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Applications of Semi $^{\#}$gα-closed Sets in Topological Spaces

Research Article

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Abstract: In this paper, we define some new sets namely semi $^{\#}$gα-border, semi $^{\#}$gα-frontier and semi $^{\#}$gα-exterior which are denoted by semi $^{\#}$gα-bd(A), semi $^{\#}$gα-fr(A) and semi $^{\#}$gα-ext(A), where A is any subset of X. We also examine the basic properties of these sets

Keywords: Semi $^{\#}$gα-closed set, Semi $^{\#}$gα-open set, Semi $^{\#}$gα-closure, Semi $^{\#}$gα-interior, Semi $^{\#}$gα-border, Semi $^{\#}$gα-frontier and Semi $^{\#}$gα-exterior.

1. Introduction

Levine [7] generalized closed sets in 1970, which paved a rapid progress in research in the field of topology. Devi et.al defined and investigated the notion of g$^{\#}\alpha$-closed sets [11] and $^{\#}g\alpha$-closed sets [2]. V.Kokilavani and M.Vivek Prabu [4] introduced the concepts of semi $^{\#}g\alpha$-closed sets, semi $^{\#}g\alpha$-continuous functions and semi $^{\#}g\alpha$-irresolute functions in topological spaces. In this paper, we define some new sets namely semi $^{\#}g\alpha$-border, semi $^{\#}g\alpha$-frontier and semi $^{\#}g\alpha$-exterior and study their basic properties.

2. Preliminaries

Definition 2.1. *A subset A of X is called*

(i) *α-closed [10] if cl(int(cl(A))) $\subseteq$ A. The complement of α-closed set is called α-open.*

(ii) *semi-closed [6] if int(cl(A))$\subseteq$ A. The complement of semi-closed set is called semi-open.*

(iii) *g-closed [7] if cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is open in X. The complement of g-closed set is called g-open.*

(iv) *gα-closed [8] if αcl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is α-open in X. The complement of gα-closed set is called gα-open.*

(v) *gs-closed [1] if scl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is open in X. The complement of gs-closed set is called gs-open.*

(vi) *$^{*}g\alpha$-closed [14] if αcl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is gα-open in X. The complement of $^{*}g\alpha$-closed set is called $^{*}g\alpha$-open.*

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(vii) *strongly g^*s-closed [13] if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is gs-open in X. The complement of strongly g^*s-closed set is called strongly g^*s-open.*

(viii) *$g^{\#}\alpha$-closed [11] if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g-open in X. The complement of $g^{\#}\alpha$-closed set is called $g^{\#}\alpha$-open.*

(ix) *$^{\#}g\alpha$-closed [2] if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is $g^{\#}\alpha$-open in X. The complement of $^{\#}g\alpha$-closed set is called $^{\#}g\alpha$-open.*

(x) *$g\zeta^*$-closed [3] if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is $^{\#}g\alpha$-open in X. The complement of $g\zeta^*$-closed set is called $g\zeta^*$-open.*

(xi) *semi $^{\#}g\alpha$-closed [4] if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is $^{\#}g\alpha$-open in X. The complement of semi $^{\#}g\alpha$-closed set is called semi $^{\#}g\alpha$-open. The union (respectively intersection) of all semi $^{\#}g\alpha$-open (respectively semi $^{\#}g\alpha$-closed) sets, each contained in (respectively containing) a set A of X is called the semi $^{\#}g\alpha$-interior (respectively semi $^{\#}g\alpha$-closure) of A, which is denoted by semi $^{\#}g\alpha$-int(A) (respectively semi $^{\#}g\alpha$-cl(A)).*

Theorem 2.2. *If A and B are subsets of X, then*

(i) *A is semi $^{\#}g\alpha$-open if and only if semi $^{\#}g\alpha$-int(A)=A.*

(ii) *semi $^{\#}g\alpha$-int(A) is semi $^{\#}g\alpha$-open.*

(iii) *A is semi $^{\#}g\alpha$-closed if and only if semi $^{\#}g\alpha$-cl(A)=A.*

