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Fundamental Relation on Intuitionistic Fuzzy Γ-hypermodule

Research Article

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Abstract: In this paper, we introduce and study the fundamental relation on intuitionistic fuzzy Γ-hypermodule as a generalization of the usual fuzzy Γ- hypermodule. Examples of fundamental relation on intuitionistic fuzzy Γ-hypermodule are constructed and some their proposition are proved.

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1. Introduction

The concept of Hyperstructure Theory (Hyper compositional algebra) was born in 1934, at the eighth congress of Scandinavian Mathematicians, when F. Marty first defined a hypergroup as a set equipped with an associative and reproductive hyperoperation and analysed their properties [4]. Algebraic hyperstructures represent a natural extension of classical algebraic structure. In classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructures the composition of two elements is a set. Because of extensive applications in many branches of mathematics and applied Science, the theory of algebraic hyperstructures (or hypersystems) has nowadays become a well-established branch in algebraic theory.

The theory of fuzzy sets, proposed by Zadeh [6] in 1965, has provided a useful mathematical tool for describing the behavior of systems that are too complex or ill defined to admit precise mathematical analysis by classical methods and tools. In this aspect, the concept of fuzzy groups was defined by Rosenfeld [5] and its structure was investigated.

In the year 1986 Atanassov [2] introduced intuitionistic fuzzy set as a generalization of fuzzy set. The study of Intuitionistic fuzzy hyper algebraic structures has started with the introduction of the concepts of intuitionistic fuzzy hypergroups.

In this paper, our aim is to introduce the concept of fundamental relation on intuitionistic fuzzy Γ-hypermodule as a generalization of the usual fuzzy Γ- hypermodule and Moreover, some their proposition are proved.

2. Preliminaries

In this section, some of the basic definitions are summarized that are needed in the following sequel.

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Definition 2.1 ([3]). *Let X be any non-empty set. The map $\mu : X \to [0,1]$ is called a fuzzy subset of X.*

Definition 2.2 ([1]). *Let ρ be an equivalence relation on a fuzzy Γ-hypersemigroup $(M,\circ)$ and μ,ν be two fuzzy subsets on M. We say that, $\mu\rho\nu$ if the following conditions hold,*

(i) if $\mu(a) > 0$, then $\exists\, b \in M$:$\nu(b) > 0$ and $a\rho b$,

(ii) if $\nu(x) > 0$, then $\exists\, y \in M$:$\mu(y) > 0$ and $x\rho y$.

An equivalence relation ρ on a fuzzy Γ-hypersemigroup $(M,\circ)$ is called a fuzzy regular relation (fuzzy strongly regular) on $(M,\circ)$ if, for all $a,b,c \in M$, $\gamma \in \Gamma$, the following implications holds, $a\rho b \Rightarrow (a\circ\gamma\circ c)\rho(b\circ\gamma\circ c)$ and $(c\circ\gamma\circ a)\rho(c\circ\gamma\circ b)$. This condition is equivalent to, $a\rho a', b\rho b' \Rightarrow (a\circ\gamma\circ b)\rho(a'\circ\gamma\circ b')$, for all $a,b,a',b' \in M, \gamma \in \Gamma$. $(a\rho a', b\rho b' \Rightarrow (a\circ\gamma\circ b)\bar{\rho}(a'\circ\gamma\circ b'))$.

Definition 2.3 ([1]). *An equivalence relation ρ on a fuzzy Γ-hypermodule $(M,\oplus,\odot)$ over a fuzzy Γ-hyperring $(R,\boxplus,\boxdot)$ and a canonical fuzzy hypergroup $(\Gamma,\otimes)$ is called a fuzzy regular relation on $(M,\oplus,\odot)$ if it is a fuzzy regular relation on $(M,\oplus)$ and for all $x,y \in M, r \in R$, $\alpha \in \Gamma$, then $x\rho y \Rightarrow (r\odot\alpha\odot x)\rho(r\odot\alpha\odot y)$.*

Definition 2.4 ([1]). *An equivalence relation ρ on a fuzzy Γ-hypersemigroup $(M,\circ)$ is called a fuzzy strongly regular relation on $(M,\circ)$ if, for all $a,a',b,b' \in M$, $\alpha \in \Gamma$, such that $a\rho b$ and $a'\rho b'$ then the following condition holds, $(a\circ\alpha\circ a') > 0$, $(b\circ\alpha\circ b') > 0 \Rightarrow x\rho y$ for all $x,y \in M$.*

Note 2.5 ([1]).

