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$\tilde{g}$(1,2)*-closed Sets in Bitopological Spaces

Research Article

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Abstract: In this paper, we offer a new class of sets called $\tilde{g}$(1,2)*-closed sets in bitopological spaces and we study some of its basic properties. It turns out that this class lies between the class of $\tau_{1,2}$-closed sets and the class of (1,2)*-αg-closed sets.

MSC: 54E55.

Keywords: Bitopological space, (1,2)*-$\hat{g}$-closed set, (1,2)*-$\ddot{g}$-closed set, (1,2)*-g-closed set, (1,2)*-$\ddot{g}_\alpha$-closed set.

1. Introduction

Levine [3] introduced generalized closed sets in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. Many researchers like Veerakumar [19] introduced $\hat{g}$-closed sets in topological spaces. Sheik John [17] introduced ω-closed sets in topological spaces. After the advent of these notions, many topologists introduced various types of generalized closed sets and studied their fundamental properties. Quite Recently, Ravi and Ganesan [4] introduced and studied $\ddot{g}$-closed sets in general topology as another generalization of closed sets and proved that the class of $\ddot{g}$-closed sets properly lies between the class of closed sets and the class of ω-closed sets. Ravi et al [10, 11] and Ravi and Thivagar [6] introduced (1,2)*-αg-closed sets, (1,2)*-g-closed sets, (1,2)*-sg-closed sets and (1,2)*-$\hat{g}$-closed sets respectively. Ravi and Ganesan [5] introduced (1,2)*-$\ddot{g}_\alpha$ -closed sets in bitopological spaces. In this paper, we introduce a new class of sets namely $\tilde{g}$(1,2)*-closed sets in bitopological spaces. This class lies between the class of (1,2)*-$\ddot{g}_\alpha$-closed sets and the class of (1,2)*-αg-closed sets. Properties of $\tilde{g}$(1,2)*-closed sets are studied.

2. Preliminaries

Throughout this paper, (X, τ_1, τ_2) (briefly, X) will denote bitopological space.

Definition 2.1. *Let S be a subset of X. Then S is said to be $\tau_{1,2}$-open [7] if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$. The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed. Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.*

Definition 2.2 ([7]). *Let S be a subset of a bitopological space X. Then*

1. *the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as $\cup\{F : F \subseteq S$ and F is $\tau_{1,2}$-open$\}$.*

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2. *the* $\tau_{1,2}$*-closure of* S*, denoted by* $\tau_{1,2}$*-cl(S), is defined as* $\cap \{F : S \subseteq F$ *and* F *is* $\tau_{1,2}$*-closed*$\}$.

Definition 2.3. *A subset A of a bitopological space X is called*

1. *(1,2)*-semi-open set [6] if* $A \subseteq \tau_{1,2}$*-cl(*$\tau_{1,2}$*-int(A));*
2. *(1,2)*-*α*-open set [7] if* $A \subseteq \tau_{1,2}$*-int(*$\tau_{1,2}$*-cl(*$\tau_{1,2}$*-int(A)));*
3. *(1,2)*-*β*-open set [11] if* $A \subseteq \tau_{1,2}$*-cl(*$\tau_{1,2}$*-int(*$\tau_{1,2}$*-cl(A))).*

The complements of the above mentioned open sets are called their respective closed sets. The (1,2)-semi-closure [6] (resp. (1,2)*-*α*-closure [9], (1,2)*-*β*-closure [11]) of a subset A of X, denoted by (1,2)*-scl(A) (resp. (1,2)*-*α*cl(A), (1,2)*-*β*cl(A)), is defined to be the intersection of all (1,2)*-semi-closed (resp. (1,2)*-*α*-closed, (1,2)*-*β*-closed) subsets of (X,* τ_1*,* τ_2*) containing A. It is known that (1,2)*-scl(A) (resp. (1,2)*-*α*cl(A), (1,2)*-*β*cl(A)) is a (1,2)*-semi-closed (resp. (1,2)*-*α*-closed, (1,2)*-*β*-closed) set.*

Definition 2.4. *A subset A of a bitopological space (X,* τ_1*,* τ_2*) is called*