(iv) *semi $^{\#}g\alpha$-cl(A) is semi $^{\#}g\alpha$-closed.*

(v) *semi $^{\#}g\alpha$-cl($X \setminus A$) = $X \setminus$ semi $^{\#}g\alpha$-int(A).*

(vi) *semi $^{\#}g\alpha$-int($X \setminus A$) = $X \setminus$ semi $^{\#}g\alpha$-cl(A).*

(vii) *If A is semi $^{\#}g\alpha$-open in X and B is open in X, then $A \cap B$ is semi $^{\#}g\alpha$-open in X.*

(viii) *A point $x \in$ semi $^{\#}g\alpha$-cl(A) if and only if every semi $^{\#}g\alpha$-open set in X containing x intersects A.*

(ix) *Arbitrary intersection of semi $^{\#}g\alpha$-closed sets in X is also semi $^{\#}g\alpha$-closed in X.*

Definition 2.3. *For any subset A of X,*

(i) *the border of A is defined by bd(A)=$A \setminus$ int(A).*

(ii) *the frontier of A is defined by fr(A)=cl(A) $\setminus$ int(A).*

(iii) *the exterior of A is defined by ext(A)=int($X \setminus A$).*

Definition 2.4. *A topological space X is said to be a $s^{\#}T_b$ space [5] if every gs-closed set in it is semi $^{\#}g\alpha$-closed.*

3. Semi $^{\#}g\alpha$-border of a Set

Definition 3.1. *For any subset A of X, semi $^{\#}g\alpha$-border of A is defined by semi $^{\#}g\alpha$-bd(A)=$A \setminus$ semi $^{\#}g\alpha$-int(A).*

Theorem 3.2. *In a topological space (X,τ), for any subset A of X, the following statements hold.*

(i) semi $^{\#}g\alpha$-bd(ϕ) = semi $^{\#}g\alpha$-bd(X) = ϕ.

(ii) semi $^{\#}g\alpha$-bd(A) $\subseteq$ A.

(iii) A = semi $^{\#}g\alpha$-int(A) $\cup$ semi $^{\#}g\alpha$-bd(A).

(iv) semi $^{\#}g\alpha$-int(A) $\cap$ semi $^{\#}g\alpha$-bd(A) = ϕ.

(v) semi $^{\#}g\alpha$-int(A) = A $\setminus$ semi $^{\#}g\alpha$-bd(A).

(vi) semi $^{\#}g\alpha$-int(semi $^{\#}g\alpha$-bd(A)) = ϕ.

(vii) A is semi $^{\#}g\alpha$-open if and only if semi $^{\#}g\alpha$-bd(A) = ϕ.

(viii) semi $^{\#}g\alpha$-bd(semi $^{\#}g\alpha$-int(A)) = ϕ.

(ix) semi $^{\#}g\alpha$-bd(semi $^{\#}g\alpha$-bd(A)) = semi $^{\#}g\alpha$-bd(A).

(x) semi $^{\#}g\alpha$-bd(A) = A $\cap$ semi $^{\#}g\alpha$-cl(X $\setminus$ A).

Proof. (i), (ii), (iii), (iv) and (v) follow from Definition 3.1
To prove (vi), if possible let x $\in$ semi $^{\#}g\alpha$-int(semi $^{\#}g\alpha$-bd(A)). Then x $\in$ semi $^{\#}g\alpha$-bd(A), since semi $^{\#}g\alpha$-bd(A) $\subseteq$ A, x $\in$ semi $^{\#}g\alpha$-int(semi $^{\#}g\alpha$-bd(A)) $\subseteq$ semi $^{\#}g\alpha$-int(A). Therefore x $\in$ semi $^{\#}g\alpha$-int(A) $\cap$ semi $^{\#}g\alpha$-bd(A) which is a contradiction to (iv). Thus (vi) is proved. A is semi $^{\#}g\alpha$-open if and only if semi $^{\#}g\alpha$-int(A)= A [Theorem 2.2 (i)]. But semi $^{\#}g\alpha$-bd(A)= A $\setminus$ semi $^{\#}g\alpha$-int(A) implies semi $^{\#}g\alpha$-bd(A)= ϕ. This proves (vii) and (viii). When A = semi $^{\#}g\alpha$-bd(A), Definition 3.1 becomes semi $^{\#}g\alpha$-bd(semi $^{\#}g\alpha$-bd(A))= semi $^{\#}g\alpha$-bd(A) $\setminus$ semi $^{\#}g\alpha$-int(semi $^{\#}g\alpha$-bd(A)). Using (viii), we get (ix). To prove (x), semi $^{\#}g\alpha$-bd(A)=A $\setminus$ semi $^{\#}g\alpha$-int(A)= A $\cap$ (X $\setminus$ semi $^{\#}g\alpha$-int(A))= A $\cap$ semi $^{\#}g\alpha$-cl(X $\setminus$ A)) [Theorem 2.2 (v)]. Hence (x) is proved. □

