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Semi Smooth Graceful Labeling on Some Graphs

Research Article

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Abstract: This paper contains some results on semi smooth graceful graph. We proved that $B_{m,n}$, $S^{'}(B_{m,n})$ and $S^{'}(P_n)$ are semi smooth graceful and a graph obtained by joining G and $B^2_{m,n}$ with an arbitrary path P_r is a graceful.

MSC: 05C78.

Keywords: Graceful graph, semi smooth graceful graph, bistar graph, square of bistar, path, splitting graph.

1. Introduction

We will consider a simple, undirected and finite graph $G = (V, E)$ on $p = |V|$ vertices and $q = |E|$ edges. For all terminology, notations and basic definitions we follows Harary [1]. For a Comprehensive bibliography of papers on different graph labeling are given by Gallian [2].

In 1966 Rosa [3] defined $\alpha-$labeling as a graceful labeling with an additional property. A graph which admits $\alpha-$labeling is necessarily bipartite. A natural generalization of graceful graph is the notion of k$-$graceful graph. Obviously 1$-$graceful is graceful and a graph which admits $\alpha-$labeling is always k$-$graceful graph, $\forall\ k \in N$. Ng [4] has identified some graphs that are k$-$graceful, $\forall\ k \in N$, but do not have $\alpha-$labeling.

Kaneria and Jariya [5, 6] define smooth graceful labeling and semi smooth graceful labeling. Every smooth graceful graph is also a semi smooth graceful graph. They proved cycle C_n (n $\equiv 0 \pmod 4$), path P_n, grid graph $P_n \times P_m$ and complete bipartite graph $K_{2,n}$ are smooth graceful graphs. Vaidya and Vyas [7] have proved that $D_2(P_n), B^2_{n,n}$, $S^{'}(B_{n,n})$, $M(C_n)$, Df_n are mean graphs. Here we will recall some definitions which are used in this paper.

Definition 1.1. *A function f is called graceful labeling of a graph $G = (V, E)$ if $f : V \longrightarrow \{0, 1, \dots, q\}$ is injective and the induce function $f^\star : E \longrightarrow \{1, 2, \dots, q\}$ defined as $f^\star(e) = |f(u) - f(v)|$ is bijective for every edge $e = (u, v) \in E(G)$. A graph G is called* graceful graph *if it admits a graceful labeling.*

Definition 1.2. *A semi smooth graceful graph G, we mean it is a bipartite graph with $|E(G)| = q$ and the property that for all non$-$negative integer l, there is an integer t $(1 \le t \le q)$ and an injective function $g : V(G) \longrightarrow \{0, 1, \dots, t-1, t+l, t+l+1, \dots, q+l\}$ such that the induced edge labeling function $g^\star : E(G) \longrightarrow \{1+l, 2+l, \dots, q+l\}$ defined as $g^\star(e) = |f(u) - f(v)|$ is a bijection for every edge $e = (u, v) \in E(G)$.*

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Definition 1.3. *Bistar $B_{n,n}$ is the graph obtained by joining the apex vertices of two copies of star $K_{1,n}$ by an edge. $B^2_{n,n}$ is the* square *of $B_{n,n}$ obtained by joining each pair of two vertices of $B_{n,n}$ at distance 2 by a new edge.*

Definition 1.4. *Let $G = (V, E)$ be a graph. Then the* splitting graph *of G is denoted by $S^{'}(G)$, which obtained by adding to each $v \in V$, by a new vertex $v^{'}$ such that $v^{'}$ is adjacent to those vertex of G, which are adjacent to v in G. i.e. If $V = \{v_1, v_2, \ldots, v_n\}$, then take $V^{'} = \{v^{'}_1, v^{'}_2, \ldots, v^{'}_n\}$ and $S^{'}(G) = (V \cup V^{'}, E \cup \{(u, v^{'}), (u^{'}, v)/(u, v) \in E\})$. Observe that $|V(S^{'}(G))| = 2|V(G)|$ and $|E(S^{'}(G))| = 3|E(G)|$.*

