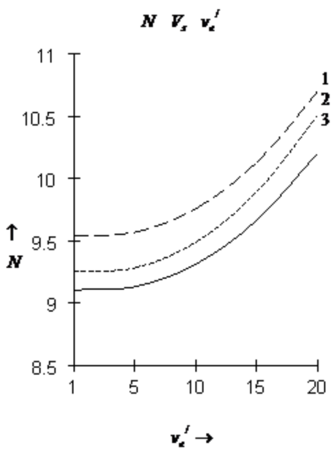

Figure 3: Amplitude N of compressive soliton in quantum plasma versus v'_e (= electrons' drift) for fixed $M = 0.6(< 1)$, $k_x = 0.5$, $H_e = 1.5$; $\alpha = 0.1(1), 0.11(2), 0.12(3)$

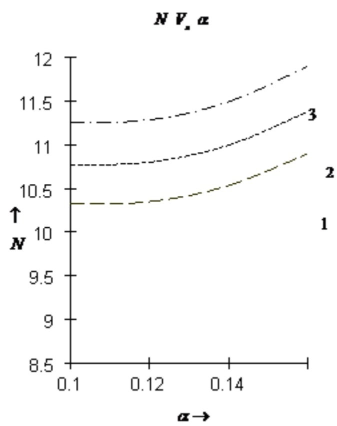

Figure 4: Amplitude N of compressive soliton in quantum plasma versus temperature α for fixed $M = 1.5(> 1)$, $k_x = 0.5$, $H_e = 1.5$ for $v'_e = 1(1), 5(2), 10(3)$

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