Theorem 3.3. *For any subset A of X,*

(i) If A is open (resp. α-open, semi-open), then semi $^{\#}g\alpha$-bd(A)=ϕ.

(ii) If A is gs-open(resp. strongly $g^{*}s$-open, $g\zeta^{*}$-open), then semi $^{\#}g\alpha$-bd(A)=ϕ.

(iii) If A is gs-open and X is a $s^{\#}T_{b}$space, then semi $^{\#}g\alpha$-bd(A)=ϕ.

Proof. (i) Since every open set is semi $^{\#}g\alpha$-open, from Theorem 3.2 (vii) semi $^{\#}g\alpha$-bd(A)=ϕ. Similarly (ii) and (iii) can be proved. □

Theorem 3.4. *A is semi $^{\#}g\alpha$-regular then semi $^{\#}g\alpha$-bd(A$\setminus$ semi $^{\#}g\alpha$-cl(A))=ϕ.*

Proof. Let A be semi $^{\#}g\alpha$-regular. Then semi $^{\#}g\alpha$-cl(A) = A = semi $^{\#}g\alpha$-int(A). Hence semi $^{\#}g\alpha$-bd(A $\setminus$ semi $^{\#}g\alpha$-cl(A))= A $\setminus$ semi $^{\#}g\alpha$-cl(A) $\setminus$ semi $^{\#}g\alpha$-int(A)= ϕ. □

4. Semi $^{\#}g\alpha$-frontier of a Set

Definition 4.1. *For any subset A of X, its semi $^{\#}g\alpha$-frontier is defined by semi $^{\#}g\alpha$-fr(A)= semi $^{\#}g\alpha$-cl(A) $\setminus$ semi $^{\#}g\alpha$-int(A).*

Theorem 4.2. *For any subset A of X, in a topological space (X,τ), the following statements hold.*

(i) *semi* $^{\#}g\alpha$*-fr*(ϕ)= *semi* $^{\#}g\alpha$*-fr*(*X*)=ϕ.

(ii) *semi* $^{\#}g\alpha$*-cl*(*A*)= *semi* $^{\#}g\alpha$*-int*(*A*) $\cap$ *semi* $^{\#}g\alpha$*-fr*(*A*).

(iii) *semi* $^{\#}g\alpha$*-int*(*A*) $\cap$ *semi* $^{\#}g\alpha$*-fr*(*A*)=ϕ.

(iv) *semi* $^{\#}g\alpha$*-bd*(*A*) $\subseteq$ *semi* $^{\#}g\alpha$*-fr*(*A*) $\subseteq$ *semi* $^{\#}g\alpha$*-cl*(*A*).

(v) *If A is semi* $^{\#}g\alpha$*-closed, then A=semi* $^{\#}g\alpha$*-int*(*A*) $\cup$ *semi* $^{\#}g\alpha$*-fr*(*A*).

(vi) *semi* $^{\#}g\alpha$*-fr*(*A*)= *semi* $^{\#}g\alpha$*-cl*(*A*) $\cap$ *semi* $^{\#}g\alpha$*-cl*($X \setminus A$).

(vii) *A point* $x \in$ *semi* $^{\#}g\alpha$*-fr*(*A*), *if and only if every semi* $^{\#}g\alpha$*-open set containing x intersects both A and its complement* $X \setminus A$.

