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Common Fixed Point Theorems in Intuitionistic Generalized Fuzzy Metric Spaces

Research Article

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Abstract: In this paper, we prove Common fixed point theorems in intuitionistic generalized fuzzy metric spaces. We also, discuss result related to R-weakly commuting mappings.

MSC: 47H10, 54H25.

Keywords: R-weakly commuting mappings and intuitionstic fuzzy metric spaces, R-weakly commuting mappings of type $(P-1)$ and type $(P-2)$.

1. Introduction

The concept of fuzzy sets was introduced by zadeh [15] following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michlek [6] and George and Veeramani [5] modified the notion of fuzzy metric spaces with the help of continuous t-norms.

As a generalization of fuzzy sets, Atanassove [2] introduced and studied the concept of intuitionistic fuzzy sets. Park [9] using the idea of intuitionistic fuzzy sets defined the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalized of fuzzy metric spaces, George and Veeramani [5] showed that every metric induces an intuitionistic fuzzy metric, every fuzzy metric space in an intuitionsitic fuzzy metric space and found a necessary and sufficient condition for an intuitionistic fuzzy metric space to be complete Choudhary [4] introduced mutually contractive sequence of self maps and proved a fixed point theorem. Kramosil and Michlek [6] introduced the notion of Cauchy sequences in an intuitionistic fuzzy metric space and proved the well known fixed point theorem of Banach [3], Turkoglu et al [14] gave the generalization of jungck's Common fixed point theorem to intuitionistic fuzzy metric spaces.

In 2006, Sedghi and Shobe [12] defined $\mathcal{M}$-fuzzy metric spaces and proved a common fixed point theorem for four weakly Compatible mappings in this spaces. In 2009, Mehra and Gugnani [7] defined the notion of an intuitionistic $\mathcal{M}$-fuzzy metric spaces due to Sedghi and Shobe and proved a common fixed point theorem for six mappings for property (E) in this newly defined space. Our result is an intuitionistic generalized fuzzy metric space in R-weakly commuting mappings.

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2. Preliminaries

Definition 2.1. *A binary operation* $* : [0,1] \times [0,1]$ *is a continuous t-norm if it satisfies the following condition.*

*1) * is associative and commutative,*

*2) * is continuous ,*

3) $a * 1 = a$ *for all* $a \in [0,1]$*,*

4) $a * b < c * d$ *whenever* $a = c$ *and* $b = d$ *for each* $a, b, c, d \in [0,1]$.

Two typical example of a continuous t-norm are $a * b = ab$ and $a * b = \min\{a, b\}$

Definition 2.2. *A binary operation* $\diamondsuit : [0,1] \times [0,1] \to [0,1]$ *is a continuous t-conorm if it satisfies the following conditions:*

1) $\diamondsuit$ *is associative and commutative,*

2) $\diamondsuit$ *is continuous,*

3) $a\diamondsuit 0 = a$ *for all* $a \in [0,1]$*,*

4) $a\diamondsuit b \leq c\diamondsuit d$ *whenever* $a \leq c$ *and* $b \leq d$*, for each* $a, b, c, d \in [0,1]$.

Two typical examples of a continuous t-conorm are $a\diamondsuit b = \min\{1, a + b\}$ and $a\diamondsuit b = \max\{a, b\}$.

Definition 2.3. *A 5-tuple* $(X, \mathcal{M}, \mathcal{N}, *, \diamondsuit)$ *is called an intuitionistic generalized fuzzy metric space if* X *is an arbitrary (non-empty) set, * is a continuous t-norm,* $\diamondsuit$ *a continuous t-conorm and* $\mathcal{M}$*,* $\mathcal{N}$ *are fuzzy sets on* $X^3 x(0,\infty)$*, satisfying the following conditions: for each* $x, y, z, a \in X$ *and* $t, s > 0$.