1. *(1,2)*-g-closed set [10] if* $\tau_{1,2}$*-cl(A)* $\subseteq U$ *whenever* $A \subseteq U$ *and U is* $\tau_{1,2}$*-open in X. The complement of (1,2)*-g-closed set is called (1,2)*-g-open set;*
2. *(1,2)*-sg-closed set [13] if (1,2)*-scl(A)* $\subseteq U$ *whenever* $A \subseteq U$ *and U is (1,2)*-semi-open in X. The complement of (1,2)*-sg-closed set is called (1,2)*-sg-open set;*
3. *(1,2)*-gs-closed set [13] if (1,2)*-scl(A)* $\subseteq U$ *whenever* $A \subseteq U$ *and U is* $\tau_{1,2}$*-open in X. The complement of (1,2)*-gs-closed set is called (1,2)*-gs-open set;*
4. *(1,2)*-*α*g-closed set [11] if (1,2)*-*α*cl(A)* $\subseteq U$ *whenever* $A \subseteq U$ *and U is* $\tau_{1,2}$*-open in X. The complement of (1,2)*-*α*g-closed set is called (1,2)*-*α*g-open set;*
5. *(1,2)*-*$\hat{g}$*-closed set [2] or (1,2)*-*ω*-closed set [2] if* $\tau_{1,2}$*-cl(A)* $\subseteq U$ *whenever* $A \subseteq U$ *and U is (1,2)*-semi-open in X. The complement of (1,2)*-*$\hat{g}$*-closed ((1,2)*-*ω*-closed) set is called (1,2)*-*$\hat{g}$*-open ((1,2)*-*ω*-open) set;*
6. *(1,2)*-*ψ*-closed set [16] if (1,2)*-scl(A)* $\subseteq U$ *whenever* $A \subseteq U$ *and U is (1,2)*-sg-open in X. The complement of (1,2)*-*ψ*-closed set is called (1,2)*-*ψ*-open set;*
7. *(1,2)*-*$\ddot{g}_\alpha$*-closed set [5] if (1,2)*-*α*cl(A)* $\subseteq U$ *whenever* $A \subseteq U$ *and U is (1,2)*-sg-open in X. The complement of (1,2)*-*$\ddot{g}_\alpha$*-closed set is called (1,2)*-*$\ddot{g}_\alpha$*-open set;*
8. *(1,2)*-gsp-closed set [16] if (1,2)*-*β*cl(A)* $\subseteq U$ *whenever* $A \subseteq U$ *and U is* $\tau_{1,2}$*-open in X. The complement of (1,2)*-gsp-closed set is called (1,2)*-gsp-open set.*

Remark 2.5. *The collection of all (1,2)*-*$\ddot{g}_\alpha$*-closed (resp. (1,2)*-*$\hat{g}$*-closed, (1,2)*-g-closed, (1,2)*-gs-closed, (1,2)*-*α*g-closed, (1,2)*-sg-closed, (1,2)*-*ψ*-closed, (1,2)*-*α*-closed, (1,2)*-semi-closed) sets is denoted by (1,2)*-*$\ddot{G}_\alpha C(X)$ *(resp. (1,2)*-*$\hat{G}C(X)$*, (1,2)*-GC(X), (1,2)*-GSC(X), (1,2)*-*$\alpha GC(X)$*, (1,2)*-SGC(X), (1,2)*-*$\psi C(X)$*, (1,2)*-*$\alpha C(X)$*, (1,2)*-SC(X)). We denote the power set of X by P(X).*

Remark 2.6.

1. *Every* $\tau_{1,2}$*-closed set is (1,2)*-semi-closed but not conversely [6].*

2. *Every $\tau_{1,2}$-closed set is (1,2)*-α-closed but not conversely [12].*

3. *Every (1,2)*-semi-closed set is (1,2)*-ψ-closed but not conversely [16].*

4. *Every (1,2)*-semi-closed set is (1,2)*-sg-closed but not conversely [13].*

5. *Every (1,2)*-$\hat{g}$-closed set is (1,2)*-g-closed but not conversely [16].*

6. *Every (1,2)*-sg-closed set is (1,2)*-gs-closed but not conversely [13].*

7. *Every (1,2)*-g-closed set is (1,2)*-αg-closed but not conversely [13].*

8. *Every (1,2)*-g-closed set is (1,2)*-gs-closed but not conversely [10].*

9. *Every $\tau_{1,2}$-closed set is (1,2)*-$\hat{g}$-closed but not conversely [2].*

10. *Every (1,2)*-$\hat{g}$-closed set is (1,2)*-sg-closed but not conversely [2].*

3. $\tilde{g}$(1,2)*-closed Sets

We introduce the following definitions.

