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Graceful Labeling for Swastik Graph

Research Article

V J Kaneria[1] and H M Makadia[2]*

1 Department of Mathematics, Saurashtra University, Rajkot, India.

2 Department of Mathematics, Government Engineering College, Rajkot, India.

Abstract: We investigate a new graph which is called swastik graph. We proved that the swastik graph is graceful. We have investigated some swastik graph related families of connected graceful graphs. We proved that path union of swastik graph, cycle of swastik graph and star of swastik graph are graceful.

MSC: 05C78.

Keywords: Graceful labeling, swastik graph, path union of graphs, cycle of graphs, star of a graph.

1. Introduction

The graceful labeling was introduced by A. Rosa [1] during 1967. Golomb [2] named such labeling as graceful labeling, which was called earlier as $\beta-$valuation. In this work we introduce a new graph which is called swastik graph and it is denoted by Sw_n.

We begin with a simple, undirected finite graph $G = (V, E)$ with $|V| = p$ vertices and $|E| = q$ edges. For all terminology and notations we follows Harary [3]. Here are some of the definitions which are useful in this paper.

Definition 1.1. *A function f is called* graceful labeling *of a graph $G = (V, E)$ if $f : V \longrightarrow \{0, 1, \dots, q\}$ is injective and the induced function $f^\star : E \longrightarrow \{1, 2, \dots, q\}$ defined as $f^\star(e) = |f(u) - f(v)|$ is bijective for every edge $e = (u, v) \in E$. A graph G is called* graceful graph *if it admits a graceful labeling.*

Definition 1.2. *Let G be a graph and $G_1, G_2, \dots, G_n$, $n \geq 2$ be n copies of graph G. Then the graph obtained by adding an edge from G_i to G_{i+1} $(1 \leq i \leq n-1)$ is called* path union *of G.*

Definition 1.3. *For a cycle C_n, each vertex of C_n is replaced by connected graphs $G_1, G_2, \dots, G_n$ and is known as* cycle of graphs. *We shall denote it by $C(G_1, G_2, \dots, G_n)$. If we replace each vertex by a graph G, i.e. $G_1 = G$, $G_2 = G$, ..., $G_n = G$, such cycle of a graph G is denoted by $C(n \cdot G)$.*

Above definition is introduced by Kaneria et al [4].

Definition 1.4. *Let G be a graph on n vertices. The graph obtained by replacing each vertex of the star $K_{1,n}$ by a copy of G is called a star of G and is denoted by $G^\star$.*

* *E-mail: makadia.hardik@yahoo.com*

Above definition is introduced by Vaidya et al [5].

Definition 1.5. *swastik graph is an union of four copies on C_{4n}. If $V_{i,j}$ ($\forall\ i = 1,2,3,4,\ \forall\ j = 1,2\dots,4n$) be vertices of i^{th} copy of $C_{4n}^{(i)}$ then we shell combine $V_{1,4t}$ & $V_{2,1}$, $V_{2,4t}$ & $V_{3,1}$, $V_{3,4t}$ & $V_{4,1}$ and $V_{4,4t}$ & $V_{1,1}$ by a single vertex. So graph seems like a plus sign. If we bend branches of graph toward clockwise at the middle then the graph looks a swastik. It is denoted as Sw_n of n size, where $n \in N - \{1\}$. Obviously $|V(Sw_n)| = 16(n) - 4$ and $|E(Sw_n)| = 16(n)$.*

In this paper we introduced gracefulness of swastik graph, path union of swastik graph, cycle of swastik graph and star of swastik graph. For detail survey of graph labeling we refer Gallian [6].