(i) If ρ is a fuzzy strongly relation on a fuzzy Γ-hypersemigroup $(M,\circ)$, then it is a fuzzy regular on $(M,\circ)$.

(ii) An equivalence relation ρ on a fuzzy Γ-hyperring $(R,\boxplus,\boxdot)$ is called a fuzzy strongly regular relation on $(R,\boxplus,\boxdot)$ if it is a fuzzy strongly regular relation both on $(R,\boxplus)$ and on $(R,\boxdot)$.

Definition 2.6 ([1]). *Let ρ be a fuzzy strongly regular relation on a fuzzy Γ-hyperring $(R,\boxplus,\boxdot)$ and θ be a fuzzy strongly regular relation on a canonical fuzzy Γ-hypergroup $(\Gamma,*)$. An equivalence relation δ on a fuzzy Γ-hypermodule $(M,\oplus,\odot)$ over a fuzzy Γ-hyperring $(R,\boxplus,\boxdot)$ and canonical fuzzy Γ-hypergroup $(\Gamma,\otimes)$ is called a fuzzy strongly regular relation on $(M,\oplus,\odot)$ if it is a fuzzy strongly regular relation on $(M,\oplus)$ and if $x\delta y, r\rho s$ and $\alpha\theta\beta$, then the following condition holds, for all $u \in M$, such that $(r\odot\alpha\odot x)(u) > 0$ and for all $v \in M$, such that $(s\odot\beta\odot y)(v) > 0$, we get $u\delta v$.*

Remark 2.7 ([1]). *We consider the following Γ- hyperoperations on the quotient set M/δ, $\bar{x}\star\bar{y} = \{\bar{z} | z \in x+y\} = \{\bar{z} | (x\oplus y)(z) > 0\}$, $\bar{r}\circledcirc\bar{\alpha}\circledcirc\bar{x} = \{\bar{z} | z \in r.\alpha.x\} = \{\bar{z} | (r\odot\alpha\odot x)(z) > 0\}$.*

Proposition 2.8 ([1]). *Let $(M,\oplus,\odot)$ be a fuzzy Γ-hypermodule over a fuzzy Γ-hyperring $(R,\boxplus,\boxdot)$ and canonical fuzzy hypergroup $(\Gamma,*)$. Let $(M,+,.)$ be the associated Γ-hypermodule over the corresponding Γ-hypergroup $(R,\uplus,\circ)$ and canonical hypergroup $(\Gamma,*)$. Then we have,*

(i) The relation δ is a fuzzy regular relation on $(M,\oplus,\odot) \Leftrightarrow (M/\delta,\star,\circledcirc)$ is a Γ-hypermodule over $(R,\uplus,\circ)$ and $(\Gamma,*)$.

(ii) The relation δ is a fuzzy strongly regular relation on $(M,\oplus,\odot)$ over $(R,\boxplus,\boxdot)$ and $(\Gamma,\otimes) \Leftrightarrow (M/\delta,\star,\circledcirc)$ is a Γ-module over R/ρ and Γ/θ.

Notation 2.9 ([1]). *If we denote by $\mathfrak{U}$ the set of all expressions consisting of finite fuzzy Γ-hyperoperations either on R,Γ,M or the external fuzzy Γ-hyperoperations applied on finite sets of elements of R,Γ,M then we have, $x\epsilon y \Leftrightarrow \exists u \in \mathfrak{U} : \{x,y\} \subset u$.*

Definition 2.10 ([1]). *An equivalence relation ϵ^* is called fundamental relation on a fuzzy Γ-hypermodule $(M, \oplus, \odot)$, if ϵ^* is a fundamental relation on the associated Γ-hypermodule $(M, +, .)$. Hence, ϵ^* is a fundamental relation on a fuzzy Γ-hypermodule $(M, \oplus, \odot)$ if and only if ϵ^* is the smallest fuzzy strongly equivalence relation on $(M, \oplus, \odot)$.*

