



# Weakly $(1,2)^*-g^*$ -closed Sets

Research Article

K.M.Dharmalingam<sup>1</sup>, G.Thanavalli<sup>2</sup> and O.Ravi<sup>3\*</sup>

1 Department of Mathematics, The Madura College, Madurai, Tamil Nadu, India.

2 Department of Mathematics, SBM College of Engineering &amp; Technology, Thamaraipadi, Dindigul, Tamil Nadu, India.

3 Department of Mathematics, P.M.Thevar College, Usilampatti, Tamil Nadu, India.

**Abstract:** The aim of this paper is to introduce a new class of  $(1,2)^*$ -generalized closed sets called weakly  $(1,2)^*-g^*$ -closed sets.

**MSC:** 54E55.

**Keywords:**  $(1,2)^*-g^*$ -closed set,  $(1,2)^*-wg^*$ -closed set,  $(1,2)^*-g^*$ -continuous map,  $(1,2)^*-g^*$ -irresolute map, weakly  $(1,2)^*-g^*$ -open map, weakly  $(1,2)^*-g^*$ -continuous map.

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## 1. Introduction

Thamilisai [21] studied and investigated the properties of the notion of  $(1,2)^*-g^*$ -closed sets. In this paper, we introduce a new class of  $(1,2)^*$ -generalized closed sets called weakly  $(1,2)^*-g^*$ -closed sets which contains the above mentioned class. Also, we investigate the relationships among the related  $(1,2)^*$ -generalized closed sets.

## 2. Preliminaries

Throughout this paper,  $X, Y$  and  $Z$  denote bitopological spaces  $(X, \tau_1, \tau_2)$ ,  $(Y, \sigma_1, \sigma_2)$  and  $(Z, \eta_1, \eta_2)$  respectively.

**Definition 2.1.** Let  $A$  be a subset of a bitopological space  $X$ . Then  $A$  is called  $\tau_{1,2}$ -open [16] if  $A = P \cup Q$ , for some  $P \in \tau_1$  and  $Q \in \tau_2$ . The complement of  $\tau_{1,2}$ -open set is called  $\tau_{1,2}$ -closed. The family of all  $\tau_{1,2}$ -open (resp.  $\tau_{1,2}$ -closed) sets of  $X$  is denoted by  $(1,2)^*-O(X)$  (resp.  $(1,2)^*-C(X)$ ).

**Definition 2.2** ([16]). Let  $A$  be a subset of a bitopological space  $X$ . Then

1. the  $\tau_{1,2}$ -interior of  $A$ , denoted by  $\tau_{1,2}\text{-int}(A)$ , is defined by  $\cup \{ U : U \subseteq A \text{ and } U \text{ is } \tau_{1,2}\text{-open} \}$ ;
2. the  $\tau_{1,2}$ -closure of  $A$ , denoted by  $\tau_{1,2}\text{-cl}(A)$ , is defined by  $\cap \{ U : A \subseteq U \text{ and } U \text{ is } \tau_{1,2}\text{-closed} \}$ .

**Remark 2.3** ([16]). Notice that  $\tau_{1,2}$ -open subsets of  $X$  need not necessarily form a topology.

**Definition 2.4.** Let  $A$  be a subset of a bitopological space  $X$ . Then  $A$  is called

\* E-mail: siingam@yahoo.com

1.  $(1,2)^*$ - $\alpha$ -open set [16] if  $A \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)))$ . The complement of  $(1,2)^*$ - $\alpha$ -open set is  $(1,2)^*$ - $\alpha$ -closed. The  $(1,2)^*$ - $\alpha$ -closure [18] of a subset  $A$  of  $X$ , denoted by  $(1,2)^*\text{-}\alpha\text{cl}(A)$ , is defined to be the intersection of all  $(1,2)^*$ - $\alpha$ -closed sets of  $X$  containing  $A$ . It is known that  $(1,2)^*\text{-}\alpha\text{cl}(A)$  is  $(1,2)^*$ - $\alpha$ -closed set.
2. regular  $(1,2)^*$ -open set [19] if  $A = \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$ . The complement of regular  $(1,2)^*$ -open set is regular  $(1,2)^*$ -closed.
3.  $(1,2)^*$ - $\pi$ -open [12] if the finite union of regular  $(1,2)^*$ -open sets.
4.  $(1,2)^*$ -semi-closed [16] if  $\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)) \subseteq A$ .
5.  $(1,2)^*$ -semi-open [16] if  $A \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))$ .

**Definition 2.5.** Let  $A$  be a subset of a bitopological space  $X$ . Then  $A$  is called

1. a  $(1,2)^*$ -generalized closed (briefly,  $(1,2)^*$ - $g$ -closed) set [17] if  $\tau_{1,2}\text{-cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_{1,2}$ -open in  $X$ . The complement of  $(1,2)^*$ - $g$ -closed set is called  $(1,2)^*$ - $g$ -open set.
2.  $(1,2)^*$ - $g^*$ -closed set [21] if  $\tau_{1,2}\text{-cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $(1,2)^*$ - $g$ -open in  $X$ . The complement of  $(1,2)^*$ - $g^*$ -closed set is called  $(1,2)^*$ - $g^*$ -open set. The family of all  $(1,2)^*$ - $g^*$ -open sets of  $X$  is denoted by  $(1,2)^*\text{-}G^*O(X)$ .