2. Main Results

Theorem 2.1. *A swastik graph Sw_n is a graceful graph, where $n \in N - \{1\}$.*

Proof. Let $G = Sw_n$ be any swastik graph of size n, where $n \in N - \{1\}$. We mention each vertices of Sw_n like $V_{i,j}$ ($i = 1,2,3,4,\ j = 1,2\dots,4n$). We see the numbers of vertices in G is $|V(Sw_n)| = p = 16(n) - 4$ and $|E(Sw_n)| = q = 16(n)$. We define labeling function $f : V(G) \longrightarrow \{0,1,\dots,q\}$ as follows

$$
\begin{aligned}
f(v_{1,j}) &= q - (\tfrac{j-1}{2}) && \text{if } j = 1,3,\dots,4n-1,\\
&= (\tfrac{j-2}{2}) && \text{if } j = 2,4,\dots,2n,\\
&= (\tfrac{j}{2}) && \text{if } j = 2n+2,2n+4,\dots,4n;\\
f(v_{2,j}) &= 2n + (\tfrac{j-1}{2}) && \text{if } j = 1,3,\dots,2n+1,\\
&= 2n + (\tfrac{j+1}{2}) && \text{if } j = 2n+3,2n+5,\dots,4n-1,\\
&= q - 2n - (\tfrac{j-2}{2}) && \text{if } j = 2,4,\dots,2n,\\
&= q - 2n - (\tfrac{j}{2}) && \text{if } j = 2n+2,2n+4,\dots,2n;\\
f(v_{3,j}) &= q - 4n - (\tfrac{j-1}{2}) && \text{if } j = 1,3,\dots,4n-1,\\
&= 4n + 1 + (\tfrac{n}{2}) && \text{if } j = 2,4,\dots,2n,\\
&= 4n + 2 + (\tfrac{n}{2}) && \text{if } j = 2n+2,2n+4,\dots,4n;\\
f(v_{4,j}) &= 6n + 2 + (\tfrac{j-1}{2}) && \text{if } j = 1,3,\dots,4n-1,\\
&= q - 6n - (\tfrac{j-2}{2}) && \text{if } j = 2,4,\dots,4n-2.
\end{aligned}
$$

Above labeling patten give rise a graceful labeling to the graph G. So G is a graceful graph. □

Illustration 2.2. *Sw_4 and its graceful labeling shown in figure 1.*

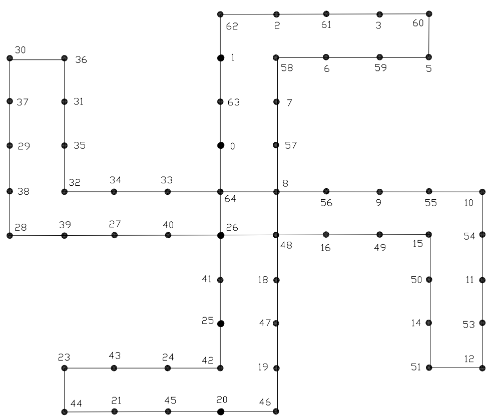

Figure 1. Sw_4, **swastik graph with** $n = 4$ **and its graceful labeling**

Theorem 2.3. *Path union of finite copies of the swastik graph Sw_n is a graceful graph, where $n \in N - \{1\}$.*

Proof. Let $G = P(r \cdot Sw_n)$ be a path union of r copies for the swastik graph Sw_n, where $n \in N - \{1\}$. Let f be the graceful labeling of Sw_n as we mentioned in Theorem 2.1. In graph G, we see that the vertices $|V(G)| = P = r(16(n) - 4)$ and the edges $|E(G)| = Q = (r-1) + r \cdot 16(n)$. Let $u_{k,i,j}$ ($\forall\ i = 1,2,3,4,\ \forall\ j = 1,2\ldots,4n$) be the vertices of k^{th} copy of Sw_n, $\forall\ k = 1,2,\ldots,r$. Where the vertices of k^{th} copy of Sw_n is $p = 16(n) - 4$ and edges of k^{th} copy of Sw_n is $q = 16n$. Join the vertices $u_{k,1,2n+1}$ with $u_{k+1,1,2n+1}$ for $k = 1,2,\ldots,r-1$ by an edge to from the path union of r copies of swastik graph. To define labeling function $g : V(G) \longrightarrow \{0,1,\ldots,Q\}$ as follows

$$g(u_{1,i,j}) = f(u_{i,j}) \qquad \text{if } f(u_{i,j}) \leq \tfrac{q}{2} + 1,$$
$$= f(u_{i,j}) + (Q - q) \qquad \text{if } f(u_{i,j}) > \tfrac{q}{2} + 1,$$