Notation 2.11 ([1]). *If we denote by $\mathfrak{UF}$ the set of all expressions consisting of finite fuzzy Γ-hyperoperations either on R, Γ, M or the external fuzzy Γ-hyperoperations applied on finite sets of elements of R, Γ, M then we have, $x\epsilon y \Leftrightarrow \exists \mu_{f\gamma} \in \mathfrak{UF} : \{x, y\} \subset \mu_{f\gamma} \Leftrightarrow \mu_{f\gamma}(x) > 0$ and $\mu_{f\gamma}(y) > 0$. The relation ϵ^* is the transitive closure of ϵ. So, in order to obtain a fuzzy Γ-module starting from a fuzzy Γ-hypermodule, we first consider the relation ϵ, then the transitive closure ϵ^* of ϵ and finally the quotient structure $(M/\epsilon^*, \star, \circledcirc)$ of the fuzzy Γ-hypermodule $(M, \oplus, \odot)$.*

Definition 2.12 ([2]). *An Intuitionistic Fuzzy Set (IFS)A in X is an object of the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\}$, where the functions $\mu_A : X \to [0, 1]$ and $\gamma_A : X \to [0, 1]$ denote the degree of membership (namely, $\mu_A(x)$) and the degree of non-membership (namely, $\gamma_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.*

Definition 2.13 ([2]). *Let A and B be Intuitionistic Fuzzy Sets of the forms $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle | x \in X\}$ and $B = \{\langle x, \mu_B(x), \gamma_B(x)\rangle | x \in X\}$. Then*

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) The complement of A is denoted by $\bar{A}$ and is defined by $\bar{A} = \{\langle x, \gamma_A(x), \mu_A(x)\rangle | x \in X\}$

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x)\rangle | x \in X\}$

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x)\rangle | x \in X\}$

The Intuitionistic Fuzzy Sets $0_\sim = \{\langle x, 0, 1\rangle | x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle | x \in X\}$ are respectively the empty set and the whole set.

3. Fundamental Relation on Intuitionistic Fuzzy Γ-hypermodule

In [1], fuzzy regular relations are introduced in the context of fuzzy Γ-hypermodules. In this section, we extend this notion to Intuitionistic fuzzy Γ-hypermodule. Let ρ be an equivalence relation on an intuitionistic fuzzy Γ-hypersemigroup $(M, \circ)$ and A, B be two intuitionistic fuzzy subsets of M. We say that, $A\rho B$ if the following conditions hold,

(i) if $\mu_A(a) > 0$, then $\exists\, b \in M : \mu_B(b) > 0$ and $\gamma_A(a) > 0$, then $\exists\, b \in M : \gamma_B(b) > 0 \Rightarrow a\rho b$,

(ii) $\mu_B(x) > 0$, then $\exists\, y \in M : \mu_A(y) > 0$ and $\gamma_B(x) > 0$, then $\exists\, y \in M : \gamma_A(y) > 0 \Rightarrow x\rho y$.

Definition 3.1. *An equivalence relation ρ on an intuitionistic fuzzy Γ-hypersemigroup $(M, \circ)$ is called a intuitionistic fuzzy regular relation (intuitionistic fuzzy strongly regular) on $(M, \circ)$ if, for all $a, b, c \in M, \alpha \in \Gamma$, the following implications holds, $a\rho b \Rightarrow (a \circ \alpha \circ c)\rho(b \circ \alpha \circ c)$ and $(c \circ \alpha \circ a)\rho(c \circ \alpha \circ b)$. This condition is equivalent to, $a\rho a', b\rho b' \Rightarrow (a \circ \alpha \circ b)\rho(a' \circ \alpha \circ b')$, for all $a, b, a', b' \in M, \alpha \in \Gamma$. [$a\rho a', b\rho b' \Rightarrow (a \circ \alpha \circ b)\bar{\rho}(a' \circ \alpha \circ b')$].*