a) $\mathcal{M}(x, y, z, t) + \mathcal{N}(x, y, z, t) = 1$*,*

b) $\mathcal{M}(x, y, z, t) > 0$*,*

c) $\mathcal{M}(x, y, z, t) = 1$ *if and only if* $x = y = z$*,*

d) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$*, where* p *is a permutation function,*

e) $\mathcal{M}(x, y, z, a, t) * \mathcal{M}(a, z, z, s) = M(x, y, z, t + s)$*,*

f) $\mathcal{M}(x, y, z, .) : (0,\infty) \to [0,1]$ *is continuous,*

g) $\mathcal{N}(x, y, z, t) > 0$*,*

h) $\mathcal{N}(x, y, z, t) = 0$*, if and only if* $x = y = z$*,*

i) $\mathcal{N}(x, y, z, t) = \mathcal{N}(p\{x, y, z\}, t)$ *where* p *is a permutation function,*

j) $\mathcal{N}(x, y, z, a, t)\diamondsuit\mathcal{N}(a, z, z, s) = \mathcal{N}(x, y, z, t + s)$*,*

k) $\mathcal{N}(x, y, z) : (0,\infty) \to [0,1]$ *is continuous.*

Then $(\mathcal{M}, \mathcal{N})$ *is called an intuitionistic generalized fuzzy metric on X.*

Example 2.4. *Let* $X = R$ *and* $\mathcal{M}(x,y,z,t) = \frac{t}{t\ +\ |x-y|+|y-z|+|z-x|}$, $\mathcal{N}(x,y,z,t) = \frac{|x-y|+|y-z|+|z-x|}{t\ +\ |x-y|+|y-z|+|z-x|}$ *for every x, y, z and* $t > 0$, *let A and B be defined as* $Ax = 2x + 1, Bx = x + 2$, *consider the sequence* $x_n = \frac{1}{n} + 1, n = 1, 2, \ldots$ *Thus we have* $\lim_{n\to\infty} \mathcal{M}\ (Ax_n,\ 3,\ 3,\ t) = \lim_{n\to\infty} \mathcal{M}\ (Bx_n,\ 3,\ 3,\ t) = 1$ *and* $\lim_{n\to\infty} \mathcal{N}\ (Ax_n,\ 3,\ 3,\ t) = \lim_{n\to\infty} \mathcal{N}\ (Bx_n,\ 3,\ 3,\ t) = 0$, *for every* $t > 0$. *Then A and B satisfying the property (E).*

Lemma 2.5. *Let* $(X, \mathcal{M}, \mathcal{N}, *, \diamondsuit)$ *be an intuitionistic generalized fuzzy metric space. Then* $\mathcal{M}(x,y,z,t)$ *and* $\mathcal{N}(x,y,z,t)$ *are non-decreasing with respect to t, for all x, y, z in X.*

Definition 2.6. *Let* $(X, \mathcal{M}, \mathcal{N}, *, \diamondsuit)$ *be an intuitionistic generalized fuzzy metric space. Then*

1) a sequence $\{x_n\}$ *in X is said to be canchy sequence if for all* $t > 0$ *and* $p > 0$, $\lim_{n\to\infty} \mathcal{M}\,(x_{n+p},\ x_n, x_n, t) = 1$ *and* $\lim_{n\to\infty} \mathcal{N}\,(x_{n+p},\ x_n, x_n, t) = 0$

2) a sequence $\{x_n\}$ *in X is said to be convergent to a point* $x \in X$, *if for all* $t > 0$, $\lim_{n\to\infty} \mathcal{M}\,(x_n, x, x, t) = 1$ *and* $\lim_{n\to\infty} \mathcal{N}\,(x_n, x, x, t) = 0$

3) An intuitionistic generalized fuzzy metric space $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *is said to be complete if and only if every Cauchy sequence in X is convergent.*

Definition 2.7. *Let A and S be maps from an intuitionistic generalized fuzzy metric space* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *into itself. The maps A and S are said to be weakly commuting if* $\mathcal{M}\,(ASz, SAz, SAz, t) \geq \mathcal{M}(Az, Sz, Sz, t)$ *and* $\mathcal{N}\,(ASz, SAz, SAz, t) \leq \mathcal{N}(Az, Sz, Sz, t)$ *for all* $z \in X$ *and* $t > 0$.

