

(1,2)*-Generalized Continuous Map in Fuzzy Bitopological Spaces

Research Article

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Abstract: In this paper, we introduce (1,2)*-Fuzzy generalized continuous maps and (1,2)*-Fuzzy \tilde{g} continuous maps and study their relations with existing generalized (1,2)*-Fuzzy continuous maps.

Keywords: Fuzzy bitopological space, (1,2)*-fuzzy g- continuous maps, (1,2)*-fuzzy \tilde{g} -continuous maps.

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1. Introduction

Levine [9] introduced the concept of Generalized closed sets in topological spaces. Sundaram et al [25] introduced and studied the concept of a class maps namely g- continuous maps which included the continuous maps and a class of gc irresolute maps. Recently Ravi et al [17] Ravi and Thivagar [16] and Duszynski et al [5] introduced (1,2)*-g-closed sets, (1,2)*-sg-closed sets and (1,2)*-f \tilde{g} -closed sets in bitopological space respectively. In this paper, we introduce (1,2)*-Fuzzy generalized continuous maps and (1,2)*-Fuzzy -f \tilde{g} continuous maps and study their relations with various generalized (1,2)*-fuzzy continuous maps.

2. Preliminaries

In this section, we list out the definitions and the results which are needed in sequel. Fuzzy sub sets in (X, τ_1, τ_2) will be denoted by λ, μ, γ and $F(X)$ denotes set of all fuzzy sub sets of (X, τ_1, τ_2) .

Definition 2.1. A fuzzy subset set λ of (X, τ) is called:

- (i) Fuzzy semi open (briefly, fs-open) if $\lambda \leq Cl(Int(\lambda))$ and a fuzzy semi closed (briefly, fs-closed) if $Int(Cl(\lambda)) \leq \lambda$ [1];
- (ii) Fuzzy pre open (briefly, fp-open) if $A \leq Int(Cl(\lambda))$ and a fuzzy pre closed (briefly, fp-closed) if $Cl(Int(\lambda)) \leq \lambda$ [4];
- (iii) Fuzzy α -open (briefly, f α -open) if $\lambda \leq IntCl(Int(\lambda))$ and a fuzzy α -closed (briefly, f α -closed) if $ClInt(Cl(\lambda)) \leq \lambda$ [4];
- (iv) Fuzzy semi-preopen (briefly, fsp-open) if $\lambda \leq ClInt(Cl(\lambda))$ and a fuzzy semi-pre closed (briefly, fsp-closed) if $IntCl(Int(\lambda)) \leq \lambda$ [26]. By FSPO (X, τ) , we denote the family of all fuzzy semi pre open sets of fts X . The semi closure (resp. α -closure, semi-pre closure) of a fuzzy set A of (X, τ) is the intersection of all fs-closed (resp. f α -closed, fsp-closed) sets that contain A and is denoted by $sCl(\lambda)$ (resp. $\alpha Cl(\lambda)$ and $spCl(\lambda)$).

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The following lemma is well-known,

Lemma 2.2. *Let λ be a fuzzy set in a fuzzy topological space (X, τ) . Then*

- (i) $\alpha Cl\lambda = \lambda \vee ClIntCl\lambda$.
- (ii) $sCl\lambda = \lambda \vee IntCl\lambda$.
- (iii) $pCl\lambda \geq \lambda \vee ClInt\lambda$.
- (iii) $spCl\lambda \geq \lambda \vee IntClInt\lambda$.

Definition 2.3. *A fuzzy set λ in a fuzzy topological space (X, τ) is called:*

- (i) [2] Fuzzy generalized closed set if $Cl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. We briefly denote it as fg-closed set.
- (ii) [11] Fuzzy semi-generalized closed set if $sCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy semi open. We briefly denote it as fsg-closed set.
- (iii) [8] Fuzzy generalized almost strongly semi-closed set if $\alpha Cl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy α -open. We briefly denote it as f α g-closed set.
- (iv) [23] Fuzzy semi-pre-generalized closed set if $spCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy semi-pre-open. We briefly denote it as fspg-closed set.
- (v) [3] Fuzzy generalized strongly closed set if $\alpha Cl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. We briefly denote it as fg α -closed set.
- (vi) [6] Fuzzy \ddot{g} -closed set (briefly $\ddot{f}g$ -closed set) if $cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fsg-open.
- (vii) [7] Fuzzy α gs-closed set if $\alpha cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy semi-open. We briefly denote it as f α s-closed set.
- (viii) [22] Fuzzy generalized semi-closed set if $sCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. We briefly denote it as fgs-closed set.
- (ix) [22] Fuzzy generalized semi-pre-closed set if $spCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. We briefly denote it as fgs p -closed set.

Definition 2.4 ([19]). *Let S be a fuzzy subset of a bitopological space X . Then S is said to be $\tau_{1,2}$ -fuzzy open if $S = \lambda \cup \mu$, where $\lambda \in \tau_1$ and $\mu \in \tau_2$. The complement of $\tau_{1,2}$ -fuzzy open set is called $\tau_{1,2}$ -fuzzy-closed.*

Definition 2.5. [19] *Let S be a subset of a bitopological space X . Then*

- (i) the $\tau_{1,2}$ -closure of S , denoted by $\tau_{1,2}\text{-cl}(S)$, is defined as $\cap\{F : S \subset F \text{ and } F \text{ is } \tau_{1,2}\text{-closed}\}$.
- (ii) the $\tau_{1,2}$ -interior of S , denoted by $\tau_{1,2}\text{-int}(S)$, is defined as $\cup\{F : F \subset S \text{ and } F \text{ is } \tau_{1,2}\text{-open}\}$.

Definition 2.6. *A fuzzy subset λ of a bitopological space (X, τ_1, τ_2) is called.*

- (i) (1,2)*-sg-closed set [16] if $(1,2)^*\text{-scl}(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is (1,2)*- semi open in X . The complement of (1,2)*-sg-closed set is called (1,2)*-sg-open set;
- (ii) (1,2)*-gs-closed set [16] if $(1,2)^*\text{-scl}(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $\tau_{1,2}$ -open in X . The complement of (1,2)*-gs-closed set is called (1,2)*-gs-open set;

- (iii) $(1,2)^*$ - αg -closed set [18] if $(1,2)^*\text{-}\alpha\text{cl}(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $\tau_{1,2}$ -open in X . The complement of $(1,2)^*$ - αg -closed set is called $(1,2)^*$ - αg -open set;
- (iv) $(1,2)^*$ - \hat{g} -closed set or $f\omega$ -closed set [5] if $\tau_{1,2}\text{-cl}(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $(1,2)^*$ -semi open in X . The complement of \hat{g} -closed (resp. ω -closed) set is called $(1,2)^*$ - \hat{g} -open (resp. ω -open) set;
- (v) $(1,2)^*$ - ψ -closed set [13] if $(1,2)^*\text{-scl}(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $(1,2)^*$ -sg-open in X . The complement of $(1,2)^*$ - ψ -closed set is called $(1,2)^*$ - ψ -open set;
- (vi) $(1,2)^*$ - $\check{g}\alpha$ -closed set [13] if $(1,2)^*\text{-}\alpha\text{cl}(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $(1,2)^*$ -sg-open in X . The complement of $(1,2)^*$ - $\check{g}\alpha$ -closed set is called $(1,2)^*$ - $\check{g}\alpha$ -open set;
- (vii) $(1,2)^*$ -gsp-closed set [21] if $(1,2)^*\text{-spscl}(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $\tau_{1,2}$ -open in X . The complement of $(1,2)^*$ -gsp-closed set is called $(1,2)^*$ -gsp-open set;
- (viii) $(1,2)^*$ -g-closed set [17] if $(1,2)^*\text{-}\tau_{1,2}\text{-cl}(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $\tau_{1,2}$ -open in X . The complement of $(1,2)^*$ - αg -closed set is called $(1,2)^*$ - αg -open set.

