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Some Generalized Results on G_δ-diagonal Regular Spaces

Research Article

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Abstract: In this paper we constructed a space X possessing a regular G_δ-diagonal set of full measure, then the space X has a diagonal set if $\{(x,x) : x \in X\}$ and is regular in $G_\delta - set$ for $X \times X$. We extend our result for some characterization of spaces with G_δ-diagonal set. Such a kind obtained by Borges.C.J.R [1], Cinder.J.G [2] and Zenor.P [5].

MSC: 22A15, 22A20, 22A10.

Keywords: Regular spaces, G_δ-diagonals,$w\Delta-$spaces

1. Introduction

A subset H of the space X is a regular G_δ-set if there is a sequence $\{U_n\}$ of open sets containing H such that $H = \bigcap\limits_{i=1}^{\infty} U_i = \bigcap\limits_{i=1}^{\infty} U_i^-$. A space X has a regular G_δ-diagonal if $\{(x,x) : x \in X\}$ is a regular, G_δ-set in $X \times X$.

2. Main Results

Theorem 2.1. *X has a G_δ-diagonal if and only if there is a sequence $\{G_n\}$ is open covers of X such that if $x \in X$, then $x = \bigcap\limits_{i=1}^{\infty} st(x, G_1)$.*

Theorem 2.2. *X has a regular G_δ-diagonal if and only if there is a sequence $\{G_n\}$ of open covers of X such that if x and y are distinct points of X, then there are an integer n and open sets U and V containing x and y respectively such that no member of $\{G_n\}$ intersects both U and V.*

From Theorem 2.1 and 2.2, we see that any paracompact T_2-space with a G_δ-diagonal has a regular G_δ-diagonal and a corollary to Theorem 2.2 that any space with a regular G_δ-diagonal is Hausdorff. A development $\{G_n\}$ for the space X is said to satisfy the 3-link property if it is true that if p and q are distinct points, then there is an integer n such that no member of $\{G_n\}$ intersects both at $st(p, G_n)$ and $st(q, G_n)$ (Heath [3]). According to Borges [1], a space X is a $w\Delta$-space if there is a sequence $\{G_n\}$ of open covers of X such that if x is a point and, for each n, x_n is a point of $st(x, G_n)$, then the sequence $\{x_n\}$ has a cluster point, further we extend the following results.

Theorem 2.3. *Let X be a topological space. Then the following conditions are satisfied.*

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(i) X admits a development satisfying the 3-link property.

(ii) X is a $w\Delta$-space with a regular G_δ-diagonal.

(iii) There is a semi-metric d on X such that

(A) If $\{x_n\}$ and $\{y_n\}$ are sequences both converging to x, then $\lim_{n\to\infty} d(x_n, y_n) = 0$

(B) If x and y are distinct points of X and $\{x_n\}$ and $\{y_n\}$ are sequences converging to x and y respectively.

Proof. According to Health [3] and Moore [4], a space X is an M-space if there is normal sequence $\{G_n\}$ of open covers of X such that is x is a point and, for each n, x_n is a point of $st(x, G_n)$, then the sequence $\{x_n\}$ has a cluster point. □

Theorem 2.4. *If X is a topological space, then the following conditions are equivalent:*

(a) X is a metrizable.

(b) X is a T_1-M-space such that X^2 is a perfectly normal.

(c) X is an M-space with a regular G_δ-diagonal.

(d) X is a T_1-M-space such that X^3 is hereditarily normal.

(e) X is a T_1-M-space such that X^3 is hereditarily countable paracompact.

(f) X is an M-space that admits a one-to-one continuous function onto a metricspace.

Borges [1] shows that X is paracompact, locally connected and locally peripherally compact, then X is metrizable if and only if X has G_δ-diagonal.

Theorem 2.5. *If X is locally connected and locally peripherally compact, then X is metrizable if and only if X has a regular G_δ-diagonal.*

Proof. Let $\{U_n\}$ be a sequence of open covers of X such that each member of U_n is connected and such that if p and q are distinct points, then there are open sets U and V containing p and q respectively and an integer n such that no member of U_n intersects both $st(p, G_n)$ and $st(q, G_n)$. We will first show that $\{U_n\}$ is a development for X. To this end, let $x \in X$ and let U be an open set containing x. There is an open set V with compact boundary such that $x \in V \subset U$. Suppose that, for each n, there is member ,say g_n, of U_n that contains x and intersects X-V. Then, since each g_n is connected, there is a point x_n of the boundary of V that is in g_n. Since the boundary of V is compact, the sequence $\{x_n\}$ has cluster point, say x_0. It follows that $x_0 \in \bigcap_{n=1}^{\infty} cl\ (st\,(x, U_n))$ which is a contradiction. □

By Theorem 2.3, there is a development $\{G_n\}$ for X that satisfies the 3-link property. Since X is locally connected. Let x denote, we may assume that, for each n, the members of G_n are connected. Let x denoted a point of X and let U be an open set containing x, We will show that there is an integer n such that if $g \in G_n$ and $g \bigcap st\,(x, G_n) \neq \Phi$, then $g \subset U$. It will follow that X is metrizable by Moore's [4]. To this end, let V be an open subset of U containing x with compact, suppose that, for each n, there are members U_n and V_n of G_n such that $x \in U_n$ and $U_n \bigcap V_n \neq \phi$, and $(U_n \bigcap V_n) \bigcap (X - V) \neq \phi$. Since, for each n, $U_n \bigcup V_n$ is connected, there is point x_n of $U_n \bigcup V_n$ in the boundary of V. Since the boundary of V is compact, there is a cluster point x_0, of $\{x_n\}$. But it follows that , for each n, there is a member of G_n that intersects both of $st(x, G_n)$ and $st(x_0, G_n)$ which is contradiction.

References

[1] C.J.R.Berges, *On metrizability of topological spaces*, Canad. J Math., 20(1978), 795-803.

[2] J.G.Ceder, *Some generalization of metric spaces*, Pacific J Math., 11(1961), 105-125.

[3] R.W.Health, *Metrizablity, compactness and paracompacetness in Moore spaces*, Amer. Math. Soc., 10(1978), 105-125.

[4] R.L.Moore, *Products of normal spaces with metric spaces*, Math. Ann., 154(1964), 365-382.

[5] P.Zenor, *On spaces with regular spaces*, Pacific J Math., (1986), 122-132.