**Definition 2.6.** A function  $f : X \rightarrow Y$  is called:

1.  $(1,2)^*$ -continuous [16] if  $f^{-1}(V)$  is a  $\tau_{1,2}$ -closed set in  $X$  for every  $\sigma_{1,2}$ -closed set  $V$  of  $Y$ .
2. perfectly  $(1,2)^*$ -continuous [20] if  $f^{-1}(V)$  is  $\tau_{1,2}$ -clopen in  $X$  for every regular  $(1,2)^*$ -open set  $V$  of  $Y$ .
3.  $(1,2)^*$ - $R$ -map [17] if  $f^{-1}(V)$  is regular  $(1,2)^*$ -open in  $X$  for every regular  $(1,2)^*$ -open set  $V$  of  $Y$ .
4.  $(1,2)^*$ -open [16] if  $f(V)$  is  $\sigma_{1,2}$ -open in  $Y$  for every  $\tau_{1,2}$ -open set  $V$  of  $X$ .
5.  $(1,2)^*$ -closed [16] if  $f(V)$  is  $\sigma_{1,2}$ -closed in  $Y$  for every  $\tau_{1,2}$ -closed set  $V$  of  $X$ .

**Definition 2.7** ([15]). A subset  $A$  of a bitopological space  $X$  is called:

1. a weakly  $(1,2)^*$ - $g$ -closed (briefly,  $(1,2)^*$ - $wg$ -closed) set if  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_{1,2}$ -open in  $X$ .
2. a weakly  $(1,2)^*$ - $\pi g$ -closed (briefly,  $(1,2)^*$ - $w\pi g$ -closed) set if  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $(1,2)^*$ - $\pi$ -open in  $X$ .
3. a regular  $(1,2)^*$ -weakly generalized closed (briefly,  $(1,2)^*$ - $rwg$ -closed) set if  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular  $(1,2)^*$ -open in  $X$ .

### 3. Weakly $(1,2)^*-g^*$ -closed Sets

We introduce the definition of weakly  $(1,2)^*-g^*$ -closed sets in bitopological spaces and study the relationships of such sets.

**Definition 3.1.** A subset  $A$  of a bitopological space  $X$  is called a weakly  $(1,2)^*-g^*$ -closed (briefly,  $(1,2)^*$ - $wg^*$ -closed) set if  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $(1,2)^*$ - $g$ -open in  $X$ .

**Theorem 3.2.** Every  $(1,2)^*-g^*$ -closed set is  $(1,2)^*-wg^*$ -closed but not conversely.

**Example 3.3.** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\phi, X\}$  and  $\tau_2 = \{\phi, \{a, b\}, X\}$ . Then the sets in  $\{\phi, \{a, b\}, X\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, \{c\}, X\}$  are called  $\tau_{1,2}$ -closed. Then the set  $\{a\}$  is  $(1,2)^*$ -wg\*-closed set but it is not a  $(1,2)^*$ -g\*-closed in  $X$ .

**Theorem 3.4.** Every  $(1,2)^*$ -wg\*-closed set is  $(1,2)^*$ -wg-closed but not conversely.

*Proof.* Let  $A$  be any  $(1,2)^*$ -wg\*-closed set and  $U$  be any  $\tau_{1,2}$ -open set containing  $A$ . Then  $U$  is a  $(1,2)^*$ -g-open set containing  $A$ . We have  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subseteq U$ . Thus,  $A$  is  $(1,2)^*$ -wg-closed.  $\square$

**Example 3.5.** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\phi, X\}$  and  $\tau_2 = \{\phi, \{a\}, X\}$ . Then the sets in  $\{\phi, \{a\}, X\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, \{b, c\}, X\}$  are called  $\tau_{1,2}$ -closed. Then the set  $\{a, b\}$  is  $(1,2)^*$ -wg-closed but it is not a  $(1,2)^*$ -wg\*-closed.

**Theorem 3.6.** Every  $(1,2)^*$ -wg\*-closed set is  $(1,2)^*$ -w $\pi$ g-closed but not conversely.

*Proof.* Let  $A$  be any  $(1,2)^*$ -wg\*-closed set and  $U$  be any  $(1,2)^*$ - $\pi$ -open set containing  $A$ . Then  $U$  is a  $(1,2)^*$ -g-open set containing  $A$ . We have  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subseteq U$ . Thus,  $A$  is  $(1,2)^*$ -w $\pi$ g-closed.  $\square$

**Example 3.7.** In Example 3.5, the set  $\{a, c\}$  is  $(1,2)^*$ -w $\pi$ g-closed but it is not a  $(1,2)^*$ -wg\*-closed.

**Theorem 3.8.** Every  $(1,2)^*$ -wg\*-closed set is  $(1,2)^*$ -rwg-closed but not conversely.

*Proof.* Let  $A$  be any  $(1,2)^*$ -wg\*-closed set and  $U$  be any regular  $(1,2)^*$ -open set containing  $A$ . Then  $U$  is a  $(1,2)^*$ -g-open set containing  $A$ . We have  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subseteq U$ . Thus,  $A$  is  $(1,2)^*$ -rwg-closed.  $\square$

**Example 3.9.** In Example 3.5, the set  $\{a\}$  is  $(1,2)^*$ -rwg-closed but it is not a  $(1,2)^*$ -wg\*-closed.

**Theorem 3.10.** If a subset  $A$  of a bitopological space  $X$  is both  $\tau_{1,2}$ -closed and  $(1,2)^*$ -g-closed, then it is  $(1,2)^*$ -wg\*-closed in  $X$ .

