International Journal of Mathematics And its Applications
Volume 3, Issue 3–A (2015), 41–45.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

# Further Decompositions of $rg$-Continuity

**Research Article**

**K.M.Dharmalingam[1], P.Ramesh[2] and O.Ravi[3]***

1 Department of Mathematics, The Madura College, Madurai, Tamil Nadu, India.

2 Department of Mathematics, M.Kumarasamy College of Engineering, Thalavapalayam, Karur, Tamil Nadu, India.

3 Department of Mathematics, P.M.Thevar College, Usilampatti, Tamil Nadu, India.

**Abstract:** In [12], Sundaram and Rajamani obtained three decompositions of $rg$-continuity. In this paper, we obtain three further decompositions of $rg$-continuity.

**MSC:** 54C08.

**Keywords:** $r\alpha g$-continuity, $gpr$-continuity, $C_r$-continuity, $C_r^*$-continuity and $rg$-continuity.



## 1. Introduction and Preliminaries

In 1970, Levine [6] initiated the study of so called g-closed sets in topological spaces. As a generalization, in 1997, Gnanambal [3] introduced and studied the concepts of gpr-closed sets and gpr-continuity. In 1999, Noiri [9] defined the notion of $r\alpha g$-closed sets in topological spaces. The concept of g-continuity was introduced and studied by Balachandran et. al. in 1991 [2]. In 1993, Palaniappan and Rao [10] introduced the notions of regular generalized closed ($rg$-closed) sets and $rg$-continuity in topological spaces. In 2000, Sundaram and Rajamani [12] obtained three different decompositions of $rg$-continuity by providing two types of weaker forms of continuity, namely $C_r$-continuity and $C_r^*$-continuity. In this paper, we obtain three further decompositions of $rg$-continuity. Let $(X, \tau)$ be a topological space and also cl$(A)$ and int$(A)$ denote the closure of $A$ and the interior of $A$ in (X, $\tau$), respectively.

**Definition 1.1.** *A subset $A$ of $(X, \tau)$ is said to be*

1. *regular open [11] if $A = int(cl(A))$,*
2. *$\alpha$-open [8] if $A \subseteq int(cl(int(A)))$,*
3. *preopen [7] if $A \subseteq int(cl(A))$.*

*The complements of the above mentioned open sets are called their respective closed sets. The preinterior pint(A) (resp. $\alpha$-interior, $\alpha int(A)$) of $A$ is the union of all preopen sets (resp. $\alpha$-open sets) contained in $A$. The $\alpha$-closure $\alpha cl(A)$ (resp. preclosure, $pcl(A)$) of $A$ is the intersection of all $\alpha$-closed sets (resp. preclosed sets) containing $A$.*

* *E-mail: siingam@yahoo.com*

**Definition 1.2.** *A subset $A$ of $(X, \tau)$ is said to be*

1. *$rg$-closed [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $X$,*
2. *$r\alpha g$-closed [9] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $X$,*
3. *$gpr$-closed [3] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $X$.*

*The complements of the above mentioned closed sets are called their respective open sets.*

**Definition 1.3.** *A subset $A$ of $(X, \tau)$ is said to be*

1. *$rg$-open [10] iff $F \subseteq int(A)$ whenever $F \subseteq A$ and $F$ is regular closed in $(X, \tau)$,*
2. *$gpr$-open [3] iff $F \subseteq pint(A)$ whenever $F \subseteq A$ and $F$ is regular closed in $(X, \tau)$,*
3. *a t-set [13] if $int(A) = int(cl(A))$,*
4. *an $\alpha^*$-set [4] if $int(A) = int(cl(int(A)))$.*

**Lemma 1.4** ([1])**.** *If $A$ is a subset of $X$, then*

1. *$pint(A) = A \cap int(cl(A))$,*
2. *$\alpha int(A) = A \cap int(cl(int(A)))$ and $\alpha cl(A) = A \cup cl(int(cl(A)))$.*

**Remark 1.5.** *The following holds in a topological space. Every $rg$-open set is $gpr$-open but not conversely [3].*

## 2. $r\alpha g$-open Sets

**Proposition 2.1.** *For a subset of a topological space, the following hold:*

1. *Every $r\alpha g$-open set is $gpr$-open.*
2. *Every $rg$-open set is $r\alpha g$-open.*

*Proof.* It follows from the definitions. □

**Remark 2.2.** *The converses of Proposition 2.1 is not true, in general.*

**Example 2.3.** *Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$. Then $\{b, e\}$ is $gpr$-open set but not $r\alpha g$-open.*