Example 3.2. *Let $M = \{0, 1\}$, then the four Intuitionistic fuzzy set of I defines an intuitionistic fuzzy Γ-hypersemigroup structure on I as follows,*

$0 \circ \alpha \circ 0 = A = \langle x, \mu_A(x), \gamma_A(x)\rangle$ *where* $\mu_A(x) = \frac{a_1}{0} + \frac{a_2}{1}$, $\gamma_A(x) = \frac{b_1}{0} + \frac{b_2}{1}$ *with* $0 \leq a_1 + b_1 \leq 1$ *and* $0 \leq a_2 + b_2 \leq 1$

$0 \circ \alpha \circ 1 = B = \langle x, \mu_B(x), \gamma_B(x)\rangle$ *where* $\mu_B(x) = \frac{a_3}{0} + \frac{a_4}{1}$, $\gamma_B(x) = \frac{b_3}{0} + \frac{b_4}{1}$ *with* $0 \leq a_3 + b_3 \leq 1$ *and* $0 \leq a_4 + b_4 \leq 1$

$1 \circ \alpha \circ 0 = C = \langle x, \mu_C(x), \gamma_C(x)\rangle$ *where* $\mu_C(x) = \frac{a_5}{0} + \frac{a_6}{1}$,$\gamma_C(x) = \frac{b_5}{0} + \frac{b_6}{1}$ *with* $0 \leq a_5 + b_5 \leq 1$ *and* $0 \leq a_6 + b_6 \leq 1$

$1 \circ \alpha \circ 1 = D = \langle x, \mu_D(x), \gamma_D(x)\rangle$ *where* $\mu_D(x) = \frac{a_7}{0} + \frac{a_8}{1}$,$\gamma_D(x) = \frac{b_7}{0} + \frac{b_8}{1}$ *with* $0 \leq a_7 + b_7 \leq 1$ *and* $0 \leq a_8 + b_8 \leq 1$

Let ρ *be an equivalence relation on* M*. Clearly,* $A\rho B$ *since,* $\mu_A(0) = a_1 > 0$, $\exists\, 1 \in M : \mu_B(1) = a_4 > 0$ *and* $\gamma_A(0) = b_1 > 0$, $\exists\, 1 \in M : \gamma_B(1) = b_4 > 0 \Rightarrow 0\rho 1$*. Also,* $\mu_B(1) = a_4 > 0$, $\exists\, 0 \in M : \mu_A(0) = a_1 > 0$ *and* $\gamma_B(1) = b_4 > 0$, $\exists\, 0 \in M : \gamma_A(0) = b_1 > 0 \Rightarrow 1\rho 0$.

Definition 3.3. *An equivalence relation* ρ *is an intuitionistic fuzzy* Γ*-hypermodule* $(M, \oplus, *)$ *over an intuitionistic fuzzy* Γ*-hyperring* $(R, +, .)$ *and an intuitionistic fuzzy canonical hypergroup* $(\Gamma, \otimes)$ *is called an intuitionistic fuzzy regular relation on* $(M, \oplus, *)$ *if it is an intuitionistic fuzzy regular relation on* $(M, \oplus)$ *and for all* $x, y \in M, r \in R, \alpha \in \Gamma$*, then* $x\rho y \Rightarrow (r * \alpha * x)\rho(r * \alpha * y)$ *where* $\mu_{r*\alpha*x}(a) > 0$*, then* $\exists\, b \in M : \mu_{r*\alpha*y}(b) > 0$ *and* $\gamma_{r*\alpha*x}(a) > 0$*, then* $\exists\, b \in M : \gamma_{r*\alpha*y}(b) > 0 \Rightarrow a\rho b$*.*

Definition 3.4. *Let* $(\Gamma, \times)$ *be an intuitionistic fuzzy canonical hypergroups. Let* $(R, +, .)$ *be an intuitionistic fuzzy* Γ*-hyperring. A nonempty set* M *together with two intuitionistic fuzzy* Γ*- hyperoperation* $\oplus, *$ *is called an intuitionistic fuzzy left* Γ*- hypermodule over* $(R, +, .)$ *if the following conditions hold,*