*Proof.* Let  $A$  be a  $(1,2)^*$ -g-closed set in  $X$  and  $U$  be any  $\tau_{1,2}$ -open set containing  $A$ . Then  $U \supseteq \tau_{1,2}\text{-cl}(A) \supseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)))$ . Since  $A$  is  $\tau_{1,2}$ -closed,  $U \supseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))$  and hence  $(1,2)^*$ -wg\*-closed in  $X$ .  $\square$

**Theorem 3.11.** If a subset  $A$  of a bitopological space  $X$  is both  $\tau_{1,2}$ -open and  $(1,2)^*$ -wg\*-closed, then it is  $\tau_{1,2}$ -closed.

*Proof.* Since  $A$  is both  $\tau_{1,2}$ -open and  $(1,2)^*$ -wg\*-closed,  $A \supseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) = \tau_{1,2}\text{-cl}(A)$  and hence  $A$  is  $\tau_{1,2}$ -closed in  $X$ .  $\square$

**Corollary 3.12.** If a subset  $A$  of a bitopological space  $X$  is both  $\tau_{1,2}$ -open and  $(1,2)^*$ -wg\*-closed, then it is both regular  $(1,2)^*$ -open and regular  $(1,2)^*$ -closed in  $X$ .

**Theorem 3.13.** Let  $X$  be a bitopological space and  $A \subseteq X$  be  $\tau_{1,2}$ -open. Then,  $A$  is  $(1,2)^*$ -wg\*-closed if and only if  $A$  is  $(1,2)^*$ -g\*-closed.

*Proof.* Let  $A$  be  $(1,2)^*$ -g\*-closed. By Theorem 3.2, it is  $(1,2)^*$ -wg\*-closed. Conversely, let  $A$  be  $(1,2)^*$ -wg\*-closed. Since  $A$  is  $\tau_{1,2}$ -open, by Theorem 3.11,  $A$  is  $\tau_{1,2}$ -closed. Hence  $A$  is  $(1,2)^*$ -g\*-closed.  $\square$

**Theorem 3.14.** If a set  $A$  of  $X$  is  $(1,2)^*$ -wg\*-closed, then  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) - A$  contains no non-empty  $(1,2)^*$ -g-closed set.

*Proof.* Let  $F$  be a  $(1,2)^*$ -g-closed set such that  $F \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) - A$ . Since  $F^c$  is  $(1,2)^*$ -g-open and  $A \subseteq F^c$ , from the definition of  $(1,2)^*$ -wg\*-closedness it follows that  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) \subseteq F^c$ . i.e.,  $F \subseteq (\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)))^c$ . This implies that  $F \subseteq (\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A))) \cap (\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)))^c = \phi$ .  $\square$

**Theorem 3.15.** *If a subset  $A$  of a bitopological space  $X$  is (1,2)\*-nowhere dense, then it is (1,2)\*-wg\*-closed.*

*Proof.* We know that a subset  $A$  of  $X$  is (1,2)\*-nowhere dense if  $\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)) = \emptyset$ . Since  $\tau_{1,2}\text{-int}(A) \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$  and  $A$  is (1,2)\*-nowhere dense,  $\tau_{1,2}\text{-int}(A) = \emptyset$ . Therefore  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)) = \emptyset$  and hence  $A$  is (1,2)\*-wg\*-closed in  $X$ .  $\square$

The converse of Theorem 3.15 need not be true as seen in the following example.

**Example 3.16.** *Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, \{a\}, X\}$  and  $\tau_2 = \{\emptyset, \{b, c\}, X\}$ . Then the sets in  $\{\emptyset, \{a\}, \{b, c\}, X\}$  are called  $\tau_{1,2}$ -open and  $\tau_{1,2}$ -closed. Then the set  $\{a\}$  is (1,2)\*-wg\*-closed set but not (1,2)\*-nowhere dense in  $X$ .*

**Remark 3.17.** *The following examples show that (1,2)\*-wg\*-closedness and (1,2)\*-semi-closedness are independent.*

**Example 3.18.** *In Example 3.3, we have the set  $\{a, c\}$  is (1,2)\*-wg\*-closed set but not (1,2)\*-semi-closed in  $X$ .*

**Example 3.19.** *Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, \{a\}, X\}$  and  $\tau_2 = \{\emptyset, \{b\}, X\}$ . Then the sets in  $\{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}$  are called  $\tau_{1,2}$ -closed. Then the set  $\{a\}$  is (1,2)\*-semi-closed set but not (1,2)\*-wg\*-closed in  $X$ .*

**Remark 3.20.** *From the above discussions and known results, we obtain the following diagram for a subset of a bitopological space, where  $A \rightarrow B$  represents  $A$  implies  $B$  but not conversely.*

**Diagram**

$$\tau_{1,2}\text{-closed} \Rightarrow (1,2)^*\text{-wg}^*\text{-closed} \Rightarrow (1,2)^*\text{-wg-closed} \Rightarrow (1,2)^*\text{-w}\pi\text{g-closed} \Rightarrow (1,2)^*\text{-rwg-closed}$$

**Definition 3.21.** *A subset  $A$  of a bitopological space  $X$  is called (1,2)\*-wg\*-open set if  $A^c$  is (1,2)\*-wg\*-closed in  $X$ .*

**Proposition 3.22.** *Every (1,2)\*-g\*-open set is (1,2)\*-wg\*-open but not conversely.*

**Theorem 3.23.** *A subset  $A$  of a bitopological space  $X$  is (1,2)\*-wg\*-open if  $G \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$  whenever  $G \subseteq A$  and  $G$  is (1,2)\*-g-closed.*

*Proof.* Let  $A$  be any (1,2)\*-wg\*-open. Then  $A^c$  is (1,2)\*-wg\*-closed. Let  $G$  be a (1,2)\*-g-closed set contained in  $A$ . Then  $G^c$  is a (1,2)\*-g-open set containing  $A^c$ . Since  $A^c$  is (1,2)\*-wg\*-closed, we have  $\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A^c)) \subseteq G^c$ . Therefore  $G \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$ .