(viii) *semi* $^{\#}g\alpha$*-cl*(*semi* $^{\#}g\alpha$*-fr*(*A*))= *semi* $^{\#}g\alpha$*-fr*(*A*), *i.e, semi* $^{\#}g\alpha$*-fr*(*A*) *is semi* $^{\#}g\alpha$*-closed.*

(ix) *semi* $^{\#}g\alpha$*-fr*(*A*) = *semi* $^{\#}g\alpha$*-fr*($X \setminus A$).

(x) *A is semi* $^{\#}g\alpha$*-closed if and only if semi* $^{\#}g\alpha$*-fr*(*A*) = *semi* $^{\#}g\alpha$*-bd*(*A*), *i.e, A is semi* $^{\#}g\alpha$*-closed if and only if A contains its semi* $^{\#}g\alpha$*-frontier.*

(xi) *A is semi* $^{\#}g\alpha$*-regular if and only if semi* $^{\#}g\alpha$*-fr*(*A*)=ϕ.

(xii) *semi* $^{\#}g\alpha$*-fr*(*semi* $^{\#}g\alpha$*-int*(*A*)) $\subseteq$ *semi* $^{\#}g\alpha$*-fr*(*A*).

(xiii) *semi* $^{\#}g\alpha$*-fr*(*semi* $^{\#}g\alpha$*-cl*(*A*)) $\subseteq$ *semi* $^{\#}g\alpha$*-fr*(*A*).

(xiv) *semi* $^{\#}g\alpha$*-fr*(*semi* $^{\#}g\alpha$*-fr*(*A*)) $\subseteq$ *semi* $^{\#}g\alpha$*-fr*(*A*).

(xv) *X=semi* $^{\#}g\alpha$*-int*(*A*) $\cup$ *semi* $^{\#}g\alpha$*-int*($X\setminus A$) $\cup$ *semi* $^{\#}g\alpha$*-fr*(*A*).

(xvi) *semi* $^{\#}g\alpha$*-int*(*A*) = $A \setminus$ *semi* $^{\#}g\alpha$*-fr*(*A*).

(xvii) *If A is semi* $^{\#}g\alpha$*-open, then* $A \cap$ *semi* $^{\#}g\alpha$*-fr*(*A*) = ϕ, *i.e, semi* $^{\#}g\alpha$*-fr*(*A*) $\subseteq X \setminus A$.

Proof. (i), (ii), (iii) and (iv) follows from Definition 4.1 (v) follows from (ii) and Theorem 2.2 (ii). (vi) follows from Theorem 2.2 (v). (vii) can be proved using (vi) and Theorem 2.2 (viii). From (vi), we can prove (viii) by applying the results of Theorem 2.2 (iii) and (ix). Proof of (ix) is similar. To prove (x): If A is semi $^{\#}g\alpha$-closed, then A = semi $^{\#}g\alpha$-cl(A).Hence Definition 4.1 reduces to semi $^{\#}g\alpha$-fr(A)= A $\setminus$ semi $^{\#}g\alpha$-int(A) = semi $^{\#}g\alpha$-bd(A).

Conversely, suppose that semi $^{\#}g\alpha$-fr(A) = semi $^{\#}g\alpha$-bd(A),using Definitions 4.1 and 3.1, we get semi $^{\#}g\alpha$-cl(A) = A, which proves the sufficient part. From Theorem 2.2 (i) and (iii) and Definition 4.1, (xi) can be proved. Since semi $^{\#}g\alpha$-int(A) is semi $^{\#}g\alpha$-open, (xii) holds. Similarly (xiii) can also be proved. Since semi $^{\#}g\alpha$-fr(A) is semi $^{\#}g\alpha$-closed,invoking (x), (xiv) can be proved. To prove (xv), since X=semi $^{\#}g\alpha$-cl(A) $\cup$ (X$\setminus$ semi $^{\#}g\alpha$-cl(A)), but from (ii) semi $^{\#}g\alpha$-cl(A) = semi $^{\#}g\alpha$-int(A) $\cup$ semi $^{\#}g\alpha$-fr(A). Also X $\setminus$ semi $^{\#}g\alpha$-cl(A)=semi $^{\#}g\alpha$-int(X $\setminus$ A). Hence X = semi $^{\#}g\alpha$-int(A) $\cup$ semi $^{\#}g\alpha$-fr(A) $\cup$ semi $^{\#}g\alpha$-int(X $\setminus$ A). Thus (xv) is proved. Proof of (vi) is obvious. If A is semi $^{\#}g\alpha$-open, A= semi $^{\#}g\alpha$-int(A). Hence (xvii) follows from (iii). □