(1) $(M, \oplus)$ *is an intuitionistic fuzzy canonical* Γ*-hypergroup,*

(2) $* : R \times \Gamma \times M \to F^*(M)$ *is such that for all* $a, b \in M$, $r, s \in R$, $\alpha, \beta \in \Gamma$ *we have,*

(i) $r * \alpha * (a \oplus b) = (r * \alpha * a) \oplus (r * \alpha * b)$,

(ii) $(r + s) * \alpha * a = (r * \alpha * a) \oplus (s * \alpha * a)$,

(iii) $r * (\alpha \times \beta) * a = (r * \alpha * a) \oplus (r * \beta * a)$,

(iv) $r * \alpha * (s * \beta * a) = (r * \alpha * s) * \beta * a$.

Proposition 3.5. *An equivalence relation* ρ *is an intuitionistic fuzzy* Γ*-hypermodule* $(M, \oplus, *)$ *over an intuitionistic fuzzy* Γ*-hyperring* $(R, +, .)$ *and an intuitionistic fuzzy canonical hypergroup* $(\Gamma, \otimes) \Leftrightarrow \rho$ *is a regular relation on* $(M, +, .)$ *over the* Γ*-hyperring* $(R, \uplus, \circ)$ *and canonical hypergroup* $(\Gamma, \times)$*.*

Proof. Let us assume that, the equivalence relation ρ is an intuitionistic fuzzy regular relation on $(M, \oplus, *)$ over an intuitionistic fuzzy Γ-hyperring $(R, +, .)$

Claim: ρ is a regular relation on $(M, +, .)$. For that, let $x, y \in M, r \in R, \alpha \in \Gamma$. We have $x\rho y \Rightarrow (r * \alpha * x)\rho(r * \alpha * y)$, then by definition 3.3 $\mu_{r*\alpha*x}(a) > 0$, then $\exists\, b \in M : \mu_{r*\alpha*y}(b) > 0$ and $\gamma_{r*\alpha*x}(a) > 0$, then $\exists\, b \in M : \gamma_{r*\alpha*y}(b) > 0 \Rightarrow a\rho b$ also if $\mu_{r*\alpha*y}(a^{'}) > 0$, then $\exists\, b' \in M : \mu_{r*\alpha*x}(b') > 0$ and $\gamma_{r*\alpha*y}(a^{'}) > 0$, then $\exists\, b' \in M : \gamma_{r*\alpha*x}(b') > 0 \Rightarrow a'\rho b'$. Let $a\rho b$ and $a^{'}\rho b^{'}$, where $a, b, a^{'}, b^{'} \in M$. we have, $(a \oplus a')\rho(b \oplus b^{'}) \Leftrightarrow$ the following conditions hold,

$\mu_{a\oplus a'}(u) > 0$, then $\exists\, v \in M : \mu_{b\oplus b'}(v) > 0$ and

$\gamma_{a\oplus a'}(u) > 0$, then $\exists\, v \in M : \gamma_{b\oplus b'}(v) > 0 \Rightarrow u\rho v$, also if $\mu_{b\oplus b'}(t) > 0$, then $\exists\, w \in M : \mu_{a\oplus a'}(w) > 0$ and $\gamma_{b\oplus b'}(t) > 0$, then $\exists\, w \in M : \gamma_{a\oplus a'}(w) > 0 \Rightarrow t\rho w$. If $u \in a + a'$, then $\exists\, v \in b + b' : u\rho v$ and if $t \in b + b^{'}$, then $\exists\, w \in a + a' : t\rho w$ (ie) $(a + a^{'})\bar{\rho}(b + b^{'})$. Thus, $a\rho b, a'\rho b' \Rightarrow (a + a')\bar{\rho}(b + b')$. Hence, ρ is an intuitionistic fuzzy regular relation on $(M, \oplus) \Leftrightarrow \rho$ is a regular relation on $(M, +)$. Conversely, let us assume that ρ is a regular relation on $(M, +, .)$

