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$\pi g\alpha$ Closed Mappings in Intuitionistic Fuzzy Topological Spaces

Research Article

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Abstract: In this paper, we introduce the concepts of intuitionistic fuzzy $\pi g\alpha$ closed mappings and intuitionistic fuzzy i-$\pi g\alpha$ closed mappings. Further, we study some of their properties.

MSC: 54A40, 03F55.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy $\pi g\alpha$ closed set, Intuitionistic fuzzy $\pi g\alpha$ closed mapping and Intuitionistic fuzzy i-$\pi g\alpha$ closed mapping.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [14] and later Atanassov [1] generalized this idea to intuitionistic fuzzy set using the notion of fuzzy sets. On the other hand Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. The concept of generalized closed sets in topological spaces was introduced by Levine [6]. In this paper we introduce intuitionistic fuzzy $\pi g\alpha$ closed mappings and intuitionistic fuzzy i-$\pi g\alpha$ closed mappings. The relations between intuitionistic fuzzy $\pi g\alpha$ closed mappings and other intuitionistic fuzzy closed mappings are given.

2. Preliminaries

Definition 2.1 ([1])**.** *Let X be a non empty set. An intuitionistic fuzzy set (IFS in short) A in X can be described in the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$$

where the functions $\mu_A : X \to [0, 1]$ is called the membership function and $\mu_A(x)$ denotes the degree to which $x \in A$ and the function $\nu_A : X \to [0, 1]$ is called the non-membership function and $\nu_A(x)$ denotes the degree to which $x \notin A$ and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Denote by IFS(X), the set of all intuitionistic fuzzy sets in X. Throughout this paper, X denotes a non empty set.

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Definition 2.2 ([1]). *Let A and B be any two IFSs of the form* $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *and* $B=\{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. *Then*

(i) $A \subseteq B$ *if and only if* $\mu_A(x) \leq \mu_B(x)$ *and* $\nu_A(x) \geq \nu_B(x)$ *for all* $x \in X$,

(ii) $A = B$ *if and only if* $A \subseteq B$ *and* $B \subseteq A$,

(iii) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$,

(iv) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,

(v) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$.

Definition 2.3 ([1]). *The intuitionistic fuzzy sets* $0_\sim = \{\langle x, 0, 1\rangle \mid x \in X\}$ *and* $1_\sim = \{\langle x, 1, 0\rangle \mid x \in X\}$ *are the empty set and the whole set of X respectively.*

Definition 2.4 ([1]). *Let A and B be any two IFSs of the form* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *and* $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$. *Then*

(i) $A \subseteq B$ *and* $A \subseteq C \Rightarrow A \subseteq B \cap C$,

(ii) $A \subseteq C$ *and* $B \subseteq C \Rightarrow A \cup B \subseteq C$,

(iii) $A \subseteq B$ *and* $B \subseteq C \Rightarrow A \subseteq C$,

(iv) $(A \cup B)^c = A^c \cap B^c$ *and* $(A \cap B)^c = A^c \cup B^c$,

(v) $((A)^c)^c = A$,

(vi) $(1_\sim)^c = 0_\sim$ *and* $(0_\sim)^c = 1_\sim$.

Definition 2.5 ([3]). *An intuitionistic fuzzy topology (IFT in short) on X is a family* τ *of IFSs in X satisfying the following axioms :*

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ *for any family* $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) *is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in* τ *is known as an intuitionistic fuzzy open set (IFOS in short) in X.*
The complement A^c *of an IFOS A in an IFTS* (X, τ) *is called an intuitionistic fuzzy closed set (IFCS in short) in X.*

Definition 2.6 ([3]). *Let* (X, τ) *be an IFTS and* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:*

$$int(A) = \cup \{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\},$$
$$cl(A) = \cap \{K \mid K \text{ is an IFCS in } X \text{ and } A \subseteq K\}.$$

Proposition 2.7 ([3]). *For any two IFSs A and B in* (X, τ), *we have*

(i) $int(A) \subseteq A$,

(ii) $A \subseteq cl(A)$,

(iii) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$ and $cl(A) \subseteq cl(B)$,

(iv) $int(int(A)) = int(A)$,

(v) $cl(cl(A)) = cl(A)$,

(vi) $cl(A \cup B) = cl(A) \cup cl(B)$,

(vii) $int(A \cap B) = int(A) \cap int(B)$.