$\forall\ i = 1,2,3,4,\ \forall\ j = 1,2,\ldots,4n;$

$$g(u_{2,i,j}) = g(u_{1,i,j}) + (Q - q) \qquad \text{if } g(u_{1,i,j}) < \tfrac{Q}{2},$$
$$= g(u_{1,i,j}) - (Q - q) \qquad \text{if } g(u_{1,i,j}) > \tfrac{Q}{2},$$

$\forall\ i = 1,2,3,4,\ \forall\ j = 1,2,\ldots,4n;$

$$g(u_{k,i,j}) = g(u_{k-2,i,j}) + (q + 1) \qquad \text{if } g(u_{k-2,i,j}) < \tfrac{Q}{2},$$
$$= g(u_{k-2,i,j}) - (q + 1) \qquad \text{if } g(u_{k-2,i,j}) > \tfrac{Q}{2},$$

$\forall\ i = 1,2,3,4,\ \forall\ j = 1,2,\ldots,4n;$ $\qquad \forall\ k = 3,4,\ldots,r.$

Above labeling patten give rise a graceful labeling to given graph G. So path union of finite copies of the swastik graph is graceful graph. □

Illustration 2.4. *Path union of 3 copies of Sw_3 and its graceful labeling shown in figure 2.*

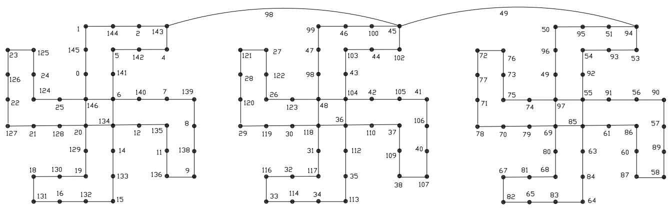

Figure 2. A Path union of 3 copies of Sw_3 and its graceful labeling

Theorem 2.5. *Cycle of r copies of swastik graph $C(r \cdot Sw_n)$ is a graceful graph, where $n \in N - \{1\}$ and $r \equiv 0,3$ (mod 4).*

Proof. Let $G = C(r \cdot Sw_n)$ be a cycle of swastik graph Sw_n, where $n \in N - \{1\}$. Let f be the graceful labeling for Sw_n as we mentioned in Theorem 2.1. In graph G, we see that the vertices $|V(G)| = P = r(16(n) - 4)$ and the edges $|E(G)| = Q = r(16(n) + 1)$. Let $u_{k,i,j}$ ($\forall\ i = 1,2,3,4,\ \forall\ j = 1,2\ldots,4n$) be the vertices of k copy of Sw_n, $\forall\ k = 1,2,\ldots,r$. Where the vertices of k^{th} copy of Sw_n is $p = 16(n) - 4$ and edges of k^{th} copy of Sw_n is $q = 16n$. Join the vertices $u_{k,1,2n+1}$ with $u_{k+1,1,2n+1}$ for $k = 1,2,\ldots,r-1$ and $u_{r,1,2n+1}$ with $u_{1,1,2n+1}$ by an edge to from $C(r \cdot Sw_n)$. We define labeling function $g : V(G) \longrightarrow \{0,1,\ldots,Q\}$ as follows

$$g(u_{1,i,j}) = f(u_{i,j}) \qquad \text{if } f(u_{i,j}) \leq \tfrac{q}{2} + 1,$$
$$= f(u_{i,j}) + (Q - q) \qquad \text{if } f(u_{i,j}) > \tfrac{q}{2} + 1,$$