Definition 3.1. *A subset A of a bitopological space X is called*

1. *(1,2)*-$\ddot{g}$-closed set if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-sg-open in X. The complement of (1,2)*-$\ddot{g}$-closed set is called (1,2)*-$\ddot{g}$-open set.*

2. *$\tilde{g}$(1,2)*-closed if (1,2)*-αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-$\hat{g}$-open in X. The collection of all (1,2)*-$\ddot{g}$-closed (resp. $\tilde{g}$(1,2)*-closed) sets in X is denoted by (1,2)*-$\ddot{G}C(X)$ (resp. (1,2)*- $\tilde{G}C(X)$).*

Proposition 3.2. *Every $\tau_{1,2}$-closed set is (1,2)*-$\ddot{g}$-closed.*

Proof. If A is a $\tau_{1,2}$-closed subset of X and G is any (1,2)*-sg-open set containing A, then G $\supseteq$ A = $\tau_{1,2}$-cl(A). Hence A is (1,2)*-$\ddot{g}$-closed in X. □

The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3. *Let X = $\{a, b, c\}$, τ_1 = $\{\emptyset, X, \{a, b\}\}$ and τ_2 = $\{\emptyset, X, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{a, b\}, \{b, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{a\}, \{c\}\}$ are called $\tau_{1,2}$-closed. Then (1,2)*-$\ddot{G}C(X)$ = $\{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$. Clearly, the set $\{a, c\}$ is a (1,2)*-$\ddot{g}$-closed but it is not a $\tau_{1,2}$-closed set in X.*

Proposition 3.4. *Every (1,2)*-$\ddot{g}$-closed set is (1,2)*-$\ddot{g}_\alpha$-closed.*

Proof. If A is a (1,2)*-$\ddot{g}$-closed subset of X and G is any (1,2)*-sg-open set containing A, then G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*-αcl(A). Hence A is (1,2)*-$\ddot{g}_\alpha$-closed in X. □

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5. *Let X = $\{a, b, c\}$, τ_1 = $\{\emptyset, X, \{b\}\}$ and τ_2 = $\{\emptyset, X\}$. Then the sets in $\{\emptyset, X, \{b\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{a, c\}\}$ are called $\tau_{1,2}$-closed. Then (1,2)*-$\ddot{G}C(X)$ = $\{\emptyset, \{a, c\}, X\}$ and (1,2)*-$\ddot{G}_\alpha C(X)$ = $\{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$. Clearly, the set $\{a\}$ is an (1,2)*-$\ddot{g}_\alpha$-closed but not a (1,2)*-$\ddot{g}$-closed set in X.*

Proposition 3.6. *Every (1,2)*-$\ddot{g}$-closed set is (1,2)*-ψ-closed.*

Proof. If A is a (1,2)*-$\ddot{g}$-closed subset of X and G is any (1,2)*-sg-open set containing A, then G $\supseteq \tau_{1,2}$-cl(A) $\supseteq$ (1,2)*-scl(A). Hence A is (1,2)*-ψ-closed in X. □

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then the sets in $\{\emptyset, X, \{a\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{b, c\}\}$ are called $\tau_{1,2}$-closed. Then (1,2)*-$\ddot{G}C(X) = \{\emptyset, \{b, c\}, X\}$ and (1,2)*-$\psi C(X) = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$. Clearly, the set $\{b\}$ is a (1,2)*-ψ-closed but not a (1,2)*-$\ddot{g}$-closed set in X.*

Proposition 3.8. *Every (1,2)*-$\ddot{g}$-closed set is (1,2)*-$\hat{g}$-closed.*

Proof. Suppose that A $\subseteq$ G and G is (1,2)*-semi-open in X. Since every (1,2)*-semi-open set is (1,2)*-sg-open and A is (1,2)*-$\ddot{g}$-closed, therefore $\tau_{1,2}$-cl(A) $\subseteq$ G. Hence A is (1,2)*-$\hat{g}$-closed in X. □

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9. *Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{d\}, \{b, c, d\}\}$ and $\tau_2 = \{\emptyset, X, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{d\}, \{b, c\}, \{b, c, d\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{a\}, \{a, d\}, \{a, b, c\}\}$ are called $\tau_{1,2}$-closed. Clearly, the set $\{a, c, d\}$ is a (1,2)*-$\hat{g}$-closed but not a (1,2)*-$\ddot{g}$ -closed set in X.*

