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Higher Order Duality in Non-Differentiable Minimax Fractional Programming With Generalized $(F,\ \alpha,\ \rho,\ d)$–Type I Function

Research Article

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Abstract: Higher order dual for Minimax fractional programming problem is formulated. Weak duality strong duality and converse duality theorems are discussed involving generalized higher order $(F,\ \alpha,\ \rho,\ d)$– Type–I functions.

Keywords: Non differentiable fractional programming, Minimax programming, Higher order duality.

1. Introduction and Preliminaries

Several authors [1–7] have shown their interest in developing optimality conditions and duality results for minimax programming problems. Ahmad & Husain [8] considered the following minimax programming problem

$$\min \sup_{y\in Y} f\left(x,\ y\right) + \left(x^T\ B\ x\right)^{1/2}$$

Subject to $g\left(x\right) \leq 0,\ x \in R^n$

In this paper, we consider the following non differentiable minimax fractional programming problem (HFP)

$$\min \sup_{y\in Y} \frac{f\left(x\right) + y^T h\left(x\right)}{g\left(x\right)} + \left(x^T B\, x\right)^{1/2}$$

Subject to $h\left(x\right) \leq 0,\ x \in R^n$, where Y is a compact subset of R^l, $f\ g : R^n \to R\ \left(g\left(x\right) > 0\right)$. $h : R^n \to R^m$ are continuously differentiable function at $x \in R^n$ and B is an n x n positive semidefinite symmetric matrix. In this paper, we formulate a higher order fractional dual of (HFP) and establish weak, strong, and converse-duality theorems under higher order $(F,\ \alpha,\ \rho,\ d)$-Type–I assumptions.

2. Notations and Preliminary Results

Definition 2.1. *$(f,\ g,\ h_i)$ is said to be higher order $(F,\ \alpha,\ \rho,\ d)$ - Type I at $\overline{x} \in X$ with respect to $p \in R^n$ for all $x \in S$ and $\overline{y}_i \in Y\left(x\right)$,*

$$\frac{f\left(x\right) + \overline{y}_j^T h_j\left(x\right)}{g\left(x\right)} \geq \frac{f\left(\overline{x}\right) + \overline{y}_j^T h_j\left(\overline{x}\right)}{g\left(\overline{x}\right)}$$

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$$+\left[P^T\nabla\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)+\frac{1}{2}p^T\nabla^2\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)P\right]$$

$$-p^T\nabla_p\left[p^T\nabla\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)+\frac{1}{2}p^T\nabla^2\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)p\right]$$

$$+F\left(x,\bar{x},\ \alpha'(x,\ \overline{x})\left(\nabla_p\left[P^T\nabla\right]\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)+\frac{1}{2}p^T\nabla^2\frac{\left(f(\overline{x})+\overline{y}^T h(\overline{x})\right)}{g(\overline{x})}P\right)+\rho_i^1\left(d^2(x,\overline{x})\right),\ i=1,2,...s$$

$$-\left[h_j(\overline{x})+\left(p^T\nabla h_j(\overline{x})+\frac{1}{2}p^T\nabla^2 h_j(\overline{x})p\right)-p^T\nabla_p\left(p^T\nabla h_j(\overline{x})+\frac{1}{2}p^T\nabla^2 h_j(\overline{x})p\right)\right]\geq$$

$$F\left(x,\ \overline{x},\ \alpha^2(x,\ \overline{x})\nabla_p\left(p^T\nabla h_j(\bar{x})+\frac{1}{2}p^T\nabla^2 h_j(\overline{x})p\right)\right)+p_j^2 d^2(x,\ \overline{x})$$

Definition 2.2. *(f,g_j) is said to be higher order $(F,\ \alpha,\ \rho,\ d)$ pseudo quasi type – I at $x\in X$ with respect to $p\in R^n$ if for all $x\in S$ and $\overline{y}_i\in Y(x)$.*

$$\frac{f(x)+\overline{y}_j^T h_j(x)}{g(x)}<\frac{f(\overline{x})+\overline{y}_j^T h_j(\overline{x})}{g(\overline{x})}+\left[p^T\nabla\left(\frac{f(\overline{x})+\overline{y}^T h_j(\overline{x})}{g(\overline{x})}\right)+\frac{1}{2}p^T\nabla^2\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)p\right]$$

$$-p^T\nabla_p\left[p^T\nabla\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)+\frac{1}{2}p^T\nabla^2\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)p\right]$$