Definition 2.7. A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called;

- (i) $(1,2)^*$ - g_α -continuous [12] if $f^{-1}(V)$ is a $(1,2)^*$ - g_α -closed set of X for every $\sigma_{1,2}$ -closed set V of Y .
- (ii) $(1,2)^*$ - ψ -continuous [14] if $f^{-1}(V)$ is a $(1,2)^*$ - ψ -closed set of X for every $\sigma_{1,2}$ -closed set V of Y .
- (iii) $(1,2)^*$ - \hat{g} -continuous [5] if $f^{-1}(V)$ is a $(1,2)^*$ - \hat{g} -closed set of X for every $\sigma_{1,2}$ -closed set V of Y .
- (iv) $(1,2)^*$ -g-continuous [22] if $f^{-1}(V)$ is a $(1,2)^*$ -g-closed set of X for every $\sigma_{1,2}$ -closed set V of Y .
- (v) $(1,2)^*$ - αg_s -continuous [20] if $f^{-1}(V)$ is an $(1,2)^*$ - αg_s -closed set of X for every $\sigma_{1,2}$ -closed set V of Y .
- (vi) $(1,2)^*$ - αg -continuous [10] if $f^{-1}(V)$ is an $(1,2)^*$ - αg -closed set of X for every $\sigma_{1,2}$ -closed set V of Y .
- (vii) $(1,2)^*$ -gs-continuous [15] if $f^{-1}(V)$ is a $(1,2)^*$ -gs-closed set of X for every $\sigma_{1,2}$ -closed set V of Y .
- (viii) $(1,2)^*$ -gsp-continuous [15] if $f^{-1}(V)$ is a $(1,2)^*$ -gsp-closed set of X for every $\sigma_{1,2}$ -closed set V of Y .
- (ix) $(1,2)^*$ -sg-continuous [16] if $f^{-1}(V)$ is a $(1,2)^*$ -sg-closed set of X for every $\sigma_{1,2}$ -closed set V of Y .
- (x) $(1,2)^*$ -semi-continuous [16] if $f^{-1}(V)$ is a $(1,2)^*$ -semi-open set of X for every $\sigma_{1,2}$ -open set V of Y .
- (xi) $(1,2)^*$ - α -continuous [19] if $f^{-1}(V)$ is an $(1,2)^*$ - α -closed set of X for every $\sigma_{1,2}$ -closed set V of Y .
- (xii) $(1,2)^*$ -continuous [21] if $f^{-1}(V)$ is a $(1,2)^*$ -closed set of X for every $\sigma_{1,2}$ -closed set V of Y .

3. $(1,2)^*$ -Fuzzy generalized closed set

Definition 3.1 ([24]). A fuzzy subset λ of a fpts (X, τ_1, τ_2) called.

- (i) $(1,2)^*$ -fsg-closed set if $(1,2)^*\text{-scl}(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $(1,2)^*$ -fuzzy semi open in X . The complement of $(1,2)^*$ -fsg-closed set is called $(1,2)^*$ -fsg-open set;
- (ii) $(1,2)^*$ -fgs-closed set if $(1,2)^*\text{-scl}(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $\tau_{1,2}$ -open in X . The complement of $(1,2)^*$ -fgs-closed set is called $(1,2)^*$ -fgs-open set;

- (iii) $(1,2)^*$ - $f\alpha g$ -closed set if $(1,2)^*\text{-}\alpha\text{cl}(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $\tau_{1,2}$ -open in X . The complement of $(1,2)^*$ - $f\alpha g$ -closed set is called $(1,2)^*$ - $f\alpha g$ -open set;
- (iv) $(1,2)^*$ - $f\hat{g}$ -closed set or $f\omega$ -closed set if $\tau_{1,2}\text{-cl}(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $(1,2)^*$ - f s-open in X . The complement of $f\hat{g}$ -closed (resp. ω -closed) set is called $(1,2)^*$ - $f\hat{g}$ -open (resp. ω -open) set;
- (v) $(1,2)^*$ - $f\psi$ -closed set if $(1,2)^*\text{-scl}(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $(1,2)^*$ - f sg-open in X . The complement of $(1,2)^*$ - $f\psi$ -closed set is called $(1,2)^*$ - $f\psi$ -open set;
- (vi) $(1,2)^*$ - $f\hat{g}\alpha$ -closed set if $(1,2)^*\text{-}\alpha\text{cl}(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $(1,2)^*$ - f sg-open in X . The complement of $(1,2)^*$ - $f\hat{g}\alpha$ -closed set is called $(1,2)^*$ - Fg_α -open set;
- (vii) $(1,2)^*$ - f gsp-closed set if $(1,2)^*\text{-s}p\text{cl}(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $\tau_{1,2}$ -open in X . The complement of $(1,2)^*$ - f gsp-closed set is called $(1,2)^*$ - f gsp-open set;
- (viii) $(1,2)^*$ - $f\alpha g$ s-closed set if $(1,2)^*\text{-}\alpha\text{cl}(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $(1,2)^*$ - f s-open in X . The complement of $(1,2)^*$ - $f\alpha g$ s-closed set is called $(1,2)^*$ - $f\alpha g$ s-open set.

Definition 3.2 ([24]). A fuzzy subset λ of a fbts X is called $(1,2)^*$ - f g closed set if $\tau_{1,2}\text{-cl}(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $\tau_{1,2}$ -open subset in X . The complement of $(1,2)^*$ - f g-closed set is called $(1,2)^*$ - f g-open set.

Definition 3.3 ([24]). A fuzzy subset λ of a fbts X is called $(1,2)^*$ - $f\hat{g}$ -closed set if $\tau_{1,2}\text{-cl}(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $(1,2)^*$ - f sg-open in X . The complement of $(1,2)^*$ - $f\hat{g}$ -closed set is called $(1,2)^*$ - $f\hat{g}$ -open set.

Proposition 3.4. Every $(1,2)^*$ - $f\alpha$ -closed set is $(1,2)^*$ - $f\hat{g}\alpha$ -closed.