2. Main Results

Theorem 2.1. *$B_{m,n}$ is a semi smooth graceful graph.*

Proof. We know that $B_{m,n}$, bistar graph obtained by joining apex vertices of $K_{1,m}$ and $K_{1,n}$ by a new edge. Let $v_0, v_1, \ldots, v_m, u_0, u_1, \ldots, u_n$ be vertices of $B_{m,n}$. Obviously $E(B_{m,n}) = \{(u_0, v_0), (v_0, v_1), \ldots, (v_0, v_m), (u_0, u_1), \ldots, (u_0, u_n)\}$ i.e. $|V(B_{m,n})| = m + n + 2$ and $|E(B_{m,n})| = m + n + 1$. We shall define vertex labeling function $f : V(B_{m,n}) \longrightarrow \{0, 1, \ldots, t-1, t+l, t+l+1, \ldots, q+l\}$, where $q = |E(B_{m,n})| = m + n + 1$, $t = m + 1$ and l is the arbitrary non-negative integer as follows,

$f(v_0) = q = m + n + 1 + l;$

$f(u_0) = m;$

$f(v_i) = i - 1, \quad \forall\ i = 1, 2, \ldots, m;$

$f(u_j) = q - j + l, \quad \forall\ j = 1, 2, \ldots, n.$

Above labeling pattern give rise a semi smooth graceful labeling to the graph $B_{m,n}$, as it will produce edge labels $(v_0, v_1) = q + l, (v_0, v_2) = q - 1 + l, \ldots, (v_0, v_m) = q - m + 1 + l = n + 2 + l, (u_0, v_0) = n + 1 + l, (u_0, u_1) = n + l, (u_0, u_2) = n - 1 + l, \ldots, (u_0, u_n) = 1 + l$. Thus f and its induced function f^* both are bijective maps. Therefore $B_{m,n}$ is a semi smooth graceful graph. □

Illustration 2.2. *$B_{4,6}$ and its semi smooth graceful labeling shown in figure 1. Here $q = |E(B_{4,6})| = 11$.*

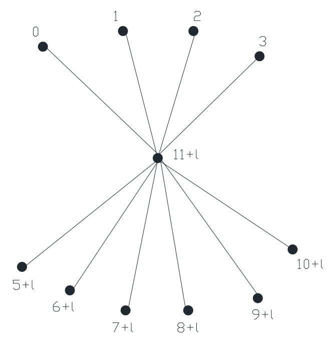

Figure 1. $B_{4,6}$ **and its semi smooth graceful labeling**

Theorem 2.3. *$S^{'}(B_{m,n})$ is a semi smooth graceful graph.*

Proof. Let $G = S^{'}(B_{m,n})$, splitting graph of $B_{m,n}$. Let $V(G) = \{v_0, v_1, \ldots, v_m, u_0, u_1, \ldots \ldots, u_n, v^{'}_0, v^{'}_1, \ldots, v^{'}_m, u^{'}_0, u^{'}_1, \ldots, u^{'}_n\}$ Obviously $E(S^{'}(B_{m,n})) = E(B_{m,n}) \bigcup \{(v_0, v^{'}_i)(i = 1, 2, \ldots, m), (v^{'}_0, v_i)(i = 1, 2, \ldots, m), (v^{'}_0, u_0), (u^{'}_0, v_0), (u^{'}_0, u_j)(j = 1, 2, \ldots, n), (u_0, u^{'}_j)(j = 1, 2, \ldots, n\}$. i.e. $|V(S^{'}(B_{m,n}))| = 2|V(B_{m,n})| = 2(m + n + 2)$ and $|E(S^{'}(B_{m,n}))| = 3|E(B_{m,n})| = 3(m + n + 1)$. We shall define vertex labeling function

$f : V(G) \longrightarrow \{0, 1, \dots, t-1,\ t+l, t+l+1, \dots, q+l\}$, where $q = |E(G)| = 3(m+n+1)$, $t = 2m+2$ and l is the arbitrary non-negative integer as follows,