Definition 2.8. *Let A and S be maps from an intuitionistic generalized fuzzy metric space* $(X,\ \mathcal{M},\ \mathcal{N}, *, \diamondsuit)$ *into itself. The maps A and S are said to be compatible if for all* $t > 0$, $\lim_{n\to\infty} \mathcal{M}\,(ASx_n, SAx_n, SAx_n, t) = 1$, *and* $\lim_{n\to\infty} \mathcal{N}\,(ASx_n, SAx_n, SAx_n, t) = 0$, *whenever* $\{x_n\}$ *is a sequence in X such that* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$ *for some* $z \in X$.

Definition 2.9. *A pair of self mappings (A, S) of a intuitionistic generalized fuzzy metric space* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *is said to be point wise R-weakly commuting, if given x in X, there exist* $R > 0$ *such that for all* $t > 0$, $\mathcal{M}\,(ASx,\ SAx,\ SAx,\ t) \geq \mathcal{M}\left(Ax,\ Sx,\ Sx,\ \frac{t}{R}\right), \mathcal{N}\,(ASx,\ SAx,\ SAx,\ t) \leq \mathcal{N}(Ax,\ Sx, Sx,\ \frac{t}{R})$ *clearly, every pair of weakly commuting mappings is point wise R-weakly commuting with R=1.*

Definition 2.10. *A pair of self mappings (A, S) of a intuitionistic generalized fuzzy metric space* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *is said to be R-weakly commuting of type (P-1) if there exists some* $R > 0$ *such that* $\mathcal{M}\,(SSx,\ ASx,\ ASx,\ t) \geq \mathcal{M}\left(Sx,\ Ax,\ Ax,\ \frac{t}{R}\right), \mathcal{N}(SSx,\ ASx,\ ASx,\ t) \leq \mathcal{N}(Sx, Ax, Ax, \frac{t}{R})$ *for all* $x \in X$ *and* $t > 0$.

Definition 2.11. *A pair of self mappings (A, S) of a intuitionistic generalized fuzzy metric space* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *is said to be R-weakly commuting of type (P-2), if there exists some* $R > 0$, *such that* $\mathcal{M}\,(AAx,\ SAx,\ SAx,\ t) \geq \mathcal{M}\left(Ax,\ Sx, Sx, \frac{t}{R}\right), \mathcal{N}(AAx,\ SAx,\ SAx,\ t) \leq \mathcal{N}(Ax,\ Sx,\ Sx,\ \frac{t}{R})$ *for all* $x \in X$ *and* $t > 0$.

Definition 2.12. *Let* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *be a intuitionistic generalized fuzzy metric space, A and S be self maps on X. A point x in X is called a coincidence point of A and S if and only if* $Ax = Sx$. *In this case,* $w = Ax = Sx$ *is called a point of coincidence of A and S.*

Definition 2.13. *A pair of self mappings (A, S) of a intuitionistic generalized fuzzy metric space* $(X,\ \mathcal{M},\ \mathcal{N},\ *,\ \diamondsuit)$ *is said to be weakly compatible if they commute at the coincidence points if* $Au = Su$ *for some u in X, then* $ASu = SAu$.