Proof. If λ is a $(1,2)^*$ - $f\alpha$ closed and μ is any $(1,2)^*$ - f sg-open set Containing λ , we have $\mu \geq \tau_{1,2}\text{-cl}(\lambda) \geq \alpha\text{cl}(\lambda)$. Hence λ is $(1,2)^*$ - $f\hat{g}\alpha$ -closed in X . \square

Proposition 3.5. Every $(1,2)^*$ - $f\hat{g}\alpha$ -closed is $(1,2)^*$ - $f\alpha g$ s-closed

Proof. If λ is a $(1,2)^*$ - $f\hat{g}\alpha$ -closed subset of X and μ is any $(1,2)^*$ - f s-open set containing λ , since every $(1,2)^*$ - f s-open set is $(1,2)^*$ - f sg-open, we have $(1,2)^*\text{-}\alpha\text{cl}(\lambda) \leq \mu$. Hence λ is $(1,2)^*$ - $f\alpha g$ s-closed in X . \square

Proposition 3.6. Every $(1,2)^*$ - $f\alpha g$ s-closed set is $(1,2)^*$ - $f\alpha g$ -closed.

Proof. If λ is $(1,2)^*$ - $f\alpha g$ s-closed subset of X and μ is any $\tau_{1,2}$ -open set containing λ , since every $\tau_{1,2}$ -open set is $(1,2)^*$ - f s-open, we have $\alpha\text{cl}(\lambda) \leq \mu$. Hence λ is $(1,2)^*$ - $f\alpha g$ -closed in X . \square

Proposition 3.7. Every $(1,2)^*$ -fuzzy closed set is $(1,2)^*$ - $f\hat{g}$ -closed.

Proof. If λ is a $(1,2)^*$ -fuzzy closed subset set of X and μ is any $(1,2)^*$ - f sg open set such that $\mu \geq \lambda = \text{cl}(\lambda)$. Hence λ is $(1,2)^*$ - $f\hat{g}$ -closed. \square

Proposition 3.8. Every $(1,2)^*$ - $f\hat{g}$ -closed set is $(1,2)^*$ - $f\psi$ -closed.

Proof. If λ is a $(1,2)^*$ - $f\hat{g}$ -closed subset of X and μ is any $(1,2)^*$ - f s open set such that $\mu \geq \lambda$, since every $(1,2)^*$ - f s open set is $(1,2)^*$ - f sg open, we have $\mu \geq \text{scl}(\lambda)$. Hence λ is $(1,2)^*$ - $f\psi$ -closed. \square

Proposition 3.9. Every $(1,2)^*$ - $f\hat{g}$ -closed set is $(1,2)^*$ - $f\hat{g}\alpha$ closed.

Proof. If λ is a $(1,2)^*$ -f \check{g} -closed subset of X and μ is any $(1,2)^*$ -fsg-open set such that $\lambda \leq \mu$, then $\mu \geq cl(\lambda) \geq \alpha cl(\lambda)$. Hence λ is $(1,2)^*$ -f $\check{g}\alpha$ closed. □

Proposition 3.10. *Every $(1, 2)^*$ -f ω -closed set is $(1,2)^*$ -fg-closed.*

Proof. If λ is a $(1,2)^*$ -f ω -closed subset of X and μ is any $(1,2)^*$ -fuzzy open set containing λ , then $\mu \geq \tau_{1,2}\text{-cl}(\lambda)$. Hence λ is $(1,2)^*$ -fg-closed in X . □

Proposition 3.11. *Every $(1,2)^*$ -f ψ -closed set is $(1,2)^*$ -fsg closed.*

Proof. If λ is a $(1,2)^*$ -f ψ -closed subset of X and μ is any $(1,2)^*$ -fs-open set containing λ , we have $(1,2)^*\text{-scl}(\lambda) \leq \mu$. Hence λ is $(1,2)^*$ -fsg-closed. □

Proposition 3.12. *Every $(1,2)^*$ -fg-closed set is $(1,2)^*$ -fsg-closed.*

Proof. If λ is a $(1,2)^*$ -fg-closed subset of X and μ is any $\tau_{1,2}$ -open set containing λ , since every $\tau_{1,2}$ -open set is $(1,2)^*$ -fsg-open, we have $\mu \geq \tau_{1,2}\text{-cl}(\lambda) \geq (1,2)^*\text{-scl}(\lambda)$. Hence λ is $(1,2)^*$ -fsg-closed in X . □

Proposition 3.13. *Every $(1,2)^*$ -fg-closed set is $(1,2)^*$ -f α g-closed.*

Proof. If λ is a $(1,2)^*$ -fg-closed subset of X and μ is any $\tau_{1,2}$ -open set containing λ . we have $\mu \geq \tau_{1,2}\text{-cl}(\lambda) \geq (1,2)^*\text{-}\alpha cl(\lambda)$. Hence λ is $(1,2)^*$ -f α g-closed in X . □

Proposition 3.14. *Every $(1,2)^*$ -f ω -closed set is $(1,2)^*$ -fsg-closed.*

Proof. If λ is a $(1,2)^*$ -f ω -closed subset of X and μ is any $(1,2)^*$ -fs open set containing λ , we have $\mu \geq \tau_{1,2}\text{-cl}(\lambda) \geq (1,2)^*\text{-scl}(\lambda)$. Hence λ is $(1,2)^*$ -fsg-closed in X . □

Proposition 3.15. *Every $(1,2)^*$ -fuzzy sg-closed set is $(1,2)^*$ -fuzzy gs-closed.*

Proof. If λ is $(1,2)^*$ -fuzzy sg-closed subset of X and μ is any $\tau_{1,2}$ -open set Containing λ , since every $\tau_{1,2}$ -open set is $(1,2)^*$ -fs open, we have $scl(\lambda) \leq \mu$. Hence λ is $(1,2)^*$ -fuzzy gs-closed. □

Proposition 3.16. *Every $(1,2)^*$ -fuzzy ω - closed set is $(1,2)^*$ -fuzzy α gs-closed.*

Proof. If λ is $(1,2)^*$ -fuzzy ω -closed and μ is any $(1,2)^*$ -fuzzy semi open set containing λ , we have $\alpha cl(\lambda) \leq \tau_{1,2}\text{-cl}(\lambda) \leq \mu$. Hence λ is $(1,2)^*$ -fuzzy α gs-closed. □

Proposition 3.17. *Every $(1,2)^*$ -fuzzy semi-pre-closed set is $(1,2)^*$ -fuzzy pre-semi-closed.*

Proof. If λ is a $(1,2)^*$ -fuzzy semi pre closed and μ is any $(1,2)^*$ -fg-open set such that $\lambda \leq \mu$, we have $spCl(\lambda) = \lambda$, it follows that $spCl(\lambda) = \lambda \leq \mu$. Hence λ is fuzzy pre-semi-closed in X . □

Proposition 3.18. *Every fuzzy pre-semi-closed set is fgsp-closed.*

Proof. If λ is a $(1,2)^*$ -fuzzy pre semi closed and μ is any $(1,2)^*$ -fg-open set such that $\lambda \leq \mu$, we have $spCl(\lambda) = \lambda \leq \mu$. Hence λ is fgsp-closed in X . □

4. (1,2)*- fuzzy g-continuous Maps and (1,2)*-fuzzy \ddot{g} -continuous Maps.