Proposition 3.10. *Every (1,2)*-α-closed set is (1,2)*-$\ddot{g}_\alpha$-closed.*

Proof. If A is an (1,2)*-α-closed subset of X and G is any (1,2)*-sg-open set containing A, we have (1,2)*-αcl(A) = A $\subseteq$ G. Hence A is (1,2)*-$\ddot{g}_\alpha$-closed in X. □

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$ and $\tau_2 = \{\emptyset, X\}$. Then the sets in $\{\emptyset, X, \{a, b\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{c\}\}$ are called $\tau_{1,2}$-closed. Then (1,2)*-$\alpha C(X) = \{\emptyset, \{c\}, X\}$ and (1,2)*-$\ddot{G}_\alpha C(X) = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}$. Clearly, the set $\{a, c\}$ is an (1,2)*-$\ddot{g}_\alpha$-closed but not an (1,2)*-α-closed set in X.*

Remark 3.12. *(1,2)*-$\hat{g}$-closed set is different from $\tilde{g}$(1,2)*-closed.*

Example 3.13.

1. *) Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, \{a, b\}, X\}$. Then $\{b\}$ is $\tilde{g}$(1,2)*-closed set but not (1,2)*-$\hat{g}$ -closed.*
2. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, \{b, c\}, X\}$. Then $\{b\}$ is (1,2)*-$\hat{g}$ -closed set but not $\tilde{g}$(1,2)*-closed.*

Proposition 3.14. *Every (1,2)*-$\ddot{g}$-closed set is (1,2)*-g-closed.*

Proof. If A is a (1,2)*-$\ddot{g}$-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is (1,2)*-sg-open, we have G $\supseteq \tau_{1,2}$-cl(A). Hence A is (1,2)*-g-closed in X. □

The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b, c\}\}$ are called both $\tau_{1,2}$-open and $\tau_{1,2}$-closed. Then (1,2)*-$\ddot{G}C(X) = \{\emptyset, \{a\}, \{b, c\}, X\}$ and (1,2)*-$GC(X) = P(X)$. Clearly, the set $\{a, b\}$ is a (1,2)*-g-closed but not a (1,2)*-$\ddot{g}$-closed set in X.*

Proposition 3.16. *Every $\tilde{g}$(1,2)*-closed set is (1,2)*-αg-closed.*

Proof. If A is a $\tilde{g}$(1,2)*-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is (1,2)*-$\hat{g}$-open, we have (1,2)*-αcl(A) $\subseteq$U. Hence A is (1,2)*-αg-closed in X. □

The converse of Proposition 3.16 need not be true as seen from the following example.

Example 3.17. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ *and* $\tau_2 = \{\emptyset, \{b, c\}, X\}$. *Then* $\{a, c\}$ *is (1,2)*-*α*g-closed set but not* $\tilde{g}$*(1,2)*-closed.*

Proposition 3.18. *Every (1,2)*-*$\ddot{g}$*-closed set is (1,2)*-*α*g-closed.*

Proof. If A is a (1,2)*-$\ddot{g}$-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is (1,2)*-sg-open, we have G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*-αcl(A). Hence A is (1,2)*-αg-closed in X. □

The converse of Proposition 3.18 need not be true as seen from the following example.

Example 3.19. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$ *and* $\tau_2 = \{\emptyset, X, \{c\}\}$. *Then the sets in* $\{\emptyset, X, \{c\}, \{a, b\}\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\emptyset, X, \{c\}, \{a, b\}\}$ *are called* $\tau_{1,2}$*-closed. Then (1,2)*-*$\ddot{G}C(X) = \{\emptyset, \{c\}, \{a, b\}, X\}$*. and (1,2)*-*$\alpha GC(X) = P(X)$*. Clearly, the set* $\{a, c\}$ *is an (1,2)*-*α*g-closed but not a (1,2)*-*$\ddot{g}$*-closed set in X.*

Proposition 3.20. *Every (1,2)*-*$\ddot{g}$*-closed set is (1,2)*-gs-closed.*

Proof. If A is a (1,2)*-$\ddot{g}$-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is (1,2)*-sg-open, we have G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*-scl(A). Hence A is (1,2)*-gs-closed in X. □

The converse of Proposition 3.20 need not be true as seen from the following example.