$f(v_0) = q + l,\ f(u_0) = 2m,\ f(v_0^{'}) = 2m + 3n + 1 + l,\ f(u_0^{'}) = 2m + 1;$

$f(v_i) = m + (i-1), \qquad \forall\ i = 1, 2, \dots, m;$

$f(v_i^{'}) = m - i, \qquad \forall\ i = 1, 2, \dots, m;$

$f(u_j) = 2m + 2j + l, \qquad \forall\ j = 1, 2, \dots, n;$

$f(u_j^{'}) = 2(m+n) + j + l, \qquad \forall\ j = 1, 2, \dots, n.$

Above defined labeling pattern give rise a semi smooth graceful labeling to the graph G. □

Illustration 2.4. *$S^{'}(B_{3,5})$ and its semi smooth graceful labeling shown in figure 2. Here $q = |E(S^{'}B_{3,5})| = 27$.*

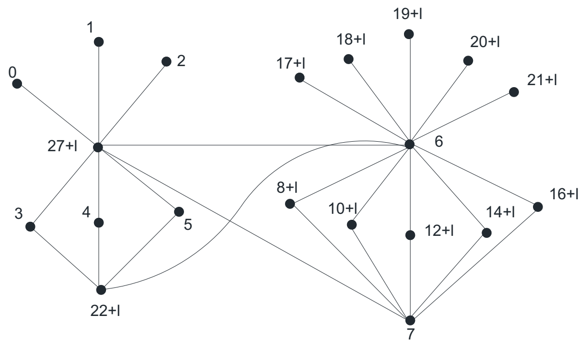

Figure 2. $S^{'}(B_{3,5})$ **and its semi smooth graceful labeling.**

Theorem 2.5. *$S^{'}(P_n)$ is a semi smooth graceful graph.*

Proof. Let P_n be the path on consecutive $v_1, v_2, \dots, v_n$ vertices. For the graph $S^{'}(P_n)$ added vertices corresponding to $v_1, v_2, \dots, v_n$ are $v_1^{'}, v_2^{'}, \dots, v_n^{'}$. Obviously $|V(S(P_n))| = 2n$ and $|E(S^{'}(P_n))| = 3n - 3$.

We shall define labeling function $f : V(S^{'}(P_n)) \longrightarrow \{0, 1, \dots, t-1, t+l, t+l+1, \dots\dots, q+l\}$, where $q = |E(S^{'}(P_n))| = 3n-3$, $t = 3\lfloor \frac{n}{2} \rfloor - 1$ and l is the arbitrary non-negative integer as follows,

$f(v_i) = q - \frac{3}{2}(i-1) + l, \qquad$ when i is odd,

$\qquad = (\frac{3i}{2}) - 2, \qquad$ when i is even, $\forall\ i = 1, 2, \dots, n;$

$f(v_i^{'}) = f(v_i) - 1, \qquad \forall\ i = 1, 2, \dots, n.$

Above defined labeling pattern give rise a semi smooth graceful labeling to the graph $S^{'}(P_n)$ and so $S^{'}(P_n)$ is a semi smooth graceful graph. □

Illustration 2.6. *$S^{'}(P_7)$ and its semi smooth graceful labeling shown in figure 3. Here $q = |E(S^{'}(P_7))| = 18$.*

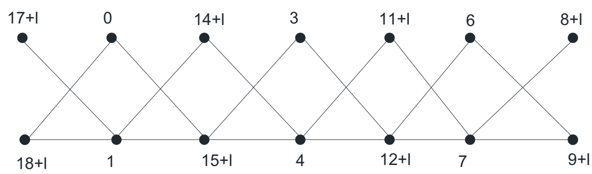

Figure 3. $S^{'}(P_7)$ **and its semi smooth graceful labeling.**

Theorem 2.7. *Let G be a semi smooth graceful graph. A graph obtained by joining G and $B^2_{m,n}$ with an arbitrary path P_r is graceful.*