In this section we introduce the following definitions.

Definition 4.1. A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called;

- (i) (1,2)*-fuzzy $\ddot{\alpha}$ continuous if $f^{-1}(V)$ is a (1,2)*- fuzzy $\ddot{\alpha}$ closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y .
- (ii) (1,2)*-fuzzy ψ -continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy ψ -closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y .
- (iii) (1,2)*-fuzzy \hat{g} -continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy \hat{g} -closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y .
- (iv) (1,2)*-fuzzy α gs-continuous if $f^{-1}(V)$ is an (1,2)*- fuzzy α gs-closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y .
- (v) (1,2)*-fuzzy α g-continuous if $f^{-1}(V)$ is an (1,2)*-fuzzy α g-closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y .
- (vi) (1,2)* fuzzy -gs-continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy gs-closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y .
- (vii) (1,2)*-fuzzy gsp-continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy gsp-closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y .
- (viii) (1,2)*-fuzzy sg-continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy sg-closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y .
- (ix) (1,2)*-fuzzy semi-continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy semi-open set of X for every $\sigma_{1,2}$ -fuzzy open set V of Y .
- (x) (1,2)*-fuzzy α -continuous if $f^{-1}(V)$ is an (1,2)*-fuzzy α -closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y .
- (xi) (1,2)*-fuzzy continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y .
- (xii) (1,2)*-fuzzy pre semi continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy pre semi closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y .
- (xiii) (1,2)*-fuzzy semi pre continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy semi pre closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y .

Definition 4.2. A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called (1,2)*-fuzzy g continuous if every $f^{-1}(\lambda)$ is (1,2)* -fuzzy g -closed in (X, τ_1, τ_2) for every $\sigma_{1,2}$ closed set λ of (Y, σ_1, σ_2) .

Definition 4.3. A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called (1,2)*-fuzzy g - irresolute if every $f^{-1}(\lambda)$ is (1,2)* -fuzzy g -closed in (X, τ_1, τ_2) for every (1,2)*-fuzzy g -closed set λ of (Y, σ_1, σ_2) .

Definition 4.4. A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called (1,2)*-fuzzy \ddot{g} continuous if every $f^{-1}(\lambda)$ is (1,2)* -fuzzy \ddot{g} closed in (X, τ_1, τ_2) for every $\sigma_{1,2}$ closed set λ of (Y, σ_1, σ_2) .

Proposition 4.5. Every -fuzzy continuous is (1,2)*-fuzzy g -continuous but not conversely.

Proof. It is obvious. The converse of the proposition 4.5 is not true as seen from the following example. □

Example 4.6. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{0, 1, \lambda = \frac{0.7}{a} + \frac{0.3}{b} + \frac{0.8}{c}\}$, $\tau_2 = \{0, 1, \mu = \frac{0.3}{a} + \frac{0.5}{b} + \frac{0.7}{c}\}$. $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is (1,2)*-fuzzy g -continuous but not (1,2)*-fuzzy continuous, since $f^{-1}(\{\gamma = \frac{0.8}{a} + \frac{0.8}{b} + \frac{0.8}{c}\}) = \{\gamma\}$ is not $\tau_{1,2}$ -closed in X .

Proposition 4.7. Every (1,2)*-fuzzy α -continuous map is (1,2)*-fuzzy $\ddot{\alpha}$ -continuous but not conversely.

Proof. The proof follows from Proposition 3.4. The converse of the proposition 4.7 is not true as seen from the following example. □

Example 4.8. Let $X = Y = \{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.3}{a} + \frac{0.4}{b}\}$, $\tau_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy $\check{g}\alpha$ -continuous but not $(1,2)^*$ -fuzzy α -continuous, since $f^{-1}(\{\gamma = \frac{0.4}{a} + \frac{0.5}{b}\}) = \{\gamma\}$ is not $(1,2)^*$ - $f\alpha$ -closed in X .

Proposition 4.9. Every $(1,2)^*$ -fuzzy $\check{g}\alpha$ -continuous map is $(1,2)^*$ -fuzzy α gs-continuous but not conversely.

Proof. The proof follows from Proposition 3.5. The converse of the proposition 4.9 is not true as seen from the following example. □

Example 4.10. Let $X=Y=\{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\}$, $\tau_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy α gs-continuous but not $(1,2)^*$ -fuzzy $\check{g}\alpha$ -continuous, since $f^{-1}(\{\gamma = \frac{0.4}{a} + \frac{0.5}{b}\}) = \{\gamma\}$ is not $(1,2)^*$ - $f\check{g}\alpha$ -closed in X .

Proposition 4.11. Every $(1,2)^*$ -fuzzy g -continuous map is $(1,2)^*$ -fuzzy gs -continuous but not conversely.

Proof. The proof follows from Proposition 3.12. The converse of the proposition 4.11 is not true as seen from the following example. □

Example 4.12. Let $X = Y = \{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\}$, $\tau_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy- gs continuous but not $(1,2)^*$ -fuzzy g -continuous, since $f^{-1}(\{\gamma = \frac{0.4}{a} + \frac{0.4}{b}\}) = \{\gamma\}$ is not $(1,2)^*$ -fuzzy g -closed in X .

Proposition 4.13. Every $(1,2)^*$ -fuzzy α gs-continuous is $(1,2)^*$ -fuzzy αg -continuous but not conversely.

Proof. The proof follows from Proposition 3.6. The converse of the proposition 4.13 is not true as seen from the following example. □

Example 4.14. Let $X = Y = \{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.3}{a} + \frac{0.6}{b}\}$; $\tau_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy αg -continuous but not $(1,2)^*$ -fuzzy α gs-continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.5}{b}\}) = \{\lambda\}$ is not $(1,2)^*$ -fuzzy α gs-closed in X .

Proposition 4.15. Every $(1,2)^*$ -fuzzy continuous is $(1,2)^*$ -fuzzy \check{g} -continuous but not conversely.

Proof. The proof follows from proposition 3.7. The converse of the proposition 4.15 is not true as seen from the following example. □

Example 4.16. Let $X = Y = \{a, b, c\}$, $\tau_1 = \{0, 1, \lambda = \frac{0.7}{a} + \frac{0.3}{b} + \frac{0.8}{c}\}$, $\tau_2 = \{0, 1, \mu = \frac{0.3}{a} + \frac{0.5}{b} + \frac{0.7}{c}\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy \check{g} -continuous but not $(1,2)^*$ -fuzzy continuous, since $f^{-1}(\{\lambda = \frac{0.8}{a} + \frac{0.9}{b} + \frac{0.8}{c}\}) = \{\lambda\}$ is not $(1,2)^*$ -fuzzy closed set in X .