Example 3.21. *In Example 3.7, we have (1,2)*-*$\ddot{G}C(X) = \{\emptyset, \{b, c\}, X\}$ *and (1,2)*-*$GSC(X) = \{\emptyset, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$*. Clearly, the set* $\{c\}$ *is a (1,2)*-gs-closed but not a (1,2)*-*$\ddot{g}$*-closed set in X.*

Proposition 3.22. *Every (1,2)*-*$\ddot{g}$*-closed set is (1,2)*-sg-closed.*

Proof. If A is a (1,2)*-$\ddot{g}$-closed subset of X and G is any (1,2)*-semi-open set containing A, since every (1,2)*-semi-open set is (1,2)*-sg-open, we have G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*-scl(A). Hence A is (1,2)*-sg-closed in X. □

The converse of Proposition 3.22 need not be true as seen from the following example.

Example 3.23. *In Example 3.7, we have (1,2)*-*$\ddot{G}C(X) = \{\emptyset, \{b, c\}, X\}$ *and (1,2)*-*$SGC(X) = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$*. Clearly, the set* $\{b\}$ *is a (1,2)*-sg-closed but not a (1,2)*-*$\ddot{g}$*-closed set in X.*

Proposition 3.24. *Every (1,2)*-*$\ddot{g}_\alpha$*-closed set is* $\tilde{g}$*(1,2)*-closed.*

Proof. If A is an (1,2)*-$\ddot{g}_\alpha$-closed subset of X and G is any (1,2)*-$\hat{g}$-open set containing A, since every (1,2)*-$\hat{g}$-open set is (1,2)*-sg-open, we have (1,2)*-αcl(A) $\subseteq$ G. Hence A is $\tilde{g}$(1,2)*-closed in X. □

The converse of Proposition 3.24 need not be true as seen from the following example.

Example 3.25. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ *and* $\tau_2 = \{\emptyset, \{a\}, \{b, c\}, X\}$. *Then* $\{c\}$ *is* $\tilde{g}$*(1,2)*-closed set but not (1,2)*-*$\ddot{g}_\alpha$*-closed.*

Proposition 3.26. *Every (1,2)*-*α*-closed set is* $\tilde{g}$*(1,2)*-closed.*

Proof. If A is an (1,2)*-α-closed subset of X and G is any (1,2)*-$\hat{g}$-open set containing A, we have (1,2)*-αcl(A) = A $\subseteq$ G. Hence A is $\tilde{g}$(1,2)*-closed in X. □

The converse of Proposition 3.26 need not be true as seen from the following example.

Example 3.27. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a, b\}\}$ *and* $\tau_2 = \{\emptyset, \{a, c\}, X\}$. *Then* $\{b, c\}$ *is* $\tilde{g}$*(1,2)*-closed set but not (1,2)*-*α*-closed.*

Proposition 3.28. *Every (1,2)*-*ψ*-closed set is (1,2)*-sg-closed.*

Proof. Suppose that A $\subseteq$ G and G is (1,2)*-semi-open in X. Since every (1,2)*-semi-open set is (1,2)*-sg-open and A is (1,2)*-ψ-closed, therefore (1,2)*-scl(A) $\subseteq$ G. Hence A is (1,2)*-sg-closed in X. □

The converse of Proposition 3.28 need not be true as seen from the following example.

Example 3.29. *In Example 3.15, we have (1,2)*-*$\psi C(X) = \{\emptyset, X, \{a\}, \{b, c\}\}$ *and (1,2)*-*$SGC(X) = P(X)$. *Clearly, the set* $\{a, b\}$ *is a (1,2)*-sg-closed but not a (1,2)*-*ψ*-closed set in X.*

Proposition 3.30. *Every (1,2)*-*$\ddot{g}$*-closed set is (1,2)*-gsp-closed.*

Proof. If A is a (1,2)*-$\ddot{g}$-closed subset of X and G is any $\tau_{1,2}$-open set containing A, since every $\tau_{1,2}$-open set is (1,2)*-sg-open, we have G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*-βcl(A). Hence A is (1,2)*-gsp-closed in X. □

The converse of Proposition 3.30 need not be true as seen from the following example.