$$\Rightarrow F\left(x,\overline{x};\ \alpha^1(x,\overline{x})\left(\nabla_p\left[p^T\nabla\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)\right]+\frac{1}{2}p^T\nabla^2\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)p\right)\right)$$

$$<-p_i^1\left(d^2(x,\ \overline{x})\right),\ i=1,\ 2,.....3$$

$$-\left[h_i(\overline{x})+\left(p^T\nabla h_j(\overline{x})+\frac{1}{2}p^T\nabla^2 h_j(\overline{x})p\right)-p^T\nabla_p\left(p^T\nabla h_j(\overline{x})-\frac{1}{2}p^T\nabla^2 h_j(\overline{x})p\right)\right]\leq 0$$

$$\Rightarrow F\left(x,\ \overline{x},\ \alpha^2(x,\ \overline{x})\left(\nabla_p\left(p^T\nabla h_j(\overline{x})+\frac{1}{2}p^T\nabla^2 h_j(\overline{x})p\right)\right)\right)\leq -p_j^2(x,\overline{x})$$

In the above definition, if

$$F\left(x,\ \overline{x};\ \alpha^1(x,\ \overline{x})\left(\nabla_p\left[p^T\nabla\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)\right]+\frac{1}{2}p^T\nabla^2\left(\frac{f(\overline{x})+\overline{y}^T h(x)}{g(\overline{x})}\right)p\right)\right)\geq -p_i^1\left(x^2(x,\ \overline{x})\right)$$

$$\Rightarrow\ \frac{f(x)+\overline{y}_j^T h_j(x)}{g(x)}>\frac{f(\overline{x})+\overline{y}_j^T h_j(\overline{x})}{g(\overline{x})}+\left[p^T\nabla\left(\frac{f(\overline{x})+\overline{y}_j^T h_j(\overline{x})}{g(\overline{x})}\right)\right]$$

$$+\frac{1}{2}p^T\nabla^2\left(\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)P\right)-p^T\nabla_p\left[p^T\nabla\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)+\frac{1}{2}p^T\nabla^2\left(\frac{f(\overline{x})+\overline{y}^T h(\overline{x})}{g(\overline{x})}\right)p\right]$$

Lemma 2.1 (Generalized schwartz Inequality)**.** *Let B be a positive semidefinite symmetric matrix of order n. Then for all $x,\ u\in R^n$,*

$$x^T Bu\leq\left(x^T Bx\right)^{1/2}\left(u^T Bu\right)^{1/2}$$

We observe that equality holds if $Bx=\lambda Bu$ for some $\lambda\ \geq 0$. If $u^T Bu\leq 1$, we have $x^T Bu\leq\left(x^T Bx\right)^{1/2}$

Theorem 2.1 (Necessary conditions)**.** *If x^* is a local or global solution of (HFP) satisfying ${x^*}^T Bx^*>0$ and if $\nabla g_j(x^*),j\in J(x^*)$ are linearly independent then there exist $(s^*,t^*,\overline{y}^*)\in K,\ u^*\ \in R^n$ and $\mu^*\in R_+^m$ such that*

$$\sum_{i=1}^{s^*}t_i^*\nabla\left(\frac{f(x*)+\overline{y}_j^* h_j(x*)}{g(x*)}\right)+Bu*+\sum_{j=1}^{m}\nabla\mu_j^* h_j(x^*)=0$$

$$\sum_{j=1}^{m}M_j^* h_j(x^*)=0$$

$$t_i^*\geq 0,\ i=1,\ 2,\ ....,5^*,\ \sum_{i=1}^{s^*}t_i^*=1$$

$$u^{*\,T}Bu^*\ \leq\ 1,\ \left(x^{*T}Bx^*\right)^{\frac{5}{2}}\ =\ x^{*\,T}Bu^*$$

3. High Order Dual Problem

We formulate higher order dual

(HFD)$(s,\ \overset{\max}{y},\ \overline{y}) \in k\ \ (z,\ u,\ \mu,\ \overset{\sup}{p}) \in H\,(s,\ t,\ \overline{y})$

$$\sum_{i=1}^{s} t_i \left[\frac{f\,(z) + \overline{y}_i^T\, h_i\,(z)}{y\,(z)} + \left\{p^T \Delta \left(\frac{f\,(z) + y_i^T h_i\,(z)}{g\,(z)}\right)\right.\right.$$