Proposition 4.17. Every $(1,2)^*$ -fuzzy \check{g} -continuous is $(1,2)^*$ -fuzzy ψ -continuous but not conversely.

Proof. The proof follows from proposition 3.8. The converse of the proposition 4.17 is not true as seen from the following example. □

Example 4.18. Let $X=Y=\{a, b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.7}{b}\}$; $\tau_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy ψ -continuous but not $(1,2)^*$ -fuzzy \check{g} -continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.3}{b}\}) = \{\lambda\}$ is not $(1,2)^*$ -fuzzy \check{g} -closed set in X .

Proposition 4.19. Every $(1,2)^*$ -fuzzy ψ -continuous is $(1,2)^*$ -fuzzy sg continuous but not conversely.

Proof. The proof follows from proposition 3.11. The converse of the proposition 4.19 is not true as seen from the following example. \square

Example 4.20. Let $X=Y=\{a,b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.7}{b}\}$; $\tau_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy sg-continuous but not $(1,2)^*$ -fuzzy ψ -continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.3}{b}\}) = \{\lambda\}$ is not $(1,2)^*$ -fuzzy ψ closed in X

Proposition 4.21. Every $(1,2)^*$ -fuzzy ω -continuous is $(1,2)^*$ -fuzzy g -continuous but not conversely.

Proof. The proof follows from proposition 3.10. \square

The converse of the proposition 4.21 is not true as seen from the following example.

Example 4.22. Let $X=Y=\{a,b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.5}{a} + \frac{0.5}{b}\}$; $\tau_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy g -continuous but not $(1,2)^*$ -fuzzy ω -continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.4}{b}\}) = \{\lambda\}$ is not $(1,2)^*$ -fuzzy ω closed in X .

Proposition 4.23. Every $(1,2)^*$ -fuzzy \ddot{g} -continuous is $(1,2)^*$ -fuzzy $\ddot{g}\alpha$ -continuous but not conversely.

Proof. The proof follows from proposition 3.9 \square

The converse of the proposition 4.23 is not true as seen from the following example.

Example 4.24. Let $X=Y=\{a,b\}$, $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.7}{b}\}$; $\tau_2 = \{0, 1\}$, $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy $\ddot{g}\alpha$ -continuous but not $(1,2)^*$ -fuzzy \ddot{g} -continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.3}{b}\}) = \{\lambda\}$ is not $(1,2)^*$ -fuzzy \ddot{g} -closed.

Proposition 4.25. Every $(1,2)^*$ -fuzzy g -continuous is $(1,2)^*$ -fuzzy αg -continuous but not conversely.

Proof. The proof follows from Proposition 3.13. The converse of the proposition 4.25 is not true as seen from the following example. \square

Example 4.26. Let $X=Y=\{a,b\}$ and $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.7}{b}\}$; $\tau_2 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy αg -continuous but not fuzzy $(1,2)^*$ -fuzzy g -continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.3}{b}\}) = \{\lambda\}$ is not $(1,2)^*$ -fuzzy g -closed in X

Proposition 4.27. Every $(1,2)^*$ -fuzzy ω -continuous is $(1,2)^*$ -fuzzy sg-continuous but not conversely.

Proof. The proof follows from Proposition 3.14. The converse of the proposition 4.27 is not true as seen from the following example. \square

Example 4.28. Let $X=Y=\{a,b\}$ and $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\}$; $\tau_2 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy sg-continuous but not fuzzy $(1,2)^*$ -fuzzy ω -continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.4}{b}\}) = \{\lambda\}$ is not $(1,2)^*$ -fuzzy ω -closed X .

Proposition 4.29. Every $(1,2)^*$ -fuzzy sg-continuous is $(1,2)^*$ -fuzzy gs -continuous but not conversely.

Proof. The proof follows from Proposition 3.15. The converse of the proposition 4.29 is not true as seen from the following example. \square

Example 4.30. Let $X=Y=\{ a,b\}$ and $\tau_1 = \{0,1, \mu = \frac{0.3}{a} + \frac{0.6}{b}\}; \tau_2 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy gs -continuous but not fuzzy $(1,2)^*$ -fuzzy sg -continuous, since $f^{-1} \{ \lambda = \frac{0.3}{a} + \frac{0.4}{b} \} = \{ \lambda \}$ is not $(1,2)^*$ -fuzzy sg -closed X .

Proposition 4.31. Every $(1,2)^*$ -fuzzy ω -continuous is $(1,2)^*$ -fuzzy αgs -continuous but not conversely.

Proof. The proof follows from Proposition 3.16. The converse of the proposition 4.31 is not true as seen from the following example. □

Example 4.32. Let $X=Y=\{ a,b\}$ and $\tau_1 = \{0,1, \mu = \frac{0.4}{a} + \frac{0.7}{b}\}; \tau_2 = F(x)$ and $\sigma_2 = \tau_2$.

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy αgs -continuous but not fuzzy $(1,2)^*$ -fuzzy ω -continuous, since $f^{-1} \{ \lambda = \frac{0.4}{a} + \frac{0.3}{b} \} = \{ \lambda \}$ is not $(1,2)^*$ -fuzzy ω -closed X .

Proposition 4.33. Every $(1,2)^*$ -fuzzy semi pre continuous is $(1,2)^*$ -fuzzy pre semi continuous but not conversely.

Proof. The proof follows from Proposition 3.17. □

The converse of the proposition 4.33 is not true as seen from the following example.

Example 4.34. Let $X=Y= \{ a, b ,c\}$, $\tau_1 = \{0,1, \lambda = \frac{0.7}{a} + \frac{0.3}{b} + \frac{1}{c}\}$, $\tau_2 = \{0,1, \mu = \frac{0.7}{a} + \frac{0}{b} + \frac{0}{c}\}$. $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ -fuzzy pre semi continuous but not $(1,2)^*$ -fuzzy semi pre continuous, since $f^{-1}(\{ \gamma = \frac{1}{a} + \frac{0.3}{b} + \frac{0}{c} \}) = \{ \gamma \}$ is not fuzzy semi pre closed in X .