Definition 2.14. *A pair of self mappings (A, S) of a intuitionistic generalized fuzzy metric space $(X, \mathcal{M}, \mathcal{N}, *, \diamondsuit)$ is said to be R-weakly commuting of type (P) if there exists some $R > 0$ such that $\mathcal{M}(AAx, SSx, SSx, t) \geq \mathcal{M}\left(Ax, Sx, Sx, \frac{t}{R}\right), \mathcal{N}(AAx, SSx, SSx, t) \leq \mathcal{N}(Ax, Sx, Sx, \frac{t}{R})$ for all $x \in X$ and $t > 0$.*

Lemma 2.15. *Let $\{x_n\}$ be a sequence in an intuitionistic generalized fuzzy metric space $(X, \mathcal{M}, \mathcal{N}, *, \diamondsuit)$, if there exists a constant $k \in (0,1)$ such that $\mathcal{M}(x_n, x_{n+1}, x_{n+1}, kt) \geq \mathcal{M}(x_{n-1}, x_n, x_n, t)$ and $\mathcal{N}(x_n, x_{n+1}, x_{n+1}, kt) \leq \mathcal{N}(x_{n-1}, x_n, x_n, t)$ for all $t > 0$. Then $\{x_n\}$ is Cauchy sequence in X.*

Lemma 2.16. *Let $(X, \mathcal{M}, \mathcal{N}, *, \diamondsuit)$ be an intuitionistic generalized fuzzy metric space and for all $x, y, z, \in X$, $t > 0$, and if for a number $k \in (0, 1)$, $\mathcal{M}(x, y, z, kt) \geq \mathcal{M}(x, y, z, t)$ and $\mathcal{N}(x, y, z, kt) \leq \mathcal{N}(x, y, z, t)$ then $x = y = z$.*

3. Main Results

Theorem 3.1. *Let $(X, \mathcal{M}, \mathcal{N}, *, \diamondsuit)$ be a complete intuitionistic generalized metric space. Let f and g be weakly compatible self maps of X satisfying*

$$\mathcal{M}(gx, gy, gz, kt) \geq \mathcal{M}(fx, fy, fz, t), \mathcal{N}(gx, gy, gz, kt) \leq \mathcal{N}(fx, fy, fz, t) \quad where 0 < k < 1, \tag{1}$$

$$g(X) \subseteq f(X). \tag{2}$$

If one of g(X) or f(X) is complete then f and g have a unique common fixed point.

Proof. Let $x_0 \in X$. Since $g(X) \subseteq f(X)$, choose $x_1 \in X$ such that $g(x_0) = f(x_1)$. In general, choose x_{n+1} such that $y_n = fx_{n+1} = gx_n$. Then by (1),

$$\begin{aligned}\mathcal{M}(fx_n, fx_{n+1}, fx_{n+1}, t) &= \mathcal{M}(gx_{n-1}, gx_n, gx_n, t)\\ &\geq \mathcal{M}(fx_{n-1}, fx_n, fx_n, \frac{t}{k}) = \mathcal{M}(gx_{n-2}, gx_{n-1}, gx_{n-1}, \frac{t}{k}) \cdots \geq \mathcal{M}(fx_0, fx_n, fx_n, \frac{t}{k^n})\\ \mathcal{N}(fx_n, fx_{n+1}, fx_{n+1}, t) &= \mathcal{N}(gx_{n-1}, gx_n, gx_n, t)\\ &\leq \mathcal{N}(fx_{n-1}, fx_n, fx_n, \frac{t}{k}) = \mathcal{N}(gx_{n-2}, gx_{n-1}, gx_{n-1}, \frac{t}{k}) \cdots \leq \mathcal{N}(fx_0, fx_1, x_1, \frac{t}{k^n})\end{aligned}$$