Theorem 4.3. *If a subset A of X is semi* $^{\#}g\alpha$*-open or semi* $^{\#}g\alpha$*-closed in* (X,τ), *then semi* $^{\#}g\alpha$*-fr*(*semi* $^{\#}g\alpha$*-fr*(*A*))= *semi* $^{\#}g\alpha$*-fr*(*A*).

Proof. By Theorem 4.2 (vi), we have semi $^{\#}g\alpha$-fr(semi $^{\#}g\alpha$-fr(A))= semi $^{\#}g\alpha$-cl(semi $^{\#}g\alpha$-fr(A)) $\cap$ semi $^{\#}g\alpha$-cl(X $\setminus$ semi $^{\#}g\alpha$-fr(A))= semi $^{\#}g\alpha$-fr(A) $\cap$ semi $^{\#}g\alpha$-cl(X $\setminus$ semi $^{\#}g\alpha$-fr(A))= semi $^{\#}g\alpha$-cl(A) $\cap$ semi $^{\#}g\alpha$-cl(X$\setminus$ A) $\cap$ semi $^{\#}g\alpha$-cl(X $\setminus$ semi $^{\#}g\alpha$-fr(A)). If A is semi $^{\#}g\alpha$-open in X, by Theorem 4.2 (xvii), we have semi $^{\#}g\alpha$-fr(A) $\cap$ A = ϕ. Therefore A $\subseteq$ X $\setminus$ semi $^{\#}g\alpha$-fr(A). Hence semi $^{\#}g\alpha$-cl(A) $\subseteq$ semi $^{\#}g\alpha$-cl(X $\setminus$ semi $^{\#}g\alpha$-fr(A)). i.e, semi $^{\#}g\alpha$-cl(A) $\cap$ semi $^{\#}g\alpha$-cl(X $\setminus$ semi $^{\#}g\alpha$-fr(A))= semi $^{\#}g\alpha$-cl(A). If A is semi $^{\#}g\alpha$-closed in X, then X $\setminus$ A is semi $^{\#}g\alpha$-open and hence from the above case, we have semi $^{\#}g\alpha$-cl(X $\setminus$ A) $\cap$ semi $^{\#}g\alpha$-cl(X $\setminus$ semi $^{\#}g\alpha$-fr(X $\setminus$ A))= semi $^{\#}g\alpha$-cl(X $\setminus$ A). In both the cases using Theorem 4.2(vi), we get semi $^{\#}g\alpha$-fr(semi $^{\#}g\alpha$-fr(A))= semi $^{\#}g\alpha$-cl(A) $\cap$ semi $^{\#}g\alpha$-cl(X $\setminus$ A)= semi $^{\#}g\alpha$-fr(A). □

Theorem 4.4. *If A is any subset of X, then semi $^{\#}g\alpha$-fr(semi $^{\#}g\alpha$-fr(semi $^{\#}g\alpha$-fr(A)))= semi $^{\#}g\alpha$-fr(semi $^{\#}g\alpha$-fr(A)).*

Proof. It follows from Theorem 4.2 (viii) and Theorem 4.3. □

Theorem 4.5. *If A and B are subsets of X such that A $\cap$ B = ϕ, where A is semi $^{\#}g\alpha$-open in X, then A $\cap$ semi $^{\#}g\alpha$-cl(B)=ϕ.*

Proof. If possible, let x $\in$ A $\cap$ semi $^{\#}g\alpha$-cl(B). Then A is a semi $^{\#}g\alpha$-open set containing x and also x $\in$ semi $^{\#}g\alpha$-cl(B). By Theorem 2.2(viii) A $\cap$ B = ϕ, which is a contradiction. Thus A $\cap$ semi $^{\#}g\alpha$-cl(B)=ϕ. □