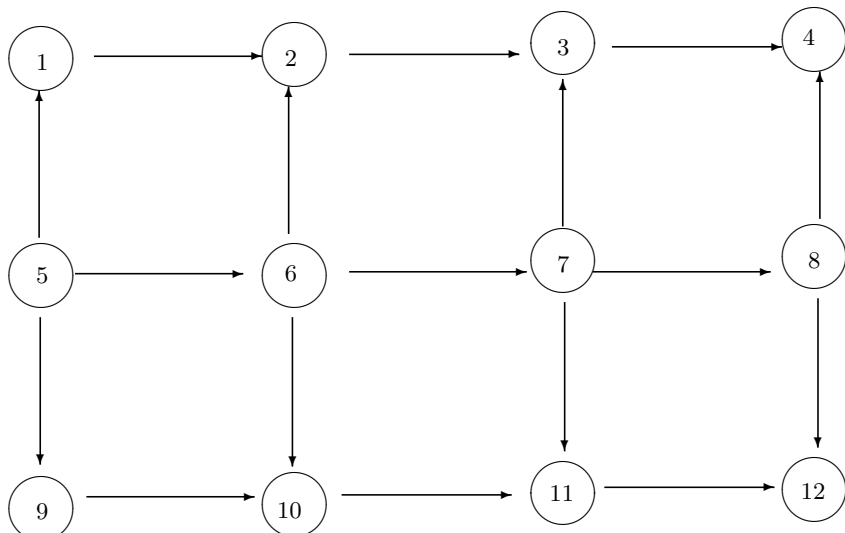
Proposition 4.35. Every $(1,2)^*$ -fuzzy pre semi continuous is $fgsp$ continuous but not conversely.

Proof. The proof follows from Proposition 3.18. □

The converse of the proposition 4.35 is not true as seen from the following example.

Example 4.36. Let $X=Y= \{ a, b ,c\}$, $\tau_1 = \{0,1, \lambda = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}\}$, $\tau_2 = \{0,1\}$. $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity map. Then f is $(1,2)^*$ - $fgsp$ continuous but not $(1,2)^*$ -fuzzy pre semi continuous, since $f^{-1}(\{ \gamma = \frac{1}{a} + \frac{1}{b} + \frac{0}{c} \}) = \{ \gamma \}$ is not fuzzy pre semi closed in X .

Remark 4.37. From the above Examples 4.6, 4.8, 4.10, 4.12, 4.14, 4.16, 4.18, 4.20, 4.22, 4.24, 4.26, 4.28, 4.30, 4.32, 4.34, 4.36 and propositions 4.5, 4.7, 4.9, 4.11, 4.13, 4.15, 4.17, 4.19, 4.21, 4.23, 4.25, 4.27, 4.29, 4.31, 4.33, 4.35 we obtain the following diagram, where $A \rightarrow B$ (resp. $A \leftrightarrow B$) represents A implies B but not conversely (resp. A and B are independent of each other)



Where

- | | |
|---|------------------------------------|
| (1) (1,2)*-f α -continuous | (7) (1,2)*-f ω - continuous |
| (2) (1,2)*- f $\check{g}\alpha$ -continuous | (8) (1,2)*-f g-continuous |
| (3) (1,2)*- f α gs-continuous | (9) (1,2)*-fs-continuous |
| (4) (1,2)*- f α g-continuous | (10) (1,2)*-f ψ -continuous |
| (5) (1,2)*- fuzzy -continuous | (11) (1,2)*-fsg-continuous |
| (6) (1,2)*-f \check{g} -continuous | (12) (1,2)*-fgs-continuous |

Definition 4.38. For every set $A \leq X$, we define the (1,2)*-g-closure of A to be the intersection of all (1,2)*-fg-closed sets containing A . In symbols, $(1,2)^*g-cl(A) = \cap \{ F : A \leq F \in (1,2)^*FGC(X) \}$.

Proposition 4.39. A map $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*-fg-continuous if and only if $f^{-1}(U)$ is (1,2)*-fg-open in X for every $\sigma_{1,2}$ -open set U in Y .

Proof. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be (1,2)*-fg-continuous and U be an $\sigma_{1,2}$ -open set in Y . Then U^c is $\sigma_{1,2}$ -closed in Y and since f is (1,2)*-fg-continuous, $f^{-1}(U^c)$ is (1,2)*-fg-closed in X . But $f^{-1}(U^c) = (f^{-1}(U))^c$ and so $f^{-1}(U)$ is (1,2)*-fg-open in X .

Conversely, assume that $f^{-1}(U)$ is (1,2)*-fg-open in X for each $\sigma_{1,2}$ -open set U in Y . Let F be a $\sigma_{1,2}$ -closed set in Y . Then F^c is $\sigma_{1,2}$ -open in Y and by assumption, $f^{-1}(F^c)$ is (1,2)*-fg-open in X . Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have $f^{-1}(F)$ is (1,2)*-fg-closed in X and so f is (1,2)*-fg-continuous. \square

Remark 4.40. The composition of two (1,2)*-fg-continuous maps need not be (1,2)*-fg-continuous and this is shown by the following example.

Example 4.41. Let $X = Y = Z = \{ a, b \}$, $\tau_1 = 0, 1, \lambda = \frac{0.4}{a} + \frac{0.6}{b}, \mu = \frac{0.2}{a} + \frac{0.3}{b}, \tau_2 = 0, 1$. Then τ_{12} -open sets are $0, 1, \lambda = \frac{0.4}{a} + \frac{0.6}{b}, \mu = \frac{0.2}{a} + \frac{0.3}{b}$ and

τ_{12} -closed sets are $0, 1, \lambda' = \frac{0.6}{a} + \frac{0.4}{b}, \mu' = \frac{0.8}{a} + \frac{0.7}{b}$.

Let $\sigma_1 = 0, 1, \eta = \frac{0.4}{a} + \frac{0.5}{b}$. Then σ_{12} -open sets are $0, 1, \eta = \frac{0.4}{a} + \frac{0.5}{b}$

and σ_{12} -closed sets are $0, 1, \eta' = \frac{0.6}{a} + \frac{0.5}{b}$. Let $\gamma_1 = 0, 1, \phi = \frac{0.7}{a} + \frac{0.4}{b}$,

$\gamma_2 = 0, 1$. Then γ_{12} -open sets are $0, 1, \phi = \frac{0.7}{a} + \frac{0.4}{b}$ and γ_{12} -closed sets are $0, 1, \phi' = \frac{0.3}{a} + \frac{0.6}{b}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$

be the identity maps. Then f and g are (1,2)*-fg-continuous but their

$g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is not (1,2)*-fg-continuous, because

$V = \frac{0.3}{a} + \frac{0.6}{b}$ is $\eta_{1,2}$ -closed in Z but $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) = f^{-1}(V) = V = \frac{0.3}{a} + \frac{0.6}{b}$ which is not (1,2)*-fg-closed in X .

Proposition 4.42. If $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*-fg-continuous and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*-continuous, then their composition $g \circ f : (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*-fg-continuous.

Proof. Let F be any $\eta_{1,2}$ -closed set in (Z, η_1, η_2) . Since $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ is (1,2)*-continuous, $g^{-1}(F)$ is $\sigma_{1,2}$ -closed in (Y, σ_1, σ_2) . Since $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*-fg-continuous, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is (1,2)*-fg-closed in X and so $g \circ f$ is (1,2)*-fg-continuous. \square

Proposition 4.43. If A is (1,2)*-fg-closed in X and if $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is (1,2)*-fsg-irresolute and (1,2)*-fuzzy closed, then $f(A)$ is (1,2)*-fg-closed in Y .