$\forall\ i = 1,2,3,4,\ \forall\ j = 1,2,\ldots,4n;$

$$g(u_{2,i,j}) = g(u_{1,i,j}) + (Q - q) \qquad \text{if } g(u_{1,i,j}) < \tfrac{Q}{2},$$
$$= g(u_{1,i,j}) - (Q - q) \qquad \text{if } g(u_{1,i,j}) > \tfrac{Q}{2},$$

$\forall\ i = 1,2,3,4,\ \forall\ j = 1,2,\ldots,4n;$

$$\begin{aligned} g(u_{k,i,j}) &= g(u_{k-2,i,j}) + (q+1) && \text{if } g(u_{k-2,i,j}) < \tfrac{Q}{2}, \\ &= g(u_{k-2,i,j}) - (q+1) && \text{if } g(u_{k-2,i,j}) > \tfrac{Q}{2}, \end{aligned}$$

$\forall\ i = 1,2,3,4,\ \forall\ j = 1,2,\ldots,4n;$

$\forall\ k = 3,4,\ldots,\lceil \frac{r}{2} \rceil;$

$$\begin{aligned} g(u_{\lceil \frac{k}{2} \rceil+1,i,j}) &= g(u_{\lceil \frac{k}{2} \rceil-1,i,j}) + (q+2) && \text{if } g(u_{\lceil \frac{k}{2} \rceil-1,i,j}) < \tfrac{Q}{2}, \\ &= g(u_{\lceil \frac{k}{2} \rceil-1,i,j}) - (q+1) && \text{if } g(u_{\lceil \frac{k}{2} \rceil-1,i,j}) > \tfrac{Q}{2}, \end{aligned}$$

$\forall\ i = 1,2,3,4,\ \forall\ j = 1,2,\ldots,4n;$

$$\begin{aligned} g(u_{\lceil \frac{r}{2} \rceil+2,i,j}) &= g(u_{\lceil \frac{r}{2} \rceil,i,j}) + (q+2) && \text{if } g(u_{\lceil \frac{r}{2} \rceil,i,j}) < \tfrac{Q}{2}, \\ &= g(u_{\lceil \frac{r}{2} \rceil,i,j}) - (q+1) && \text{if } g(u_{\lceil \frac{r}{2} \rceil,i,j}) > \tfrac{Q}{2}, \end{aligned}$$

$\forall\ i = 1,2,3,4,\ \forall\ j = 1,2,\ldots,4n;$

$$\begin{aligned} g(u_{k,i,j}) &= g(u_{k-2,i,j}) + (q+1) && \text{if } g(u_{k-2,i,j}) < \tfrac{Q}{2}, \\ &= g(u_{k-2,i,j}) - (q+1) && \text{if } g(u_{k-2,i,j}) > \tfrac{Q}{2}, \end{aligned}$$

$\forall\ i = 1,2,3,4,\ \forall\ j = 1,2,\ldots,4n;$

$\forall\ k = \lceil \frac{r}{2} \rceil + 3, \lceil \frac{r}{2} \rceil + 4, \ldots, r.$

Above labeling patten give rise a graceful labeling to cycle of r copies for swastik graph. □

Illustration 2.6. *$C(4 \cdot Sw_2)$ and its graceful labeling shown in figure 3.*

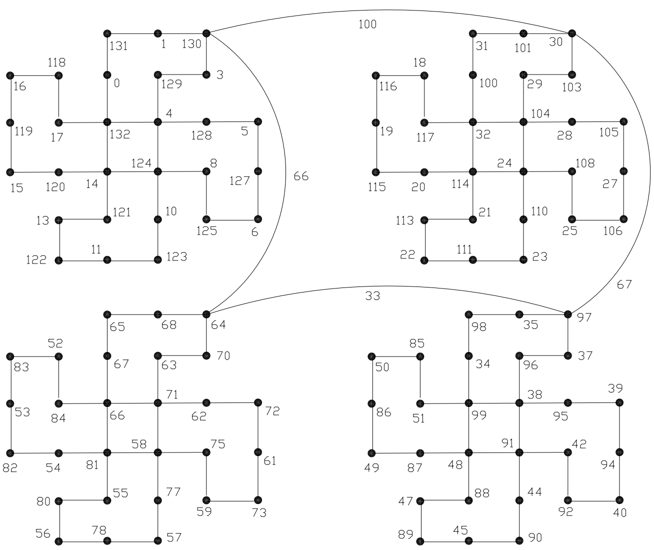

Figure 3. **A cycle of four copies for Sw_2 and its graceful labeling**

Theorem 2.7. *Star of swastik graph $(Sw_n)^\star$ is graceful,where $n \in N - \{1\}$.*