Claim: ρ is an intuitionistic fuzzy regular relation on $(M, \oplus, *)$. If $x\rho y, r \in R, \alpha \in \Gamma$, then we have $(r * \alpha * x)\rho(r * \alpha * y)$ (ie)$\mu_{r*\alpha*x}(u) > 0$, then $\exists\, v \in M : \mu_{r*\alpha*y}(v) > 0$ and $\gamma_{r*\alpha*x}(u) > 0$, then $\exists\, v \in M : \gamma_{r*\alpha*y}(v) > 0 \Rightarrow u\rho v$, also if $\mu_{r*\alpha*y}(t) > 0$, then $\exists\, w \in M : \mu_{r*\alpha*x}(w) > 0$ and $\gamma_{r*\alpha*y}(t) > 0$, then $\exists\, w \in M : \gamma_{r*\alpha*x}(w) > 0 \Rightarrow t\rho w$. If $u \in r.\alpha.x$, then $\exists\, v \in r.\alpha.y : u\rho v$ and $t \in r.\alpha.y$, then $\exists\, w \in r.\alpha.x : t\rho w$ (ie) $(r.\alpha.x)\rho(r.\alpha.y)$. Thus, $x\rho y \Rightarrow (r.\alpha.x)\rho(r.\alpha.y)$. Hence, ρ is an intuitionistic fuzzy regular relation on $(M, \oplus, *)$. □

Definition 3.6. *An equivalence relation ρ is an intuitionistic fuzzy Γ-hypersemigroup $(M, \circ)$ is called an intuitionistic fuzzy strongly regular relation on $(M, \circ)$ if for all $a, b, a', b' \in M, \alpha \in \Gamma, r \in R, \alpha \in \Gamma$, such that $a\rho a'$ and $b\rho b'$, the following conditions hold, $\mu_{a*\circ*a'}(x) > 0$ and $\gamma_{a*\circ*a'}(x) > 0$ and $\mu_{b*\circ*b'}(y) > 0$ and $\gamma_{b*\circ*b'}(y) > 0 \Rightarrow x\rho y$ for all $x, y \in M$.*

Remark 3.7. *If ρ is an intuitionistic fuzzy strongly regular relation on an intuitionistic fuzzy Γ-hypersemigroup $(M, \circ)$, then it is an intuitionistic fuzzy regular relation on $(M, \circ)$.*

Definition 3.8. *An equivalence relation ρ on an intuitionistic fuzzy Γ-hyperring $(R, +, .)$ is called an intuitionistic fuzzy strongly regular relation on $(R, +, .)$ if it is an intuitionistic fuzzy strongly regular relation on both $(R, +)$ and $(R, .)$.*

Definition 3.9. *Let ρ be an intuitionistic fuzzy strongly regular relation on an intuitionistic fuzzy Γ-hyperring $(R, +, .)$ and θ be an intuitionistic fuzzy strongly regular relation on an intuitionistic fuzzy canonical hypergroup $(\Gamma, \otimes)$. An equivalence relation δ on an intuitionistic fuzzy Γ-hypermodule $(M, \oplus, *)$ over an intuitionistic fuzzy Γ-hyperring $(R, +, .)$ and intuitionistic fuzzy canonical hypergroup $(\Gamma, \otimes)$ is called an intuitionistic fuzzy strongly regular relation on $(M, \oplus, *)$ if it is an intuitionistic fuzzy strongly regular relation on $(M, \oplus)$ and if $x\delta y, r\rho s, \alpha\theta\beta$, then the following conditions hold, for all $u \in M$ such that $\mu_{r*\alpha*x}(u) > 0$ and $\gamma_{r*\alpha*x}(v) > 0$ and for all $v \in M$ such that $\mu_{s*\beta*y}(v) > 0$ and $\gamma_{s*\beta*y}(u) > 0 \Rightarrow u\delta v$.*

Proposition 3.10. *An equivalence relation δ is an intuitionistic fuzzy strongly regular relation on $(M, \oplus, *) \Leftrightarrow \delta$ is a strongly regular relation on $(M, +, .)$.*