Therefore for any p,

$$\begin{aligned}\mathcal{M}(fx_n, fx_{n+p}, fx_{n+p}, t) &\geq \mathcal{M}(fx_n, fx_{n+p}, fx_{n+p}, \frac{t}{p}) \geq \ldots (p-times) \geq \mathcal{M}(fx_{n+p-1}, fx_{n+p}, fx_{n+p}, \frac{t}{p})\\ &\geq \mathcal{M}\left(fx_0, fx_1, fx_1, \frac{t}{pk^n}\right) \geq \ldots (p-times) \geq \mathcal{M}(fx_0, fx_1, fx_1, \frac{t}{pk^{n+p-1}})\\ \mathcal{N}(fx_n, fx_{n+p}, fx_{n+p}, t) &\leq \mathcal{N}(fx_n, fx_{n+1}, fx_{n+1}, \frac{t}{p}) \leq \ldots (p-times) \leq \mathcal{N}(fx_{n+p-1}, fx_{n+p}, fx_{n+p}, \frac{t}{p})\\ &\leq \mathcal{N}(fx_0, fx_1, fx_1, \frac{t}{pk^n}) \leq \ldots (p-times) \leq \mathcal{N}(fx_0, fx_1, fx_1, \frac{t}{pk^{n+p-1}})\end{aligned}$$

As $n \to \infty$, $\{fx_n\} = \{y_n\}$ is Cauchy sequence and so, by completeness of X, $\{y_n\} = \{fx_n\}$ is convergent. Call the limit u, then $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = u$. As, f(X) is complete, so there exist a point p in X such that fp = u. Now, from (1)

$$\begin{aligned}&\mathcal{M}(gp, gx_n, gx_n, kt) \geq \mathcal{M}(fp, fx_n, fx_n, t) && \mathcal{N}(gp, gx_n, gx_n, kt) \leq \mathcal{N}(fp, fx_n, fx_n, t) \text{ as } n \to \infty\\ &\mathcal{M}(gp, u, u, kt) \geq \mathcal{M}(fp, u, u, t) && \mathcal{N}(gp, u, u, kt) \leq \mathcal{N}(fp, u, u, t)\\ &\mathcal{M}(gp, u, u, kt) \geq 1 && \mathcal{N}(gp, u, u, kt) \leq 0.\end{aligned}$$

This gives $gp = u = fp$. As f and g are weakly compatible, Therefore $fgp = gfp$, ie, $fu = gu$. Now, we show that u is a fixed point of f and g. From (1),

$$\mathcal{M}(gu, gx_n, gx_n, kt) \geq \mathcal{M}(fu, fx_n, fx_n, t) \qquad \mathcal{N}(gu, gx_n, gx_n, kt) \leq \mathcal{N}(fu, fx_n, fx_n, t) \text{ as } n \to \infty,$$
$$\mathcal{M}(gu, u, u, kt) \geq \mathcal{M}(fu, u, u, t) \qquad \mathcal{N}(gu, u, u, kt) \leq \mathcal{N}(fu, u, u, t)$$
$$\mathcal{M}(gu, u, u, kt) \geq \mathcal{M}(gu, u, u, t) \qquad \mathcal{N}(gu, u, u, kt) \leq \mathcal{N}(gu, u, u, t) . gu = u = fu.$$

Hence, u is a common fixed point of f and g. For uniqueness, let w be another fixed point of f and g, then by (1),

$$\mathcal{M}(gu, gw, gw, kt) \geq \mathcal{M}(fu, fw, fw, t) \qquad \mathcal{N}(gu, gw, gw, kt) \leq \mathcal{N}(fu, fw, fw, t)$$
$$\mathcal{M}(u, w, w, kt) \geq \mathcal{M}(u, w, w, t) \qquad \mathcal{N}(u, w, w, kt) \leq \mathcal{N}(u, w, w, t)$$

Therefore u is unique common fixed point of f and g. ☐

Example 3.2. *Let $X = [0, 1]$. Define $(\mathcal{M}, \mathcal{N})$ by*

$$\mathcal{M}(x, y, z, t) = \begin{cases} \frac{t}{t + |x-y|+|y-z|+|z-x|}, & t > 0; \\ 0, & t = 0. \end{cases} \text{ and } \mathcal{N}(x, y, z, t) = \begin{cases} \frac{|x-y|+|y-z|+|z-x|}{t + |x-y|+|y-z|+|z-x|}, & t > 0; \\ 1, & t = 0. \end{cases}$$