Conversely, we suppose that  $G \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$  whenever  $G \subseteq A$  and  $G$  is (1,2)\*-g-closed. Then  $G^c$  is a (1,2)\*-g-open set containing  $A^c$  and  $G^c \supseteq (\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)))^c$ . It follows that  $G^c \supseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A^c))$ . Hence  $A^c$  is (1,2)\*-wg\*-closed and so  $A$  is (1,2)\*-wg\*-open.  $\square$

**Definition 3.24.** *Let  $f : X \rightarrow Y$  be a function. Then  $f$  is said to be*

1. *contra (1,2)\*-g\*-continuous if the inverse image of every  $\sigma_{1,2}$ -open set in  $Y$  is (1,2)\*-g\*-closed set in  $X$ .*
2. *(1,2)\*-g\*-irresolute if the inverse image of every (1,2)\*-g\*-closed set in  $Y$  is (1,2)\*-g\*-closed set in  $X$ .*

**Theorem 3.25.** *The following are equivalent for a function  $f : X \rightarrow Y$ :*

1.  *$f$  is contra (1,2)\*-g\*-continuous.*
2. *the inverse image of every  $\sigma_{1,2}$ -closed set of  $Y$  is (1,2)\*-g\*-open in  $X$ .*

*Proof.* Let  $U$  be any  $\sigma_{1,2}$ -closed set of  $Y$ . Since  $Y \setminus U$  is  $\sigma_{1,2}$ -open, then by (1), it follows that  $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$  is (1,2)\*-g\*-closed. This shows that  $f^{-1}(U)$  is (1,2)\*-g\*-open in  $X$ . Converse is similar.  $\square$

## 4. Weakly $(1,2)^*-g^*$ -continuous Functions

**Definition 4.1.** Let  $X$  and  $Y$  be two bitopological spaces. A function  $f : X \rightarrow Y$  is called weakly  $(1,2)^*-g^*$ -continuous (briefly,  $(1,2)^*-wg^*$ -continuous) if  $f^{-1}(U)$  is a  $(1,2)^*-wg^*$ -open set in  $X$  for each  $\sigma_{1,2}$ -open set  $U$  of  $Y$ .

**Example 4.2.** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\phi, \{a\}, X\}$  and  $\tau_2 = \{\phi, \{b, c\}, X\}$ . Then the sets in  $\{\phi, \{a\}, \{b, c\}, X\}$  are called  $\tau_{1,2}$ -open and  $\tau_{1,2}$ -closed. Let  $Y = \{a, b, c\}$ ,  $\sigma_1 = \{\phi, Y\}$  and  $\sigma_2 = \{\phi, \{a\}, Y\}$ . Then the sets in  $\{\phi, \{a\}, Y\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, \{b, c\}, Y\}$  are called  $\sigma_{1,2}$ -closed. The function  $f : X \rightarrow Y$  defined by  $f(a) = b$ ,  $f(b) = c$  and  $f(c) = a$  is  $(1,2)^*-wg^*$ -continuous, because every  $\sigma_{1,2}$ -open subset of  $Y$  is  $(1,2)^*-wg^*$ -closed in  $Y$ .

**Theorem 4.3.** Every  $(1,2)^*-g^*$ -continuous function is  $(1,2)^*-wg^*$ -continuous.

*Proof.* It follows from Theorem 3.2. □

The converse of Theorem 4.3 need not be true as seen in the following example.

**Example 4.4.** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\phi, \{a\}, X\}$  and  $\tau_2 = \{\phi, \{b, c\}, X\}$ . Then the sets in  $\{\phi, \{a\}, \{b, c\}, X\}$  are called  $\tau_{1,2}$ -open and  $\tau_{1,2}$ -closed. Let  $Y = \{a, b, c\}$ ,  $\sigma_1 = \{\phi, Y\}$  and  $\sigma_2 = \{\phi, \{b\}, Y\}$ . Then the sets in  $\{\phi, \{b\}, Y\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, \{a, c\}, Y\}$  are called  $\sigma_{1,2}$ -closed. Let  $f : X \rightarrow Y$  be the identity function. Then  $f$  is  $(1,2)^*-wg^*$ -continuous but not  $(1,2)^*-g^*$ -continuous.

**Theorem 4.5.** A function  $f : X \rightarrow Y$  is  $(1,2)^*-wg^*$ -continuous if and only if  $f^{-1}(U)$  is a  $(1,2)^*-wg^*$ -closed set in  $X$  for each  $\sigma_{1,2}$ -closed set  $U$  of  $Y$ .

*Proof.* Let  $U$  be any  $\sigma_{1,2}$ -closed set of  $Y$ . According to the assumption  $f^{-1}(U^c) = X \setminus f^{-1}(U)$  is  $(1,2)^*-wg^*$ -open in  $X$ , so  $f^{-1}(U)$  is  $(1,2)^*-wg^*$ -closed in  $X$ . The converse can be proved in a similar manner. □

**Definition 4.6.** A bitopological space  $X$  is said to be locally  $(1,2)^*-g^*$ -indiscrete if every  $(1,2)^*-g^*$ -open set of  $X$  is  $\tau_{1,2}$ -closed in  $X$ .