Example 3.31. *In Example 3.5, we have (1,2)*-*$\ddot{G}C(X) = \{\emptyset, \{a, c\}, X\}$ *and (1,2)*-*$GSPC(X) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. *Clearly, the set* $\{c\}$ *is a (1,2)*-gsp-closed but not a (1,2)*-*$\ddot{g}$*-closed set in X.*

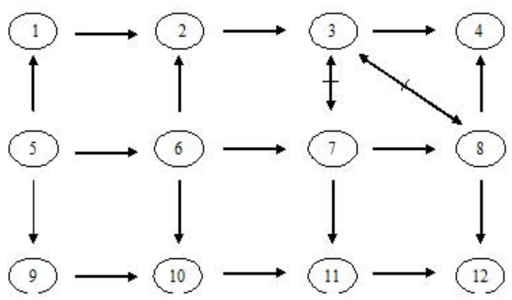
Proposition 3.32. *Every (1,2)*-*$\hat{g}$*-closed set is (1,2)*-sg-closed.*

Proof. If A is a (1,2)*-$\hat{g}$-closed subset of X and G is any (1,2)*-semi-open set containing A, then G $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ (1,2)*-scl(A). Hence A is (1,2)*-sg-closed in X. □

The converse of Proposition 3.32 need not be true as seen from the following example.

Example 3.33. *In Example 3.9, we have (1,2)*-*$\hat{G}C(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\}$ *and (1,2)*-*$SGC(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\}$. *Clearly, the set* $\{b\}$ *is a (1,2)*-sg-closed but not a (1,2)*-*$\hat{g}$*-closed set in X.*

Remark 3.34. *From the above Propositions, Examples and Remark, we obtain the following diagram, where* $A \to B$ *(resp.* $A \nleftrightarrow B$*) represents A implies B but not conversely (resp. A and B are independent of each other).*



where

(1) (1,2)-*α*-closed* *(7) (1,2)*-*$\hat{g}$*-closed*

(2) (1,2)-$\ddot{g}_\alpha$-closed*

(3) $\tilde{g}$(1,2)-closed*

(4) (1,2)-αg-closed*

(5) $\tau_{1,2}$-closed

(6) (1,2)-$\ddot{g}$-closed*

(8) (1,2)- g-closed*

(9) (1,2)-semi-closed*

(10) (1,2)-ψ-closed*

(11) (1,2)-sg-closed*

(12) (1,2)-gs-closed*

Remark 3.35. *The concepts of $\tilde{g}$(1,2)*-closed sets and (1,2)*-g-closed sets are independent.*

Example 3.36.

1. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}, \{b, c\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{a, c\}\}$. Then $\{a,b\}$ is (1,2)*-g-closed set but it is not $\tilde{g}$(1,2)*-closed set.*

2. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{a, b\}\}$. Then $\{b\}$ is $\tilde{g}$(1,2)*-closed set but it is not (1,2)*-g-closed set.*

4. Properties of $\tilde{g}$(1,2)*-closed Sets

Definition 4.1. *The intersection of all (1,2)*-$\hat{g}$-open subsets of X containing A is called the (1,2)*-$\hat{g}$-kernel of A and denoted by (1,2)*-$\hat{g}$-ker(A).*

Lemma 4.2. *A subset A of a bitopological space X is $\tilde{g}$(1,2)*-closed if and only if (1,2)*-αcl(A) $\subseteq$ (1,2)*-$\hat{g}$-ker(A).*

Proof. Suppose that A is $\tilde{g}$(1,2)*-closed. Then (1,2)*-αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-$\hat{g}$-open. Let x $\in$ (1,2)*-αcl(A). If x $\notin$ (1,2)*-$\hat{g}$-ker(A), then there is a (1,2)*-$\hat{g}$-open set U containing A such that x $\notin$ U. Since U is a (1,2)*-$\hat{g}$-open set containing A, we have x $\notin$ (1,2)*-αcl(A) and this is a contradiction.
Conversely, let (1,2)*-αcl(A) $\subseteq$ (1,2)*-$\hat{g}$-ker(A). If U is any (1,2)*-$\hat{g}$-open set containing A, then (1,2)*-αcl(A) $\subseteq$ (1,2)*-$\hat{g}$-ker(A) $\subseteq$ U. Therefore, A is $\tilde{g}$(1,2)*-closed. □

Remark 4.3. *Union of any two $\tilde{g}$(1,2)*-closed sets in X need not be a $\tilde{g}$(1,2)*-closed set as seen from the following example.*