$$\left.\left. + \frac{1}{2} p^T \nabla^2\ \left(\frac{f\,(z) + \overline{y}_i h_i\,(z)}{g\,(z)}\right) P\right)\right\} - p^T \nabla_p \left[p^T \nabla \left(\frac{f\,(z) + \overline{y}_i h_i\,(z)}{g\,(z)}\right) + \frac{1}{2} p^T \nabla^2 \left(\frac{f\,(z) + \overline{y}_i h_i\,(z)}{g\,(z)}\right) p\right]\right]$$

$$+ z^T Bu + \sum_{j\ \in j_0} \left\{\mu_j . h_j\,(z) + \mu_j \left[p^T \nabla h_j\,(z) + \frac{1}{2} p^T \nabla^2 h_j\,(z)\, p\right] - p^T \nabla_p \left[\mu_j \left(p^T \nabla h_j\,(z) + \frac{1}{2} p^T \nabla^2 h_j\,(z)\, p\right)\right]\right\},$$

where $H\,(s,\ t,\ \overline{y})$ denotes the set of all $(z,\ u,\ \mu,\ p) \in R^n \times R^n \times R_+^m \times R^n$

Satisfying $\sum\limits_{i=1}^{s} t_i \nabla_p \left[p^T \nabla \left(\frac{f(z) + \overline{y}_i^T h_i(z)}{g(z)}\right) + \frac{1}{2} p^T \nabla^2 \left(\frac{f(z) + \overline{y}_i^T h_i(z)}{g(z)}\right) p\right]$

$$+ Bu + \sum_{j=1}^{m} \nabla_p \left[\mu_j \left(p^T \nabla h_j\,(z) + \frac{1}{2} p^T \nabla^2 h_j\,(z)\, p\right)\right] = 0 \tag{1}$$

$$\sum_{j \in j} \left\{\mu_j h_j\,(z) + \mu_j \left(p^T \nabla h_j\,(z) + \frac{1}{2} p^T \nabla^2 h_j\,(z)\, p\right) - p^T \nabla_p \left(\mu_j \left[p^T \nabla h_j\,(z) + \frac{1}{2} p^T \nabla^2 h_j\,(z)\, p\right]\right)\right\} \geq 0$$

$$\beta = 1,\ 2,\ ....r, \tag{2}$$

$$u^T Bu \leq 1 \tag{3}$$

Theorem 3.1 (Weak duality)**.** *Let x and $(z,\ u,\ \mu,\ s,\ t,\ \overline{y},\ p)$ be the feasible solutions of (HFP) and (HFD) respectively. Suppose that* $\left[\sum\limits_{i=1}^{s} t_i \frac{f(.) + \overline{y}_i^T h(.)}{g(.)} + (.)^T B_u + \sum\limits_{j\ \in\ J_o} \mu_j h_j\,(.)\,,\ \sum_{j \in J_\beta} u_j h_j\,(.)\ \ \beta = 1,\ 2,\ ....,r\right]$ *is higher order $(F,\ \alpha,\ \rho,\ d)$ pseudoquasi Type I at z and* $\frac{\rho_1^1}{\alpha^1(x,\ z)} + \frac{\sum_{\beta=1}^{r} \rho_\beta^2}{\alpha^2(x,\ z)} \geq 0$*. Then* $y \overset{\sup}{\in} Y\ \frac{f(x) + y^T h(x)}{g(x)} + \left(x^T Bz\right)^{1/2}$

$$< \sum_{i=1}^{s} t_i \left[\frac{f\,(z)\ +\ \overline{y}_i^T h_i\,(z)}{g\,(z)} + \left\{p^T \left(\nabla \left(\frac{f\,(z) + \overline{y}_i^T h_i\,(z)}{g\,(z)}\right) + \frac{1}{2} p^T \nabla^2 \left(\frac{f\,(z) + \overline{y}_i^T h_i\,(z)}{g\,(z)}\right) p\right)\right\} - \right.$$
$$p^T \nabla_p \left[p^T \nabla \left(\frac{f\,(z) + \overline{y}_i^T h_i\,(z)}{g\,(z)}\right) + \frac{1}{2} p^T \nabla^2 \left(\frac{f\,(z) + \overline{y}_i^T h_i\,(z)}{g\,(z)}\right) p\right] + z^T Bu$$
$$+ \sum_{j \in j_0} \left(\mu_j h_j\,(z)\right) + \mu_j \left[p^T \nabla h_j\,(z) + \frac{1}{2} p^T \nabla^2 h_j\,(z)\, p\right] - p^T \nabla_p \left(\mu_j \left(p^T \nabla h_j\,(z) + \frac{1}{2} p^T \nabla^2 h_j\,(z)\, p\right)\right),\ j \in J,$$