**Theorem 4.7.** Let  $f : X \rightarrow Y$  be a function. If  $f$  is contra  $(1,2)^*-g^*$ -continuous and  $X$  is locally  $(1,2)^*-g^*$ -indiscrete, then  $f$  is  $(1,2)^*$ -continuous.

*Proof.* Let  $V$  be a  $\sigma_{1,2}$ -closed in  $Y$ . Since  $f$  is contra  $(1,2)^*-g^*$ -continuous,  $f^{-1}(V)$  is  $(1,2)^*-g^*$ -open in  $X$ . Since  $X$  is locally  $(1,2)^*-g^*$ -indiscrete,  $f^{-1}(V)$  is  $\tau_{1,2}$ -closed in  $X$ . Hence  $f$  is  $(1,2)^*$ -continuous. □

**Theorem 4.8.** Let  $f : X \rightarrow Y$  be a function. If  $f$  is contra  $(1,2)^*-g^*$ -continuous and  $X$  is locally  $(1,2)^*-g^*$ -indiscrete, then  $f$  is  $(1,2)^*-wg^*$ -continuous.

*Proof.* Let  $f : X \rightarrow Y$  be contra  $(1,2)^*-g^*$ -continuous and  $X$  is locally  $(1,2)^*-g^*$ -indiscrete. By Theorem 4.7,  $f$  is  $(1,2)^*$ -continuous, then  $f$  is  $(1,2)^*-wg^*$ -continuous. □

**Proposition 4.9.** If  $f : X \rightarrow Y$  is perfectly  $(1,2)^*$ -continuous and  $(1,2)^*-wg^*$ -continuous, then it is  $(1,2)^*$ -R-map.

*Proof.* Let  $V$  be any regular  $(1,2)^*$ -open subset of  $Y$ . According to the assumption,  $f^{-1}(V)$  is both  $\tau_{1,2}$ -open and  $\tau_{1,2}$ -closed in  $X$ . Since  $f^{-1}(V)$  is  $\tau_{1,2}$ -closed, it is  $(1,2)^*-wg^*$ -closed. We have  $f^{-1}(V)$  is both  $\tau_{1,2}$ -open and  $(1,2)^*-wg^*$ -closed. Hence, by Corollary 3.12, it is regular  $(1,2)^*$ -open in  $X$ , so  $f$  is  $(1,2)^*$ -R-map. □

**Definition 4.10.** A bitopological space  $X$  is called  $(1,2)^*-g^*$ -compact if every cover of  $X$  by  $(1,2)^*-g^*$ -open sets has finite subcover.

**Definition 4.11.** A bitopological space  $X$  is called weakly  $(1,2)^*-g^*$ -compact (briefly,  $(1,2)^*-wg^*$ -compact) if every  $(1,2)^*-wg^*$ -open cover of  $X$  has a finite subcover.

**Remark 4.12.** Every  $(1,2)^*-wg^*$ -compact space is  $(1,2)^*-g^*$ -compact.

**Theorem 4.13.** Let  $f : X \rightarrow Y$  be surjective  $(1,2)^*-wg^*$ -continuous function. If  $X$  is  $(1,2)^*-wg^*$ -compact, then  $Y$  is  $(1,2)^*$ -compact.

*Proof.* Let  $\{A_i : i \in I\}$  be an  $\sigma_{1,2}$ -open cover of  $Y$ . Then  $\{f^{-1}(A_i) : i \in I\}$  is a  $(1,2)^*-wg^*$ -open cover in  $X$ . Since  $X$  is  $(1,2)^*-wg^*$ -compact, it has a finite subcover, say  $\{f^{-1}(A_1), f^{-1}(A_2), \dots, f^{-1}(A_n)\}$ . Since  $f$  is surjective  $\{A_1, A_2, \dots, A_n\}$  is a finite subcover of  $Y$  and hence  $Y$  is  $(1,2)^*$ -compact.  $\square$

**Definition 4.14.** A bitopological space  $X$  is called weakly  $(1,2)^*-g^*$ -connected (briefly,  $(1,2)^*-wg^*$ -connected) if  $X$  cannot be written as the disjoint union of two non-empty  $(1,2)^*-wg^*$ -open sets.

**Definition 4.15.** A bitopological space  $X$  is called  $(1,2)^*-g^*$ -connected if  $X$  cannot be written as the disjoint union of two non-empty  $(1,2)^*-g^*$ -open sets.

**Definition 4.16.** A bitopological space  $X$  is called almost  $(1,2)^*$ -connected if  $X$  cannot be written as the disjoint union of two non-empty regular  $(1,2)^*$ -open sets.

**Theorem 4.17.** If a bitopological space  $X$  is  $(1,2)^*-wg^*$ -connected, then  $X$  is almost  $(1,2)^*$ -connected (resp.  $(1,2)^*-g^*$ -connected).

*Proof.* It follows from the fact that each regular  $(1,2)^*$ -open set (resp.  $(1,2)^*-g^*$ -open set) is  $(1,2)^*-wg^*$ -open.  $\square$

**Theorem 4.18.** For a bitopological space  $X$ , the following statements are equivalent:

1.  $X$  is  $(1,2)^*-wg^*$ -connected.
2. The empty set  $\phi$  and  $X$  are only subsets which are both  $(1,2)^*-wg^*$ -open and  $(1,2)^*-wg^*$ -closed.
3. Each  $(1,2)^*-wg^*$ -continuous function from  $X$  into a discrete space  $Y$  which has at least two points is a constant function.