Example 4.4. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{a, c\}\}$ and $\tau_2 = \{\emptyset, X, \{b\}, \{b, c\}\}$. Then the sets in $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ are called $\tau_{1,2}$-open and the sets in $\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$ are called $\tau_{1,2}$-closed. Then (1,2)*- $\tilde{G}C(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$. Clearly, the sets $\{a\}$ and $\{b\}$ are $\tilde{g}$(1,2)*-closed but their union $\{a, b\}$ is not a $\tilde{g}$(1,2)*-closed set in X.*

Proposition 4.5. *If a set A is $\tilde{g}$(1,2)*-closed in X then (1,2)*-$\alpha cl(A) \setminus A$ contains no nonempty $\tau_{1,2}$-closed set in X.*

Proof. Suppose that A is $\tilde{g}$(1,2)*-closed. Let F be a $\tau_{1,2}$-closed subset of (1,2)*-α $cl(A) \setminus A$. Then A $\subseteq$ F^c. But A is $\tilde{g}$(1,2)*-closed, therefore (1,2)*-αcl(A) $\subseteq$ F^c. Consequently, F $\subseteq$ $((1,2)^*\text{-}\alpha cl(A))^c$. We already have F $\subseteq$ (1,2)*-αcl(A). Thus F $\subseteq$ (1,2)*-αcl(A) $\cap$ $((1,2)^*\text{-}\alpha cl(A))^c$ and F is empty. □

The converse of Proposition 4.5 need not be true as seen from the following example.

Example 4.6. *Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b, c\}\}$. If $A = \{b\}$, then (1,2)*-$\alpha cl(A) \setminus A$ does not contain any nonempty $\tau_{1,2}$-closed set. But A is not a $\tilde{g}$(1,2)*-closed set in X.*

Theorem 4.7. *If a set A is $\tilde{g}$(1,2)*-closed in X then (1,2)*-$\alpha cl(A) - A$ contains no nonempty (1,2)*-$\hat{g}$-closed set.*

Proof. Suppose that A is $\tilde{g}$(1,2)*-closed. Let S be a (1,2)*-$\hat{g}$-closed subset of (1,2)*-αcl(A) $-$ A. Then A $\subseteq$ S^c. Since A is $\tilde{g}$(1,2)*-closed, we have (1,2)*-αcl(A) $\subseteq$ S^c. Consequently, S $\subseteq$ ((1,2)*-αcl(A))^c. Hence, S $\subseteq$ (1,2)*-αcl(A) $\cap$ ((1,2)*-αcl(A))^c = $\emptyset$. Therefore S is empty. □

Theorem 4.8. *If A is $\tilde{g}$(1,2)*-closed in X and A $\subseteq$ B $\subseteq$ (1,2)*-αcl(A), then B is $\tilde{g}$(1,2)*-closed in X.*

Proof. Let B $\subseteq$ U where U is (1,2)*-$\hat{g}$-open set in X. Then A $\subseteq$ U. Since A is $\tilde{g}$(1,2)*-closed, (1,2)*-αcl(A) $\subseteq$ U. Since B $\subseteq$ (1,2)*-αcl(A), (1,2)*-αcl(B) $\subseteq$ (1,2)*-αcl(A). Therefore (1,2)*-αcl(B) $\subseteq$ U and B is $\tilde{g}$(1,2)*-closed in X. □

Proposition 4.9. *If A is a (1,2)*-$\hat{g}$-open and $\tilde{g}$(1,2)*-closed in X, then A is (1,2)*-α-closed in X.*

Proof. Since A is (1,2)*-$\hat{g}$-open and $\tilde{g}$(1,2)*-closed, (1,2)*-αcl(A) $\subseteq$ A and hence A is (1,2)*-α-closed in X. □

Proposition 4.10. *For each $x \in X$, either $\{x\}$ is (1,2)*-$\hat{g}$-closed or $\{x\}^c$ is $\tilde{g}$(1,2)*-closed in X.*

Proof. Suppose that {x} is not (1,2)*-$\hat{g}$-closed in X. Then {x}^{c} is not (1,2)*-$\hat{g}$-open and the only (1,2)*-$\hat{g}$-open set containing {x}^{c} is the space X itself. Therefore (1,2)*-αcl({x}^{c}) $\subseteq$ X and so {x}^{c} is $\tilde{g}$(1,2)*-closed in X. □

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