Proof. Let K be the graph obtained by joining a semi smooth graph G, $B^2_{m,n}$ with an arbitrary path P_r. Let $q_1 = |E(G)|$, $q_2 = r - 1$, $q_3 = 2(m+n)+1$. Let G be a semi smooth graceful graph with semi smooth vertex labeling function $f : V(G) \longrightarrow \{0, 1, \dots, t-1, t+l, t+l+1, \dots, q_1+l\}$, whose induced edge labeling function is absolute difference of end vertices for each edge and $t \in \{1, 2, \dots, q_1\}$, l be an arbitrary non-negative integer.

Since G is a bipartite graph, we shall have following partitions of $V(G)$.

$V_1 = \{u \in V(G)/f(u) < t\ \}$

$V_2 = \{u \in V(G)/f(u) \geq t\}$

Let $w_0 \in V(G)$ be such that $f(w_0) = t-1$, such vertex would lies in V_1, otherwise $1+l$ edge label can not be produce in G. In fact $1+l$ edge label can be produce by the vertices of G whose vertex labels are $t-1$, $t+l$ and they are adjacent in G. Let $w_0 = w_1, w_2, \dots, w_r$ be the vertices of P_r. Let $u_i (i = 0, 1, \dots, m), v_j (j = 0, 1, \dots, n)$ be vertices of $B^2_{m,n}$ with $w_r = v_0$. Obviously $|E(K)| = q_1+q_2+q_3 = q_1+r+2(m+n)$. Now we define vertex labeling function $g : V(K) \longrightarrow \{0, 1, \dots, q_1+q_2+q_3\}$ as follows.

$g(w) = f(w)$, when $w \in V_1$;

$g(w) = f(w) - l + r + 2(m+n)$, when $w \in V_2$;

$g(w_i) = t + r + 2(m+n) - \frac{i}{2}$, when i is even,

$\quad = t + \frac{i-3}{2}$, when i is odd , $\forall\ i = 1, 2, \dots, r$;

Case$-$I : r is odd.

$g(v_0) = g(w_r)$, $g(u_0) = g(w_r) + q_3$;

$g(v_j) = g(w_r) + j$, $\forall\ j = 1, 2, \dots, n$;

$g(u_i) = g(w_r) + n + i$, $\forall\ i = 1, 2, \dots, m$;

Case$-$II : r is even.

$g(v_0) = g(w_{r-1}) + q_3$, $g(u_0) = g(w_{r-1}) + 1$;

$g(v_j) = g(w_{r-1}) + j + 1$, $\forall\ j = 1, 2, \dots, n$;

$g(u_i) = g(w_{r-1}) + n + i + 1$, $\forall\ i = 1, 2, \dots, m$.

Above labeling function give rise graceful labeling to the graph K and so it is graceful. □

Illustration 2.8. *A graph obtained by joining $S^{'}(P_4)$ and $B^2_{2,3}$ with path P_5 and its graceful labeling is shown in figure 4*

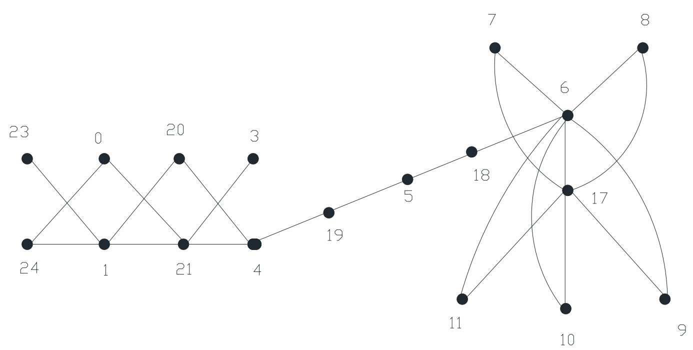

Figure 4. **A graph obtained by joining $S^{'}(P_4)$ and $B^2_{2,3}$ with path P_5 and its graceful labeling.**

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