*Proof.* (1)  $\Rightarrow$  (2). Let  $S \subseteq X$  be any proper subset, which is both  $(1,2)^*-wg^*$ -open and  $(1,2)^*-wg^*$ -closed. Its complement  $X \setminus S$  is also  $(1,2)^*-wg^*$ -open and  $(1,2)^*-wg^*$ -closed. Then  $X = S \cup (X \setminus S)$  is a disjoint union of two non-empty  $(1,2)^*-wg^*$ -open sets which is a contradiction with the fact that  $X$  is  $(1,2)^*-wg^*$ -connected. Hence,  $S = \phi$  or  $X$ .

(2)  $\Rightarrow$  (1). Let  $X = A \cup B$  where  $A \cap B = \phi$ ,  $A \neq \phi$ ,  $B \neq \phi$  and  $A, B$  are  $(1,2)^*-wg^*$ -open. Since  $A = X \setminus B$ ,  $A$  is  $(1,2)^*-wg^*$ -closed. According to the assumption  $A = \phi$ , which is a contradiction.

(2)  $\Rightarrow$  (3). Let  $f : X \rightarrow Y$  be a  $(1,2)^*-wg^*$ -continuous function where  $Y$  is a discrete bitopological space with at least two points. Then  $f^{-1}(\{y\})$  is  $(1,2)^*-wg^*$ -closed and  $(1,2)^*-wg^*$ -open for each  $y \in Y$  and  $X = \cup \{f^{-1}(\{y\}) : y \in Y\}$ . According to the assumption,  $f^{-1}(\{y\}) = \phi$  or  $f^{-1}(\{y\}) = X$ . If  $f^{-1}(\{y\}) = \phi$  for all  $y \in Y$ ,  $f$  will not be a function. Also there is no exist more than one  $y \in Y$  such that  $f^{-1}(\{y\}) = X$ . Hence, there exists only one  $y \in Y$  such that  $f^{-1}(\{y\}) = X$  and  $f^{-1}(\{y_1\}) = \phi$  where  $y \neq y_1 \in Y$ . This shows that  $f$  is a constant function.

(3)  $\Rightarrow$  (2). Let  $S \neq \phi$  be both  $(1,2)^*-wg^*$ -open and  $(1,2)^*-wg^*$ -closed in  $X$ . Let  $f : X \rightarrow Y$  be a  $(1,2)^*-wg^*$ -continuous function defined by  $f(S) = \{a\}$  and  $f(X \setminus S) = \{b\}$  where  $a \neq b$ . Since  $f$  is constant function we get  $S = X$ .  $\square$

**Theorem 4.19.** Let  $f : X \rightarrow Y$  be a  $(1,2)^*-wg^*$ -continuous surjective function. If  $X$  is  $(1,2)^*-wg^*$ -connected, then  $Y$  is  $(1,2)^*$ -connected.

*Proof.* We suppose that  $Y$  is not  $(1,2)^*$ -connected. Then  $Y = A \cup B$  where  $A \cap B = \phi$ ,  $A \neq \phi$ ,  $B \neq \phi$  and  $A, B$  are  $\sigma_{1,2}$ -open sets in  $Y$ . Since  $f$  is  $(1,2)^*$ - $wg^*$ -continuous surjective function,  $X = f^{-1}(A) \cup f^{-1}(B)$  are disjoint union of two non-empty  $(1,2)^*$ - $wg^*$ -open subsets. This is contradiction with the fact that  $X$  is  $(1,2)^*$ - $wg^*$ -connected.  $\square$

## 5. Weakly $(1,2)^*$ - $g^*$ -open and Weakly $(1,2)^*$ - $g^*$ -closed Functions

**Definition 5.1.** Let  $X$  and  $Y$  be bitopological spaces. A function  $f : X \rightarrow Y$  is called weakly  $(1,2)^*$ - $g^*$ -open (briefly,  $(1,2)^*$ - $wg^*$ -open) if  $f(V)$  is a  $(1,2)^*$ - $wg^*$ -open set in  $Y$  for each  $\tau_{1,2}$ -open set  $V$  of  $X$ .

**Remark 5.2.** Every  $(1,2)^*$ - $g^*$ -open function is  $(1,2)^*$ - $wg^*$ -open but not conversely.

**Example 5.3.** Let  $X = \{a, b, c, d\}$ ,  $\tau_1 = \{\phi, \{a\}, X\}$  and  $\tau_2 = \{\phi, \{a, b, d\}, X\}$ . Then the sets in  $\{\phi, \{a\}, \{a, b, d\}, X\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, \{c\}, \{b, c, d\}, X\}$  are called  $\tau_{1,2}$ -closed. Let  $Y = \{a, b, c, d\}$ ,  $\sigma_1 = \{\phi, \{a\}, Y\}$  and  $\sigma_2 = \{\phi, \{b, c\}, \{a, b, c\}, Y\}$ . Then the sets in  $\{\phi, \{a\}, \{b, c\}, \{a, b, c\}, Y\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, \{d\}, \{a, d\}, \{b, c, d\}, Y\}$  are called  $\sigma_{1,2}$ -closed. Let  $f : X \rightarrow Y$  be the identity function. Then  $f$  is  $(1,2)^*$ - $wg^*$ -open but not  $(1,2)^*$ - $g^*$ -open.