Theorem 4.6. *If A and B are subsets of X such that A $\subseteq$ B and B is semi $^{\#}g\alpha$-closed in X, then semi $^{\#}g\alpha$-fr(A) $\subseteq$ B.*

Proof. semi $^{\#}g\alpha$-fr(A)= semi $^{\#}g\alpha$-cl(A) $\setminus$ semi $^{\#}g\alpha$-int(A) $\subseteq$ semi $^{\#}g\alpha$-cl(B) $\setminus$ semi $^{\#}g\alpha$-int(A)= B $\setminus$ semi $^{\#}g\alpha$-int(A) $\subseteq$ B. □

Theorem 4.7. *If A and B are subsets of X such that A $\cap$ B=ϕ, where A is semi $^{\#}g\alpha$-open in X, then A $\cap$ semi $^{\#}g\alpha$-fr(B)= ϕ.*

Proof. Since semi $^{\#}g\alpha$-fr(B) $\subseteq$ semi $^{\#}g\alpha$-cl(B), proof is obvious from Theorem 4.5. □

Theorem 4.8. *If A and B are subsets of X such that semi $^{\#}g\alpha$-fr(A) $\cap$ fr(B) = ϕ and fr(A) $\cap$ semi $^{\#}g\alpha$-fr(B) = ϕ, then semi $^{\#}g\alpha$-int(A $\cup$ B)= semi $^{\#}g\alpha$-int(A) $\cup$ semi $^{\#}g\alpha$-int(B).*

Proof. We know that semi $^{\#}g\alpha$-int(A) $\cup$ semi $^{\#}g\alpha$-int(B) $\subseteq$ semi $^{\#}g\alpha$-(A $\cup$ B). Let x $\in$ semi $^{\#}g\alpha$-int(A $\cup$ B). i.e, x $\in$ U $\subseteq$ A $\cup$ B, U is a semi $^{\#}g\alpha$-open set. Thus either x $\in$ semi $^{\#}g\alpha$-fr(A), x $\notin$ fr(B), since semi $^{\#}g\alpha$-fr(A) $\cap$ fr(B) = ϕ. Hence x $\in$ int(B). i.e, x $\notin$ cl(B). Since x $\in$ int(B) $\subseteq$ semi $^{\#}g\alpha$-int(B), x $\subseteq$ semi $^{\#}g\alpha$-int(B). Moreover since x $\notin$ cl(B), there exists an open set V containing x which is disjoint from B, i.e, V $\subseteq$ X $\setminus$ B. So x $\in$ U $\cap$ V $\subseteq$ A. Hence U $\cap$ V is a semi $^{\#}g\alpha$-open subset of A containing x.(By Theorem 2.2 (vii)). i.e, x $\in$ semi $^{\#}g\alpha$-int(A). Thus x $\in$ semi $^{\#}g\alpha$-int(A) $\cup$ semi $^{\#}g\alpha$-int(B). If x $\notin$ semi $^{\#}g\alpha$-fr(A), x $\in$ semi $^{\#}g\alpha$-int(A) or x $\notin$ semi $^{\#}g\alpha$-cl(A). If x $\notin$ semi $^{\#}g\alpha$-cl(A), there exists a semi $^{\#}g\alpha$-open set W containing x which is disjoint from A, i.e, W $\subseteq$ X $\setminus$ A. i.e, x $\in$ U $\cap$ W $\subseteq$ B $\subseteq$ semi $^{\#}g\alpha$-cl(B). i.e, x $\in$ semi $^{\#}g\alpha$-fr(B). Hence from the above case, we get x $\in$ semi $^{\#}g\alpha$-int(A) $\cup$ semi $^{\#}g\alpha$-int(B). So semi $^{\#}g\alpha$-int(A $\cup$ B) $\subseteq$ semi $^{\#}g\alpha$-int(A) $\cup$ semi $^{\#}g\alpha$-int(B). Thus semi $^{\#}g\alpha$-int(A $\cup$ B)= semi $^{\#}g\alpha$-int(A) $\cup$ semi $^{\#}g\alpha$-int(B). □

5. Semi $^{\#}g\alpha$-Exterior of a Set

Definition 5.1. *For any subset A of X, its semi $^{\#}g\alpha$-exterior is defined by semi $^{\#}g\alpha$-ext(A) = semi $^{\#}g\alpha$-int(X $\setminus$ A).*

Theorem 5.2. *For any subsets A and B of X, in a topological space (X,τ), the following statements hold.*

(i) semi $^{\#}g\alpha$-ext(ϕ) = semi $^{\#}g\alpha$-ext(X)=ϕ.