Proof. Let us assume that, an equivalence relation δ is an intuitionistic fuzzy strongly regular relation on $(M, \oplus, *)$. **Claim:** δ is a strongly regular relation on $(M, +, .)$. If $x\delta y, x^{'}\delta y^{'}, r\rho s, \alpha\theta\beta$, then by definition 3.9 for all $u, v \in M, \mu_{r*\alpha*x}(u) > 0$ and $\gamma_{r*\alpha*x}(v) > 0$ also, $\mu_{s*\beta*y}(v) > 0$ and $\gamma_{s*\beta*y}(u) > 0$ and for all $t, w \in M, \mu_{r*\alpha*x'}(t) > 0$ and $\gamma_{r*\alpha*x'}(w) > 0$ also, $\mu_{s*\beta*y'}(w) > 0$ and $\gamma_{s*\beta*y'}(t) > 0$. Now, for all $u, v \in M, \mu_{x\oplus x'}(u) > 0$ and $\gamma_{x\oplus x'}(v) > 0$ also, $\mu_{y\oplus y'}(v) > 0$ and $\gamma_{y\oplus y'}(u) > 0 \Rightarrow u\delta v$. Similarly, for all $t, w \in M$, we get $t\delta w$. Also, for all $u \in M : u \in x + x'$ and for all $v \in M : v \in y + y^{'}$, we get $u\delta v$ and for all $t \in M : t \in r.\alpha.x$ and for all $w \in M : w \in s.\beta.y$, we get $t\delta w$. Thus, $(x+x')\bar{\bar{\delta}}(y+y')$ and $(r.\alpha.x)\bar{\bar{\delta}}(s.\beta.y)$. Hence, δ is an intuitionistic fuzzy strongly regular relation on $(M, \oplus, *) \Leftrightarrow \delta$ is a strongly regular relation on $(M, +, .)$. ☐

Remark 3.11. *We consider the following Γ- hyperoperations on the quotient set M/δ as, $\bar{x} \star \bar{y} = \{\bar{z} | z \in x + y\} = \{\bar{z} | (x \oplus y)(z) > 0\}$ where $\mu_{x\oplus y}(z) > 0$ and $\gamma_{x\oplus y}(z) > 0 \bar{r} \circledcirc \bar{\alpha} \circledcirc \bar{x} = \{\bar{z} | z \in r.\alpha.x\} = \{\bar{z} | (r * \alpha * x)(z) > 0\}$, where $\mu_{r*\alpha*x}(z) > 0$ and $\gamma_{r*\alpha*x}(z) > 0$.*

Definition 3.12. *An equivalence relation ϵ^{**} is called fundamental relation on an intuitionistic fuzzy Γ-hypermodule $(M, \oplus, *)$, if ϵ^{**} is a fundamental relation on the associated Γ-hypermodule $(M, +, .)$. Hence, ϵ^{**} is a fundamental relation on an intuitionistic fuzzy Γ-hypermodule $(M, \oplus, *)$ if and only if ϵ^{**} is the smallest intuitionistic fuzzy strongly equivalence relation on $(M, \oplus, *)$.*

Notation 3.13. *If we denote by $\mathfrak{UF}^*$ the set of all expressions consisting of finite intuitionistic fuzzy Γ-hyperoperations either on R, Γ, M or the external fuzzy Γ-hyperoperations applied on finite sets of elements of R, Γ, M then we have, $x\epsilon^* y \Leftrightarrow \exists A \in \mathfrak{UF}^* : \{x, y\} \subset A \Leftrightarrow \mu_A(x) > 0$ and $\gamma_A(x) > 0$ also $\mu_A(y) > 0$ and $\gamma_A(y) > 0$. The relation ϵ^{**} is the transitive closure of ϵ^*. So, in order to obtain an intuitionistic fuzzy Γ-module starting from an intuitionistic fuzzy Γ-hypermodule, we first consider the relation ϵ^*, then the transitive closure ϵ^{**} of ϵ^* and finally the quotient structure $(M/\epsilon^{**}, \star, \circledcirc)$ of an intuitionistic fuzzy Γ-hypermodule $(M, \oplus, *)$.*

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