**Definition 5.4.** Let  $X$  and  $Y$  be bitopological spaces. A function  $f : X \rightarrow Y$  is called weakly  $(1,2)^*$ - $g^*$ -closed (briefly,  $(1,2)^*$ - $wg^*$ -closed) if  $f(V)$  is a  $(1,2)^*$ - $wg^*$ -closed set in  $Y$  for each  $\tau_{1,2}$ -closed set  $V$  of  $X$ . It is clear that an  $(1,2)^*$ -open function is  $(1,2)^*$ - $wg^*$ -open and a  $(1,2)^*$ -closed function is  $(1,2)^*$ - $wg^*$ -closed.

**Theorem 5.5.** Let  $X$  and  $Y$  be bitopological spaces. A function  $f : X \rightarrow Y$  is  $(1,2)^*$ - $wg^*$ -closed if and only if for each subset  $B$  of  $Y$  and for each  $\tau_{1,2}$ -open set  $G$  containing  $f^{-1}(B)$  there exists a  $(1,2)^*$ - $wg^*$ -open set  $F$  of  $Y$  such that  $B \subseteq F$  and  $f^{-1}(F) \subseteq G$ .

*Proof.* Let  $B$  be any subset of  $Y$  and let  $G$  be an  $\tau_{1,2}$ -open subset of  $X$  such that  $f^{-1}(B) \subseteq G$ . Then  $F = Y \setminus f(X \setminus G)$  is  $(1,2)^*$ - $wg^*$ -open set containing  $B$  and  $f^{-1}(F) \subseteq G$ .

Conversely, let  $U$  be any  $\tau_{1,2}$ -closed subset of  $X$ . Then  $f^{-1}(Y \setminus f(U)) \subseteq X \setminus U$  and  $X \setminus U$  is  $\tau_{1,2}$ -open. According to the assumption, there exists a  $(1,2)^*$ - $wg^*$ -open set  $F$  of  $Y$  such that  $Y \setminus f(U) \subseteq F$  and  $f^{-1}(F) \subseteq X \setminus U$ . Then  $U \subseteq X \setminus f^{-1}(F)$ . From  $Y \setminus F \subseteq f(U) \subseteq f(X \setminus f^{-1}(F)) \subseteq Y \setminus F$  it follows that  $f(U) = Y \setminus F$ , so  $f(U)$  is  $(1,2)^*$ - $wg^*$ -closed in  $Y$ . Therefore  $f$  is a  $(1,2)^*$ - $wg^*$ -closed function.  $\square$

**Remark 5.6.** The composition of two  $(1,2)^*$ - $wg^*$ -closed functions need not be a  $(1,2)^*$ - $wg^*$ -closed as we can see from the following example.

**Example 5.7.** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\phi, \{a\}, X\}$  and  $\tau_2 = \{\phi, \{a, b\}, X\}$ . Then the sets in  $\{\phi, \{a\}, \{a, b\}, X\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, \{c\}, \{b, c\}, X\}$  are called  $\tau_{1,2}$ -closed. Let  $Y = \{a, b, c\}$ ,  $\sigma_1 = \{\phi, \{a\}, Y\}$  and  $\sigma_2 = \{\phi, \{b, c\}, Y\}$ . Then the sets in  $\{\phi, \{a\}, \{b, c\}, Y\}$  are called  $\sigma_{1,2}$ -open and  $\sigma_{1,2}$ -closed. Let  $Z = \{a, b, c\}$ ,  $\eta_1 = \{\phi, Z\}$  and  $\eta_2 = \{\phi, \{a, b\}, Z\}$ . Then the sets in  $\{\phi, \{a, b\}, Z\}$  are called  $\eta_{1,2}$ -open and the sets in  $\{\phi, \{c\}, Z\}$  are called  $\eta_{1,2}$ -closed. We define  $f : X \rightarrow Y$  by  $f(a) = c$ ,  $f(b) = b$  and  $f(c) = a$  and let  $g : Y \rightarrow Z$  be the identity function. Hence both  $f$  and  $g$  are  $(1,2)^*$ - $wg^*$ -closed functions. For a  $\tau_{1,2}$ -closed set  $U = \{b, c\}$ ,  $(g \circ f)(U) = g(f(U)) = g(\{a, b\}) = \{a, b\}$  which is not  $(1,2)^*$ - $wg^*$ -closed in  $Z$ . Hence the composition of two  $(1,2)^*$ - $wg^*$ -closed functions need not be a  $(1,2)^*$ - $wg^*$ -closed.

**Theorem 5.8.** Let  $X, Y$  and  $Z$  be bitopological spaces. If  $f : X \rightarrow Y$  is a  $(1,2)^*$ -closed function and  $g : Y \rightarrow Z$  is a  $(1,2)^*$ - $wg^*$ -closed function, then  $g \circ f : X \rightarrow Z$  is a  $(1,2)^*$ - $wg^*$ -closed function.



**Definition 5.9.** A function  $f : X \rightarrow Y$  is called a weakly  $(1,2)^*-g^*$ -irresolute (briefly,  $(1,2)^*-wg^*$ -irresolute) if  $f^{-1}(U)$  is a  $(1,2)^*-wg^*$ -open set in  $X$  for each  $(1,2)^*-wg^*$ -open set  $U$  of  $Y$ .