**Example 2.4.** *Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$. Then $\{a, b, d\}$ is $r\alpha g$-open set but not $rg$-open.*

**Remark 2.5.** *By Proposition 2.1 and Remark 1.5, we have the following diagram. In this diagram, there is no implication which is reversible as shown by examples above.*

$rg$-open
↙
$r\alpha g$-open ↓
↘
$gpr$-open

**Lemma 2.6.** *Let $A$ be a subset of $(X, \tau)$. Then $A$ is $r\alpha g$-open iff $F \subseteq \alpha int(A)$ whenever $F \subseteq A$ and $F$ is regular closed in $(X, \tau)$.*

# 3. $C_r$-sets and $C_r^*$-sets

**Definition 3.1** ([12]). *A subset $A$ of a topological space $(X, \tau)$ is called*

1. *a $C_r$-set if $A = U \cap V$, where $U$ is $rg$-open and $V$ is a $t$-set,*
2. *a $C_r^*$-set if $A = U \cap V$, where $U$ is $rg$-open and $V$ is an $\alpha^*$-set.*

We have the following proposition:

**Proposition 3.2.** *For a subset of a topological space, the following hold:*

1. *Every $t$-set is an $\alpha^*$-set [4] and a $C_r$-set.*
2. *Every $\alpha^*$-set is a $C_r^*$-set.*
3. *Every $C_r$-set is a $C_r^*$-set.*
4. *Every $rg$-open set is a $C_r$-set.*

From Proposition 3.2, We have the following diagram.

$$\begin{array}{ccccc} rg\text{-open set} & \longrightarrow & C_r\text{-set} & \longleftarrow & t\text{-set} \\ & & \downarrow & & \downarrow \\ & & C_r^*\text{-set} & \longleftarrow & \alpha^*\text{-set} \end{array}$$

**Remark 3.3.** *The converses of implications in Diagram II need not be true as the following examples show.*

**Example 3.4.** *Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$. Then $\{b\}$ is $C_r$-set but not $rg$-open.*

**Example 3.5.** *In Example 3.4, $\{c\}$ is $C_r$-set but not $t$-set.*

**Example 3.6.** *In Example 3.4, $\{b, c, e\}$ is $C_r^*$-set but not $C_r$-set.*

**Example 3.7.** *In Example 3.4, $\{c\}$ is $\alpha^*$-set but not $t$-set.*

**Example 3.8.** *In Example 3.4, $\{c, d, e\}$ is $C_r^*$-set but not $\alpha^*$-set.*

**Proposition 3.9.** *A subset $A$ of $X$ is $rg$-open if and only if it is both $gpr$-open and a $C_r$-set in $X$.*

*Proof.* Necessity is trivial. We prove the sufficiency. Assume that $A$ is $gpr$-open and a $C_r$-set in $X$. Let $F \subseteq A$ and $F$ is regular closed in $X$. Since $A$ is a $C_r$-set in $X$, $A = U \cap V$, where $U$ is $rg$-open and $V$ is a $t$-set. Since $A$ is $gpr$-open, $F \subseteq p\text{int}(A) = A \cap \text{int}(\text{cl}(A)) = (U \cap V) \cap \text{int}(\text{cl}(U \cap V)) \subseteq (U \cap V) \cap \text{int}(\text{cl}(U) \cap \text{cl}(V)) = (U \cap V) \cap \text{int}(\text{cl}(U)) \cap \text{int}(\text{cl}(V))$. This implies $F \subseteq \text{int}(\text{cl}(V)) = \text{int}(V)$ since $V$ is a $t$-set. Since $F$ is regular closed, $U$ is $rg$-open and $F \subseteq U$, we have $F \subseteq \text{int}(U)$. Therefore, $F \subseteq \text{int}(U) \cap \text{int}(V) = \text{int}(U \cap V) = \text{int}(A)$. Hence $A$ is $rg$-open in $X$. □

**Corollary 3.10.** *A subset $A$ of $X$ is $rg$-open if and only if it is both $r\alpha g$-open and a $C_r$-set in $X$.*

*Proof.* This is an immediate consequence of Proposition 3.9. □

**Proposition 3.11.** *A subset $A$ of $X$ is $rg$-open if and only if it is both $r\alpha g$-open and a $C_r^*$-set in $X$.*