Clearly, $(X, \mathcal{M}, \mathcal{N}, *, \diamondsuit)$ is complete intuitionistic fuzzy metric space. Define self maps f and g on X by $f(x) = \frac{x}{2}$, $g(x) = \frac{x}{6}$. Then $g(X) \subseteq f(X)$ and for $\frac{1}{3} < q < 1$, condition (1) satisfied. However, maps are weakly compatible at $x = 0$ and $x = 0$ is unique common fixed point of f and g.

Theorem 3.3. *Let $(X, \mathcal{M}, \mathcal{N}, *, \diamondsuit)$ be an intuitionistic generalized fuzzy metric space. Let f and g be weakly compatible self maps of X satisfying condition (1) and (2)*

(1) $\mathcal{M}(gx, gy, gz, kt) \geq \mathcal{M}(fx, fy, fy, t), \mathcal{N}(gx, gy, gz, kt) \leq \mathcal{N}(fx, fy, fy, t)$ *where $0 < k < 1$*

(2) *$g(X) \subseteq f(X)$ and*

If one of g(X) or f(X) is complete, then f and g have a unique common fixed point.

Proof. From the proof of the above theorem, we conclude that $\{fx_n\}=\{y_n\}$ is Cauchy sequence in X, now, suppose that f(X) is a complete subspace of X, then the sequence of $\{y_n\}$ must get a limit, in f(X). Call it be u and $f(v) = u$. As $\{y_n\}$ is a Cauchy sequence containing a convergent subsequence, therefore, the sequence $\{y_n\}$ also converges implying there by the convergence of subsequence of the convergent subsequence. Now, from (1),

$$\mathcal{M}(gv, gx_n, gx_n, kt) \geq \mathcal{M}(fv, fx_n, fx_n, t) \qquad \mathcal{N}(gv, gx_n, gx_n, kt) \leq \mathcal{N}(fv, fx_n, fx_n, t) \text{ as } n \to \infty,$$
$$\mathcal{M}(gv, u, u, kt) \geq \mathcal{M}(fv, u, u, t) \qquad \mathcal{N}(gv, u, u, kt) \leq \mathcal{N}(fv, u, u, t),$$
$$\mathcal{M}(gv, u, u, kt) \geq \mathcal{M}(u, u, u, t) \qquad \mathcal{N}(gv, u, u, kt) \leq \mathcal{N}(u, u, u, t)$$
$$\mathcal{M}(gv, u, u, kt) \geq 1, \qquad \mathcal{N}(gv, u, u, kt) \leq 0.$$

This gives $gv = u = fv$, which shows that pair (f, g) has a point of coincidence. Since f and g are weakly compatible, therefore $fgv = gfv$ ie, $fu = gu$. Now, we show that u is a fixed point of f and g .From (1),

$$\mathcal{M}(gu, gx_n, gx_n, kt) \geq \mathcal{M}(fu, fx_n, fx_n, t) \qquad \mathcal{N}(gu, gx_n, gx_n, kt) \leq \mathcal{N}(fu, fx_n, fx_n, t) \text{ as } n \to \infty,$$
$$\mathcal{M}(gu, u, u, kt) \geq \mathcal{M}(fu, u, u, t) \qquad \mathcal{N}(gu, u, u, kt) \leq \mathcal{N}(fu, u, u, t)$$
$$\mathcal{M}(u, u, u, kt) \geq \mathcal{M}(u, u, u, t) \qquad \mathcal{N}(u, u, u, kt) \leq \mathcal{N}(u, u, u, t)$$