For all $\overline{y}_i\ \in Y\,(x)\,,\ i = 1, 2, ...s$. If follows from lemma 2.1 and (3) that

$$\sum_{i=1}^{s} t_i \frac{\left(f\,(x) + \overline{y}_i^T h_i\,(x)\right)}{g\,(x)} + x^T Bu\ + \sum_{j \in J_\theta} \mu_j h_j\ (x)$$

$$<\ \sum_{i=1}^{s} t_i \left\{\frac{f\,(z) + \overline{y}_i^T h\,(z)}{g\,(z)} + p^T \nabla \frac{f\,(z) + \overline{y}_i^T h\,(z)}{g\,(z)} + \frac{1}{2} p^T \nabla^2 \frac{f\,(z) + \overline{y}_i^T h\,(z)}{g\,(z)}\right\} p$$

$$- p^T \nabla \left(-p^T \nabla \frac{f\,(z) + \overline{y}_i^T h\,(z)}{g\,(z)}\right) + \frac{1}{2} p^T \nabla \left(\frac{f\,(z) + \overline{y}_i^T h\,(z)}{g\,(z)} p\right) + z^T Bu$$

$$+ \sum_{j \in J_0} \left(\mu_j h_j\,(z) + \mu_j \left(p^T \nabla h_j\,(z) + \frac{1}{2} p^T \nabla^2 h_j\,(z)\right) p\right) - p^T \nabla_p \left(\mu_j \left(p^T \nabla^2 h_j\,(z) + \frac{1}{2} p^T \nabla^2 h_j\,(z)\, p\right)\right) \tag{4}$$

Also from (2) *we have*

$$-\sum_{j\in J_B}\left\{\mu_j h_j(z)+\mu_j\left(p^T\nabla h_j(z)+\frac{1}{2}p^T\nabla^2 h_j(z)\,p\right)-p^T\nabla_p\left(\mu_j\left(p^T\nabla h_j(z)+\frac{1}{2}p^T\nabla^2 h_j(z)\,p\right)\right)\right\}\leq 0,\ \beta=1,\ldots,r \quad (5)$$

The higher order $(F,\ \alpha,\ \rho,\ d)$*pseudoquasi type – I assumption at* z *with* (4) *and* (5), *implies*

$$F(x,z;\alpha^1(x,z))\left\{\sum_{i=1}^{s}t_i\nabla_p\left(\nabla\left(\frac{f(z)+\bar{y}_i^T h_i(z)}{g(z)}\right)+\frac{1}{2}p^T\nabla^2\left(\frac{(fcz)+\bar{y}_i^T h(z)}{g(z)}\right)p\right)+Bu\right.$$
$$\left.+\sum_{j\in J_0}\nabla_p\left(\mu_j p^T\nabla h_j(z)+\frac{1}{2}p^T\nabla^2 h_j(z)p\right)\right\}<-\rho_1^1 d^2(x,z) \qquad (6)$$

$$F\left(x,\ z;\ \alpha^2(x,\ z)\sum_{j\in J_B}\nabla_p\left(\mu_j\left(p^T\nabla h_j(z)+\frac{1}{2}p^T\nabla^2 h_j(z)\,P\right)\right)\right)\leq-\rho_\beta^2 d^2(x,\ z),\beta=1,\ldots,r$$

By using $\alpha^1(x,\ z)>0,\ \alpha^2(x,\ z)>0$ *and the sublinearity of* F *in the above inequalities, we summarize to get*

$$F\left(x,\ z;\sum_{i=1}^{s}t_i\nabla_p\left(p^T\nabla\left(\frac{f(z)+\overline{y}_i^T h_i(z)}{g(z)}\right)+\frac{1}{2}p^T\nabla^2\left(\frac{f(z)+\overline{y}_i^T h_i(z)}{g(z)}\right)p\right)+Bu\right.$$
$$\left.+\sum_{j=1}^{m}\nabla_p\left(\mu_j\left(p^T\nabla h_j(z)+\frac{1}{2}p^T\nabla^2 h_j(z)\,p\right)\right)\right)<-\frac{\rho_1^1}{\alpha^1(x,z)}+\sum_{\beta=1}^{r}\frac{\rho_\beta^2}{\alpha^2(x,z)}d^2(x,z)$$

Since

$$\frac{\rho_1^2}{\alpha^1(x,z)}+\frac{\sum_{\beta=1}^{r}\rho_\beta^2}{\alpha^2(x,\ z)}\geq 0,$$

Inequality (6) *yields*

$$F\left(x,z;\sum_{i=1}^{s}t_i\nabla_p\left(p^T\nabla\left(\frac{f(z)+\overline{y}_i^T h_i(z)}{g(z)}\right)+\frac{1}{2}p^T\nabla^2\left(\frac{f(z)+\overline{y}_i^T h_i(z)}{g(z)}\right)p\right)+Bu\right.$$
$$\left.+\sum_{j=1}^{m}\nabla_p\left(\mu_j\left(p^T\nabla h_j(z)+\frac{1}{2}p^T\nabla^2 h_j(z)\,p\right)\right)\right)<0$$

Which contradicts (1) *as* $F(x,z,0)=0$.