**Example 5.10.** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\phi, \{b\}, X\}$  and  $\tau_2 = \{\phi, \{a, c\}, X\}$ . Then the sets in  $\{\phi, \{b\}, \{a, c\}, X\}$  are called  $\tau_{1,2}$ -open and  $\tau_{1,2}$ -closed. Let  $Y = \{a, b, c\}$ ,  $\sigma_1 = \{\phi, Y\}$  and  $\sigma_2 = \{\phi, \{b\}, Y\}$ . Then the sets in  $\{\phi, \{b\}, Y\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, \{a, c\}, Y\}$  are called  $\sigma_{1,2}$ -closed. Let  $f : X \rightarrow Y$  be the identity function. Then  $f$  is  $(1,2)^*-wg^*$ -irresolute.

**Remark 5.11.** Every  $(1,2)^*-g^*$ -irresolute function is  $(1,2)^*-wg^*$ -continuous but not conversely. Also, the concepts of  $(1,2)^*-g^*$ -irresoluteness and  $(1,2)^*-wg^*$ -irresoluteness are independent of each other.

**Example 5.12.** Let  $X = \{a, b, c, d\}$ ,  $\tau_1 = \{\phi, \{a\}, X\}$  and  $\tau_2 = \{\phi, \{b, c\}, \{a, b, c\}, X\}$ . Then the sets in  $\{\phi, \{a\}, \{b, c\}, \{a, b, c\}, X\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, \{d\}, \{a, d\}, \{b, c, d\}, X\}$  are called  $\tau_{1,2}$ -closed. Let  $Y = \{a, b, c, d\}$ ,  $\sigma_1 = \{\phi, \{a\}, Y\}$  and  $\sigma_2 = \{\phi, \{a, b, d\}, Y\}$ . Then the sets in  $\{\phi, \{a\}, \{a, b, d\}, Y\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, \{c\}, \{b, c, d\}, Y\}$  are called  $\sigma_{1,2}$ -closed. Let  $f : X \rightarrow Y$  be the identity function. Then  $f$  is  $(1,2)^*-wg^*$ -continuous but not  $(1,2)^*-g^*$ -irresolute.

**Example 5.13.** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\phi, \{a\}, X\}$  and  $\tau_2 = \{\phi, \{b, c\}, X\}$ . Then the sets in  $\{\phi, \{a\}, \{b, c\}, X\}$  are called  $\tau_{1,2}$ -open and the sets in  $\{\phi, \{a\}, \{b, c\}, X\}$  are called  $\tau_{1,2}$ -closed. Let  $Y = \{a, b, c\}$ ,  $\sigma_1 = \{\phi, \{a\}, Y\}$  and  $\sigma_2 = \{\phi, \{a, b\}, Y\}$ . Then the sets in  $\{\phi, \{a\}, \{a, b\}, Y\}$  are called  $\sigma_{1,2}$ -open and the sets in  $\{\phi, \{c\}, \{b, c\}, Y\}$  are called  $\sigma_{1,2}$ -closed. Let  $f : X \rightarrow Y$  be the identity function. Then  $f$  is  $(1,2)^*-wg^*$ -irresolute but not  $(1,2)^*-g^*$ -irresolute.

**Example 5.14.** Let  $X, \tau_1$  and  $\tau_2$  be as in Example 3.19. Let  $Y, \sigma_1$  and  $\sigma_2$  be as in Example 3.3. Let  $f$  be the identity function, then  $f$  is  $(1,2)^*-g^*$ -irresolute but not  $(1,2)^*-wg^*$ -irresolute.

**Theorem 5.15.** The composition of two  $(1,2)^*-wg^*$ -irresolute functions is also  $(1,2)^*-wg^*$ -irresolute.

**Theorem 5.16.** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be functions such that  $g \circ f : X \rightarrow Z$  is  $(1,2)^*-wg^*$ -closed function. Then the following statements hold:

1. if  $f$  is  $(1,2)^*$ -continuous and injective, then  $g$  is  $(1,2)^*-wg^*$ -closed.
2. if  $g$  is  $(1,2)^*-wg^*$ -irresolute and surjective, then  $f$  is  $(1,2)^*-wg^*$ -closed.

*Proof.*

1. Let  $F$  be a  $\sigma_{1,2}$ -closed set of  $Y$ . Since  $f^{-1}(F)$  is  $\tau_{1,2}$ -closed in  $X$ , we can conclude that  $(g \circ f)(f^{-1}(F))$  is  $(1,2)^*-wg^*$ -closed in  $Z$ . Hence  $g(F)$  is  $(1,2)^*-wg^*$ -closed in  $Z$ . Thus  $g$  is a  $(1,2)^*-wg^*$ -closed function.
2. It can be proved in a similar manner as (1).

□

**Theorem 5.17.** If  $f : X \rightarrow Y$  is an  $(1,2)^*-wg^*$ -irresolute function, then it is  $(1,2)^*-wg^*$ -continuous.

**Remark 5.18.** The converse of the above need not be true in general. The function  $f : X \rightarrow Y$  in the Example 5.14 is  $(1,2)^*-wg^*$ -continuous but not  $(1,2)^*-wg^*$ -irresolute.

**Theorem 5.19.** If  $f : X \rightarrow Y$  is surjective  $(1,2)^*-wg^*$ -irresolute function and  $X$  is  $(1,2)^*-wg^*$ -compact, then  $Y$  is  $(1,2)^*-wg^*$ -compact.

**Theorem 5.20.** If  $f : X \rightarrow Y$  is surjective  $(1,2)^*-wg^*$ -irresolute function and  $X$  is  $(1,2)^*-wg^*$ -connected, then  $Y$  is  $(1,2)^*-wg^*$ -connected.



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