$gu = u = fu$. Hence, u is a fixed point of f and g. For uniqueness, let w be another fixed point of f and g, then by (1),

$$\mathcal{M}(gu, gw, gw, kt) \geq \mathcal{M}(fu, fw, fw, t) \qquad \mathcal{N}(gu, gw, gw, kt) \leq \mathcal{N}(fu, fw, fw, t)$$
$$\mathcal{M}(u, w, w, kt) \geq \mathcal{M}(u, w, w, t) \qquad \mathcal{N}(u, w, w, kt) \leq \mathcal{N}(u, w, w, t).u = w$$

Therefore, u is a unique common fixed point of f and g. □

Theorem 3.4. *Theorem 3.3 remains true, if a weakly compatible property is replaced by any one of the following:*

(i) R-weakly commuting property,

(ii) R-weakly commuting property of type (P-1),

(iii) R-weakly commuting property of type (P-2),

(iv) R- weakly commuting property of type (P),

(v) Weakly commuting property.

Proof. (i) Since all the conditions of Theorem 3.3 are satisfied , then the existence of coincidence points for both the pairs are insured. Let x be an arbitrary point of coincidence for the pair (f, g) , then using R- weak commutativity, one gets

$$\mathcal{M}(fgx,\ gfx, gfx,\ t) \geq \mathcal{M}\left(fx,\ gx,\ gx,\ \frac{t}{R}\right) = \mathcal{M}\left(fx,\ fx,\ fx,\ \frac{t}{R}\right) = 1,$$
$$\mathcal{N}(fgx,\ gfx,\ gfx,\ t) \leq \mathcal{N}(fx,\ gx,\ gx,\ \frac{t}{R}) = \mathcal{N}(fx,\ fx,\ fx,\ \frac{t}{R}) = 0.$$

fgx = gfx. Thus, the pair (f, g) is weakly compatible. Now applying Theorem 3.3, one conclude that f and g have a unique common fixed point.

(ii) In case (f, g) is an R- weakly commuting pair of type (P-1) , then

$$\mathcal{M}(ggx,\ fgx,\ fgx,\ t) \geq \mathcal{M}\left(gx,\ fx,\ fx,\ \frac{t}{R}\right) = \mathcal{M}\left(fx,\ fx,\ fx,\ \frac{t}{R}\right) = 1,$$
$$\mathcal{N}(ggx,\ fgx,\ fgx,\ t) \leq \mathcal{N}(gx,\ fx,\ fx,\ \frac{t}{R}) = \mathcal{N}(fx,\ fx,\ fx,\ \frac{t}{R}) = 0.\ ggx = fgx.$$
$$\mathcal{M}(fgx,\ gfx,\ gfx,\ t) \geq \mathcal{M}\left(fgx,\ ggx,\ ggx,\ \frac{t}{2}\right) * \mathcal{M}\left(ggx,\ gfx,\ gfx,\ \frac{t}{2}\right)$$
$$= \mathcal{M}\left(fgx,\ fgx,\ fgx,\ \frac{t}{2}\right) * \mathcal{M}\left(x,\ x, x,\ \frac{t}{2}\right) \geq 1 * 1 = 1.$$
$$\mathcal{N}(fgx,\ gfx,\ gfx,\ t) \geq \mathcal{N}\left(fgx,\ ggx,\ ggx,\ \frac{t}{2}\right) \Diamond \mathcal{N}\left(ggx,\ gfx,\ gfx,\ \frac{t}{2}\right)$$
$$= \mathcal{N}\left(fgx,\ fgx,\ fgx,\ \frac{t}{2}\right) \Diamond \mathcal{N}\left(x,\ x, x,\ \frac{t}{2}\right) \geq 0\ \ 0{=}\ 0\ .fgx = gfx.$$

Similarly, if pair (f, g) is R- weakly commuting of type (P-2), (P) or weakly commuting property then (f, g) also commutes at their point of coincidence. Now, in view of Theorem 3.3, in all five cases, f and g have a unique common fixed point. This completes the proof. □

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