(ii) If $A \subseteq B$, then semi $^{\#}g\alpha$-ext(B) $\subseteq$ semi $^{\#}g\alpha$-ext(A).

(iii) semi $^{\#}g\alpha$-ext(A) is semi $^{\#}g\alpha$-open.

(iv) A is semi $^{\#}g\alpha$-closed if and only if semi $^{\#}g\alpha$-ext(A) $= X \setminus A$.

(v) semi $^{\#}g\alpha$-ext(A) $= X \setminus$ semi $^{\#}g\alpha$-cl(A).

(vi) semi $^{\#}g\alpha$-ext(semi $^{\#}g\alpha$-ext(A))= semi $^{\#}g\alpha$-int(semi $^{\#}g\alpha$-cl(A)).

(vii) If A is semi $^{\#}g\alpha$-regular, then semi $^{\#}g\alpha$-ext(semi $^{\#}g\alpha$-ext(A))= A.

(viii) semi $^{\#}g\alpha$-ext(A)= semi $^{\#}g\alpha$-ext($X \setminus$ semi $^{\#}g\alpha$-ext(A)).

(ix) semi $^{\#}g\alpha$-int(A) $\subseteq$ semi $^{\#}g\alpha$-ext(semi $^{\#}g\alpha$-ext(A)).

(x) X= semi $^{\#}g\alpha$-int(A) $\cup$ semi $^{\#}g\alpha$-ext(A) $\cup$ semi $^{\#}g\alpha$-fr(A).

(xi) semi $^{\#}g\alpha$-ext($A \cup B$) $\subseteq$ semi $^{\#}g\alpha$-ext(A) $\cap$ semi $^{\#}g\alpha$-ext(B).

(xii) semi $^{\#}g\alpha$-ext($A \cap B$) $\supseteq$ semi $^{\#}g\alpha$-ext(A) $\cup$ semi $^{\#}g\alpha$-ext(B).

Proof. (i) and (ii) can be proved from Definition 5.1. Since semi $^{\#}g\alpha$-int(A) is semi $^{\#}g\alpha$-open, proof of (iii) follows from Definition 5.1. Proof of (iv) is obvious. Since semi $^{\#}g\alpha$-int(X $\setminus$ A)= X $\setminus$ semi $^{\#}g\alpha$-cl(A), (v) follows from Definition 5.1. Similarly (vi) can be proved. If A is semi $^{\#}g\alpha$-regular, from (iv), we have semi $^{\#}g\alpha$-ext(A)= X $\setminus$ A which is also semi $^{\#}g\alpha$-regular. Thus semi $^{\#}g\alpha$-ext(semi $^{\#}g\alpha$-ext(A))=A, (vii) is proved. To Prove (viii), semi $^{\#}g\alpha$-ext(X $\setminus$ semi $^{\#}g\alpha$-ext(A))= semi $^{\#}g\alpha$-ext(X $\setminus$ semi $^{\#}g\alpha$-int(X $\setminus$ A))= semi $^{\#}g\alpha$-int(X $\setminus$ (X $\setminus$ semi $^{\#}g\alpha$-int(X $\setminus$ A)))= semi $^{\#}g\alpha$-int(semi $^{\#}g\alpha$-int(X $\setminus$ A))= semi $^{\#}g\alpha$-int(X $\setminus$ A)= semi $^{\#}g\alpha$-ext(A). Hence (viii) is proved. Since A $\subseteq$ semi $^{\#}g\alpha$-cl(A), using (vi), (ix) can be proved. (x) follows from Theorem 4.2 (xv) and Definition 5.1. Proof of (xi) and (xii) are obvious. □

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