*Proof.* Necessity is trivial. We prove the sufficiency. Assume that $A$ is $r\alpha g$-open and a $C_r^*$-set in $X$. Let $F \subseteq A$ and $F$ is regular closed in $X$. Since $A$ is a $C_r^*$-set in $X$, $A = U \cap V$, where $U$ is $rg$-open and $V$ is an $\alpha^*$-set. Now since $F$ is regular closed, $F \subseteq U$ and $U$ is $rg$-open, $F \subseteq \text{int(U)}$. Since $A$ is $r\alpha g$-open, $F \subseteq \alpha\text{int}(A) = A \cap \text{int}(\text{cl}(\text{int}(A))) = (U \cap V) \cap \text{int}(\text{cl}(\text{int}(U \cap V))) = (U \cap V) \cap \text{int}(\text{cl}(\text{int}(U) \cap \text{int}(V))) \subseteq (U \cap V) \cap \text{int}(\text{cl}(\text{int}(U)) \cap \text{cl}(\text{int}(V))) = (U \cap V) \cap \text{int}(\text{cl}(\text{int}(U))) \cap \text{int}(\text{cl}(\text{int}(V))) = (U \cap V) \cap \text{int}(\text{cl}(\text{int}(U))) \cap \text{int}(V)$, since $V$ is an $\alpha^*$-set. This implies $F \subseteq \text{int}(V)$. Therefore, $F \subseteq \text{int}(U) \cap \text{int}(V) = \text{int}(U \cap V) = \text{int}(A)$. Hence $A$ is $rg$-open in $X$. □

**Remark 3.12.**

*(1) The concepts of gpr-open sets and $C_r$-sets are independent of each other.*

*(2) The concepts of $r\alpha g$-open sets and $C_r$-sets are independent of each other.*

*(3) The concepts of $r\alpha g$-open sets and $C_r^*$-sets are independent of each other.*

**Example 3.13.** *Let X= $\{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$. Then $\{b, c, e\}$ is gpr-open but not $C_r$-set and $\{a, b, e\}$ is $C_r$-set but not gpr-open.*

**Example 3.14.** *Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$. Then $\{a, d\}$ is $C_r$-set but not $r\alpha g$-open set. Also $\{a, b, d\}$ is an $r\alpha g$-open set but not $C_r$-set.*

**Example 3.15.** *In Example 3.14, $\{a, d\}$ is $C_r^*$-set but not $r\alpha g$-open set and $\{a, b, d\}$ is an $r\alpha g$-open set but not $C_r^*$-set.*

## 4. Decompositions of $rg$-continuity

**Definition 4.1.** *A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be rg-continuous [10] (resp. gpr-continuous [3], $r\alpha g$-continuous, $C_r$-continuous [12] and $C_r^*$-continuous [12]) if $f^{-1}(V)$ is rg-open (resp. gpr-open, $r\alpha g$-open, $C_r$-set and $C_r^*$-set) in $(X, \tau)$ for every open set $V$ in $(Y, \sigma)$.*

From Propositions 3.9 and 3.11 and Corollary 3.10 we have the following decompositions of $rg$-continuity.

**Theorem 4.2.** *For a mapping $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

1. *$f$ is rg-continuous;*
2. *$f$ is gpr-continuous and $C_r$-continuous;*
3. *$f$ is $r\alpha g$-continuous and $C_r$-continuous;*
4. *$f$ is $r\alpha g$-continuous and $C_r^*$-continuous.*

**Remark 4.3.**

*(1) The concepts of gpr-continuity and $C_r$-continuity are independent of each other.*

*(2) The concepts of $r\alpha g$-continuity and $C_r$-continuity are independent of each other.*

*(3) The concepts of $r\alpha g$-continuity and $C_r^*$-continuity are independent of each other.*

**Example 4.4.**

*(1) Let $X = Y = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$ and $\sigma = \{\emptyset, \{b, c, e\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is gpr-continuous but not $C_r$-continuous.*

*(2) Let* $X = Y = \{a, b, c, d, e\}$, $\tau = \{\emptyset, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$ *and* $\sigma = \{\emptyset, \{a, b, e\}, Y\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then f is* $C_r$*-continuous but not gpr-continuous.*

**Example 4.5.**

*(1) Let* $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ *and* $\sigma = \{\emptyset, \{a, d\}, Y\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then f is* $C_r$*-continuous but not* $r\alpha g$*-continuous.*

*(2) Let* $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ *and* $\sigma = \{\emptyset, \{a, b, d\}, Y\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then f is* $r\alpha g$*-continuous but not* $C_r$*-continuous.*

**Example 4.6.**

*(1) Let* $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ *and* $\sigma = \{\emptyset, \{a, d\}, Y\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then f is* $C_r^*$*-continuous but not* $r\alpha g$*-continuous.*

*(2) Let* $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ *and* $\sigma = \{\emptyset, \{a, b, d\}, Y\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then f is* $r\alpha g$*-continuous but not* $C_r^*$*-continuous.*

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