Proof. Let U be any $(1,2)^*$ -fsg-open in Y such that $f(A) \leq U$. Then $A \leq f^{-1}(U)$ and by hypothesis, $\tau_{1,2}\text{-cl}(A) \leq f^{-1}(U)$. Thus $f(\tau_{1,2}\text{-cl}(A)) \leq U$ and $f(\tau_{1,2}\text{-cl}(A))$ is a $\sigma_{1,2}$ -closed set. Now, $\sigma_{1,2}\text{-cl}(f(A)) \leq \sigma_{1,2}\text{-cl}(f(\tau_{1,2}\text{-cl}(A))) = f(\tau_{1,2}\text{-cl}(A)) \leq U$. That is $\sigma_{1,2}\text{-cl}(f(A)) \leq U$ and so $f(A)$ is $(1,2)^*$ -fg-closed. \square

Definition 4.44. Let x be a point of X and A be a subset of a fuzzy bitopological space X . Then A is called an $(1,2)^*$ -fg-neighbourhood of x (briefly, $(1,2)^*$ -fg-nbhd of x) in X if there exists an $(1,2)^*$ -fg-open set U of X such that $x \in U \leq A$.

Definition 4.45. Let A be a subset of a fuzzy bitopological space X . Then $x \in (1,2)^*\text{-g-cl}(A)$ if and only if for any $(1,2)^*$ -fg-nbhd G_x of x in X , $A \cap G_x \neq \varphi$.

Proof. Necessity. Assume $x \in (1,2)^*\text{-g-cl}(A)$. Suppose that there is an $(1,2)^*$ -fg-nbhd G of the point x in X such that $G \cap A = \varphi$. Since G is $(1,2)^*$ -fg-nbhd of x in X , by Definition 4.44, there exists an $(1,2)^*$ -fg-open set U_x such that $x \in U_x \leq G$. Therefore, we have $U_x \cap A = \varphi$ and so $A \leq (U_x)^c$. Since $(U_x)^c$ is an $(1,2)^*$ -fg-closed set containing A , we have by Definition 4.38 $(1,2)^*\text{-g-cl}(A) \leq (U_x)^c$ and therefore $x \notin (1,2)^*\text{-g-cl}(A)$, which is a contradiction.

Sufficiency. Assume for each $(1,2)^*$ -fg-nbhd G_x of x in X , $A \cap G_x \neq \varphi$. Suppose $x \notin (1,2)^*\text{-g-cl}(A)$. Then by Definition 4.38, there exists a $(1,2)^*$ -fg-closed set F of X such that $A \leq F$ and $x \notin F$. Thus $x \in F^c$ and F^c is $(1,2)^*$ -fg-open in X and hence F^c is a $(1,2)^*$ -fg-nbhd of x in X . But $A \cap F^c = \varphi$, which is a contradiction. \square

In the next theorem we explore certain characterizations of $(1,2)^*$ -fg-continuous functions.

Theorem 4.46. Suppose the collection of all $(1,2)^*$ -fg-open sets of X is closed under arbitrary unions. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a map from a fuzzy bitopological space (X, τ_1, τ_2) into a fuzzy bitopological space (Y, σ_1, σ_2) . Then the following statements are equivalent.

- (i) The function f is $(1,2)^*$ -fg-continuous.
- (ii) The inverse of each $\sigma_{1,2}$ -open set is $(1,2)^*$ -fg-open.
- (iii) For each point x in X and each $\sigma_{1,2}$ -open set V in Y with $f(x) \in V$, there is an $(1,2)^*$ -fg-open set U in X such that $x \in U, f(U) \leq V$.
- (iv) The inverse of each $\sigma_{1,2}$ -closed set is $(1,2)^*$ -fg-closed.
- (v) For each x in X , the inverse of every neighborhood of $f(x)$ is an $(1,2)^*$ -fg-nbhd of x .
- (vi) For each x in X and each neighborhood N of $f(x)$, there is an $(1,2)^*$ -fg-nbhd G of x such that $f(G) \leq N$.
- (vii) For each subset A of X , $f((1,2)^*\text{-g-cl}(A)) \leq \sigma_{1,2}\text{-cl}(f(A))$.
- (viii) For each subset B of Y , $(1,2)^*\text{-g-cl}(f^{-1}(B)) \leq f^{-1}(\sigma_{1,2}\text{-cl}(B))$.

Proof. (i) \Leftrightarrow (ii). This follows from Definition 4.2

(i) \Leftrightarrow (iii). Suppose that (iii) holds and let V be an $\sigma_{1,2}$ -open set in Y and let $x \in f^{-1}(V)$. Then $f(x) \in V$ and thus there exists an $(1,2)^*$ -fg-open set U_x such that $x \in U_x$ and $f(U_x) \leq V$. Now, $x \in U_x \leq f^{-1}(V)$ and $f^{-1}(V) = \cup_{x \in f^{-1}(V)} U_x$. Then $f^{-1}(V)$ is $(1,2)^*$ -fg-open in X and therefore f is $(1,2)^*$ -fg-continuous.

Conversely, suppose that (i) holds and let $f(x) \in V$ where V is $\sigma_{1,2}$ -open in Y . Then $f^{-1}(V) \in (1,2)^*\text{-GO}(X)$, since f is $(1,2)^*$ -fg-continuous. Let $U = f^{-1}(V)$. Then $x \in U$ and $f(U) \leq V$.