Proof. Let $G = (Sw_n)^\star$ be a star of swastik graph Sw_n, where $n \in N - \{1\}$. let f be the graceful labeling for Sw_n as we mention in Theorem 2.1. In graph G, we see that the vertices $|V(G)| = P = p(p+1)$ and the edges $|E(G)| = Q = (p+1)q+p$, where $p = 16(n) - 4$ and $q = 16(n)$. Let $u_{k,i,j}$ ($\forall\ i = 1,2,3,4,\ \forall\ j = 1,2\ldots,4n$) be the vertices of k copy of Sw_n, $\forall$ $k = 1,2,\ldots,p$. Where the vertices of k^{th} copy of Sw_n is $p = 16(n) - 4$ and edges of k^{th} copy of Sw_n is $q = 16(n)$. We mention that central copy of $(Sw_n)^\star$ is $(Sw_n)^{(0)}$ and other copies of $(Sw_n)^\star$ is $(Sw_n)^{(k)}$, $\forall\ k = 1,2,\ldots,p$. We define labeling function $g : V(G) \longrightarrow \{0,1,\ldots,Q\}$ as follows

$$
\begin{aligned}
g(u_{0,i,j}) &= f(u_{i,j}) && \text{if } f(u_{i,j}) \leq \tfrac{q}{2}+1,\\
&= f(u_{i,j}) + (Q-q) && \text{if } f(u_{i,j}) > \tfrac{q}{2}+1,
\end{aligned}
$$

$\forall\ i = 1,2,3,4,\ \forall\ j = 1,2,\ldots,4n;$

$$
\begin{aligned}
g(u_{1,i,j}) &= g(u_{0,i,j}) + p(q+1) && \text{if } g(u_{0,i,j}) < \tfrac{Q}{2},\\
&= g(u_{0,i,j}) - p(q+1) && \text{if } g(u_{0,i,j}) > \tfrac{Q}{2},
\end{aligned}
$$

$\forall\ i = 1,2,3,4,\ \forall\ j = 1,2,\ldots,4n;$

$$
\begin{aligned}
g(u_{k,i,j}) &= g(u_{k-2,i,j}) + (q+1) && \text{if } g(u_{k-2,i,j}) < \tfrac{Q}{2},\\
&= g(u_{k-2,i,j}) - (q+1) && \text{if } g(u_{k-2,i,j}) > \tfrac{Q}{2},
\end{aligned}
$$

$\forall\ i = 1,2,3,4,\ \forall\ j = 1,2,\ldots,4n,\ \forall\ k = 2,3,\ldots,p.$

We see that difference of vertices for the central copy $(Sw_n)^{(0)}$ of G and its other copies $(Sw_n)^{(k)}$ $(1 \leq k \leq p)$ is precisely following sequence

$$
\begin{gathered}
p(q+1)\\
(q+1)\\
(p-1)(q+1)\\
\vdots\\
\lfloor \tfrac{p}{2} \rfloor (q+1).
\end{gathered}
$$

Using this sequence we can produce required edge label by joining corresponding vertices of $(Sw_n)^{(0)}$ with its other copy $(Sw_n)^{(k)}$ $(1 \leq k \leq p)$ in G. Thus G admits graceful labeling. □

3. Concluding Remarks

Here we introduced a new graph is called swastik graph. Present work contributes some new results. We discussed gracefulness of swastik graphs, path union of swastik graph, cycle of swastik graph and star of swastik graph. The labeling patten is demonstrated by means of illustrations which provide better understanding to derived results.

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