Theorem 3.2 (Strong duality)**.** *Let* x^* *be an optimal solution of (HFP) and let* $\nabla h_j(x^*),\ j\in J(x^*)$ *be linearly independent. Assume that* $p^T\nabla\left(\frac{f(z^*)+\overline{y}_i^T h_i(x^*)}{g(x^*)}\right)+\frac{1}{2}p^T\nabla^2\left(\frac{f(x^*)+\overline{y}_i^T h_i(x^*)}{g(x^*)}\right)p=0$ *for* $p=0$.

$$\nabla_p\left[p^T\nabla\left(\frac{f(x^*)+\overline{y}_i^T h_i(x^*)}{g(x^*)}\right)+\frac{1}{2}p^T\nabla^2\left(\frac{f(x^*)+\overline{y}_i^T h_i(x^*)}{g(x^*)}\right)p\right]=\nabla\frac{(f(x^*)+\overline{y}_i^T h_i(x^*))}{g(x^*)}$$

for $p=0,\ I=1,\ 2,\ \ldots,\ s$

$$p^T\nabla h_j(x^*)+\frac{1}{2}p^T\nabla^2 h_j(x^*)\,p=0 \ for\ p\ =\ 0$$

$$\nabla_p\left[p^T\nabla h_j(x^*)+\frac{1}{2}p^T\nabla^2 h_j(x^*)\,p\right]=\nabla h_j(x^*),\ \ for\ p=0,\ j\in J \qquad (7)$$

Then there exist $(s^*,\ t^*,\ \overline{y}^*)\in k$ *and* $(x^*,\ u^*,\ \mu^*,\ p^*)\in H(s^*,t^*,\overline{y}^*)$ *such that* $(x^*,\ u^*,\ \mu^*,\ s^*,\ t^*,\ \overline{y}^*,p^*=0)$ *is a feasible solution of (HFD) and the two objectives have the same values. If the assumption of weak duality (Theorem 3.1) hold for all feasible solutions of (HFP) and (HFD), then* $(x^*,\ u^*,\ \mu^*,\ s^*,\ t^*,\ \overline{y}^*,p^*=0)$ *is an optimal solution of (HFD).*

Proof. Since x^* is an optimal solution of (HFP) and $\nabla h_j(x^*)$, $j \in J(x^*)$ are linearly independent, by Theorem 2.1, there exist $(s^*, t^*, \overline{y}^*) \in K$ and $(x^*, u^*, \mu^*, p^*) \in H(s^*, t^*, \overline{y}^*)$ such that

$$\sum_{i=1}^{s^*} t_i^* \nabla \left(\frac{f(x^*) + \overline{y}_i^T h_i(x^*)}{g(x^*)} \right) + Bu^* + \sum_{j=1}^{m} \nabla \mu_j^* h_j(x^*) = 0 \tag{8}$$

$$\sum_{j=1}^{m} \nabla \mu_j^* h_j(x^*) = 0 \tag{9}$$

$$t_i^* \geq 0, \ i = 1, \ldots, s^*, \ \sum_{i=1}^{s} t_i^* = 1, \tag{10}$$

$$u^{*T} Bu^* \leq 1, \tag{11}$$

$$(x^* T Bx^*)^{1/2} = x^{*T} Bu^* \tag{12}$$

Thus the relations (8) to (11) along with (7) imply that $(x^*, u^*, \mu^*, s^*, t^*, \overline{y}^*, p^* = 0)$ is a feasible solution of (HFD). Also (7), (9) and (12) with p* = 0 show the equality of objective values Optimality of $(x^*, u^*, \mu^*, s^*, t^*, \overline{y}^*, p^* = 0)$ thus follows from weak duality theorem. □

4. Conclusion

The results appeared in this paper can be further generalized to the Non differentiable multi objective higher order fractional programming under generalized convexity and generalized university assumption.

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