- (ii) \Leftrightarrow (iv). This result follows from the fact if A is a subset of Y , then $f^{-1}(A^c) = (f^{-1}(A))^c$.
- (ii) \Rightarrow (v). For x in X , let N be a neighborhood of $f(x)$. Then there exists an $\sigma_{1,2}$ -open set U in Y such that $f(x) \in U \leq N$. Consequently, $f^{-1}(U)$ is an $(1,2)^*$ -fg-open set in X and $x \in f^{-1}(U) \leq f^{-1}(N)$. Thus $f^{-1}(N)$ is an $(1,2)^*$ -fg-nbhd of x .
- (v) \Rightarrow (vi). Let $x \in X$ and let N be a neighborhood of $f(x)$. Then by assumption, $G = f^{-1}(N)$ is an $(1,2)^*$ -fg-nbhd of x and $f(G) = f(f^{-1}(N)) \leq N$.
- (vi) \Rightarrow (iii). For x in X , let V be an $\sigma_{1,2}$ -open set containing $f(x)$. Then V is a neighborhood of $f(x)$. So by assumption, there exists an $(1,2)^*$ -fg-nbhd G of x such that $f(G) \leq V$. Hence there exists an $(1,2)^*$ -fg-open set U in X such that $x \in U \leq G$ and so $f(U) \leq f(G) \leq V$.
- (vii) \Leftrightarrow (iv). Suppose that (iv) holds and let A be a subset of X . Since $A \leq f^{-1}(f(A))$, we have $A \leq f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))$. Since $\sigma_{1,2}\text{-cl}(f(A))$ is a $\sigma_{1,2}$ -closed set in Y , by assumption $f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))$ is an $(1,2)^*$ -fg-closed set containing A . Consequently, $(1,2)^*\text{-g-cl}(A) \leq f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))$. Thus $f((1,2)^*\text{-g-cl}(A)) \leq f(f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))) \leq \sigma_{1,2}\text{-cl}(f(A))$.
- Conversely, suppose that (vii) holds for any subset A of X . Let F be a $\sigma_{1,2}$ -closed subset of Y . Then by assumption, $f((1,2)^*\text{-g-cl}(f^{-1}(F))) \leq \sigma_{1,2}\text{-cl}(f(f^{-1}(F))) \leq \sigma_{1,2}\text{-cl}(F) = F$. That is $(1,2)^*\text{-g-cl}(f^{-1}(F)) \leq f^{-1}(F)$ and so $f^{-1}(F)$ is $(1,2)^*$ -fg-closed.
- (vii) \Leftrightarrow (viii). Suppose that (vii) holds and B be any subset of Y . Then replacing A by $f^{-1}(B)$ in (vii), we obtain $f((1,2)^*\text{-g-cl}(f^{-1}(B))) \leq \sigma_{1,2}\text{-cl}(f(f^{-1}(B))) \leq \sigma_{1,2}\text{-cl}(B)$. That is $(1,2)^*\text{-g-cl}(f^{-1}(B)) \leq f^{-1}(\sigma_{1,2}\text{-cl}(B))$.
- Conversely, suppose that (viii) holds. Let $B = f(A)$ where A is a subset of X . Then we have, $(1,2)^*\text{-g-cl}(A) \leq (1,2)^*\text{-g-cl}(f^{-1}(B)) \leq f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))$ and so $f((1,2)^*\text{-g-cl}(A)) \leq \sigma_{1,2}\text{-cl}(f(A))$. This completes the proof of the theorem. \square

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References

- [1] K.K.Azad, *On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity*, J. Math. Anal. Appl., 82(1)(1981), 14-32.
- [2] G.Balasubramanian and P.Sundaram, *On some generalizations of fuzzy discontinuous functions*, Fuzzy Sets and Systems, 86(1)(1997), 93-100.
- [3] O.Bedre Ozbakir, *On generalized fuzzy strongly semi closed sets in fuzzy topological spaces*, Int. J. Math. Math. Sci., 30(11)(2002), 651-657.
- [4] A.S.Bin Shahna, *On fuzzy strong semicontinuity and fuzzy precontinuity*, Fuzzy Sets and Systems, 44(2)(1991), 303-308.
- [5] Z.Duszynski, M.Joseph Israel and O.Ravi, *A new generalization of closed sets in bitopology*, International Mathematical Forum (Toappear).
- [6] M.Jeyaraman, S.Vijayalakshmi and O.Ravi, *New Notion Of Closed Sets In Fuzzy Topology*, International Journal of Mathematical Archive, 4(8)(2013), 67-73.

- [7] M.Jeyaraman, J.Rajalakshmi and O.Ravi, *Another Generalization Of Closed Sets In Fuzzy Topological Spaces*, International Journal of Mathematical Archive, 4(8)(2013), 187-192.
- [8] Kandil, *Biproximities and fuzzy bitopological spaces*, Simon Stevin, 63(1989), 45-66.
- [9] N.Levine, *Generalized closed sets in topology*, Rend. Circ. Math.Palermo, 19(2)(1970), 89-96.
- [10] M.Lellis Thivagar, O.Ravi and M.E.Abd El-Monsef, *Remarks on bitopological $(1,2)^*$ -quotient mappings*, J. Egypt Math. Soc., 16(1) (2008), 17-25.
- [11] H.Maki et al., *Generalized closed sets in fuzzy topological space*, Meetings on Topological Spaces Theory and its Applications (1998), 23-36.
- [12] O.Ravi, J.Antony Rex Rodrigo, S.Ganesan and A.Kumaradhas, *On-continuous functions in topological spaces (submitted)*.
- [13] O.Ravi and S.Ganesan, *$(1,2)^*$ - $\tilde{g}\alpha$ -closed sets in bitopology (submitted)*.
- [14] O.Ravi, M.Kamaraj, A.Pandi and K.Kumaresan, *$(1,2)^*$ -closed and $(1,2)^*$ -open maps in bitopological spaces*, International Journal of Mathematical Archive, 3(2)(2012), 586-594.
- [15] O.Ravi, S.Pious Missier and T.Salai Parkunan, *On bitopological $(1,2)^*$ -generalized homeomorphisms*, Int J. Contemp. Math. Sciences, 5(11)(2010), 543-557.
- [16] O.Ravi and M.L.Thivagar, *A bitopological $(1,2)^*$ -semi-generalized continuous maps*, Bull.Malaysian Math. Sci. Soc., 2(29)(1)(2006), 76-88.
- [17] O.Ravi, M.L.Thivagar and Jinjinli, *Remarks on extensions of $(1,2)^*$ - g -closed maps*, Archimedes J. Math., 1(2)(2011), 177-187.
- [18] O.Ravi, M.L.Thivagar and A.Nagarajan, *$(1,2)^*$ - αg -closed sets and $(1,2)^*$ - $g\alpha$ -closed sets (submitted)*.
- [19] O.Ravi and M.Lellis Thivagar, *On stronger forms of $(1,2)^*$ -quotient mappings in bitopological spaces*, Internat. J. Math. Game Theory and Algebra, 14(6)(2004), 481-492.
- [20] O.Ravi, C.M.Santhana, A.Pandi and J.Antony Rex Rodrigo, *On $(1,2)^*$ -closed sets in bitopological spaces (submitted)*.
- [21] O.Ravi and M.L.Thivagar, *Remarks on λ -irresolute functions via $(1,2)^*$ -sets*, Advances in App. Math. Analysis, 5(1)(2010), 1-15.
- [22] R.K.Saraf and M.Khana, *Fuzzy generalized semi preclosed sets*, J. Tripura Math. Soc., 3(2001), 59-68.
- [23] R.K.Saraf, G.Navalagi and Meena Khanna, *On fuzzy semi-pre-generalized closed sets*, Bull. Mal. Mat. Sci. Soc., 28(1)(2005), 19-30.
- [24] P.Saravanaperumal and S.Murugesan, *$(1,2)^*$ -Generalized closed sets in fuzzy bitopological spaces (communicated)*.
- [25] P.Sundaram, H.Maki and K.Balachandran, *Semi-generalized continuous maps and semi- $T_{1/2}$ spaces*, Bull. Fukuoka Univ. Ed. III 40(1991), 3340.
- [26] M.K.R.S.Veerakumar, *\hat{g} -closed sets in topological spaces*, Bull. Allah. Math. Soc., 18(2003), 99-112.