Proposition 2.8 ([3]). *For any IFS A in (X, τ), we have*

(i) $int(0_\sim) = 0_\sim$ and $cl(0_\sim) = 0_\sim$,

(ii) $int(1_\sim) = 1_\sim$ and $cl(1_\sim) = 1_\sim$,

(iii) $(int(A))^c = cl(A^c)$,

(iv) $(cl(A))^c = int(A^c)$.

Proposition 2.9 ([3]). *If A is an IFCS in (X, τ) then $cl(A) = A$ and if A is an IFOS in (X, τ) then $int(A) = A$. The arbitrary union of IFCSs is an IFCS in (X, τ).*

Definition 2.10. *An IFS A in an IFTS (X, τ) is said to be an*

(i) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = cl(int(A))$, [4]

(ii) intuitionistic fuzzy α-closed set (IFαCS in short) if $cl(int(cl(A))) \subseteq A$, [5]

(iii) intuitionistic fuzzy semi closed set (IFSCS in short) if $int(cl(A)) \subseteq A$, [4]

(iv) intuitionistic fuzzy pre closed set (IFPCS in short) if $cl(int(A)) \subseteq A$, [4]

(v) intuitionistic fuzzy generalized closed set (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [13]

(vi) intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ), [10]

The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets.

Definition 2.11 ([7]). *An IFS A in an IFTS (X, τ) is said to be an*

(i) intuitionistic fuzzy π open set (IFπOS in short) if A is a finite union of fuzzy regular open sets,

(ii) intuitionistic fuzzy π closed set (IFπCS in short) if A^c is an intuitionistic fuzzy π open set.

Definition 2.12 ([8]). *Let A be an IFS in an IFTS (X, τ). Then the α-interior of A ($\alpha int(A)$ in short) and the α-closure of A ($\alpha cl(A)$ in short) are defined as*

$$\alpha int(A) = \cup \{G \mid G \text{ is an IF}\alpha\text{OS in } (X, \tau) \text{ and } G \subseteq A\},$$
$$\alpha cl(A) = \cap \{K \mid K \text{ is an IF}\alpha\text{CS in } (X, \tau) \text{ and } A \subseteq K\}.$$

sint(A), scl(A), pint(A) and pcl(A) are similarly defined. For any IFS A in (X, τ), we have $\alpha cl(A^c) = (\alpha int(A))^c$ *and* $\alpha int(A^c) = (\alpha cl(A))^c$.

Remark 2.13 ([8]). *Let A be an IFS in an IFTS (X, τ). Then*

(i) $\alpha cl(A) = A \cup cl(int(cl(A)))$,

(ii) $\alpha int(A) = A \cap int(cl(int(A)))$.

Remark 2.14.

(i) *Every IFπOS in (X, τ) is an IFOS in (X, τ), [7]*

(ii) *Every IFOS in (X, τ) is an IFαOS in (X, τ), [4]*

(iii) *Every IFπOS in (X, τ) is an IFπGOS. [7]*

Definition 2.15 ([12]). *An IFS A in (X, τ) is said to be an intuituionistic fuzzy $\pi g\alpha$ closed set (IFπGαCS in short) if* $\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is an IFπOS in (X, τ).*

Definition 2.16 ([12]). *An IFS A in (X, τ) is said to be an intuituionistic fuzzy $\pi g\alpha$ open set (IFπGαOS in short) if the complement A^c is an IFπGαCS in (X, τ).*

Remark 2.17 ([12]). *Every IFCS, IFαCS, IFRCS, IFGCS is an IFπGαCS, but the converses may not be true in general.*

Definition 2.18 ([3]). *Let X and Y are two nonempty sets. Let* $f : X \to Y$ *be a mapping. If* $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle \mid y \in Y\}$ *is an IFS in Y, then the preimage of B under f, denoted by* $f^{-1}(B)$, *is the IFS in X defined by* $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)\rangle \mid x \in X\}$.

Definition 2.19 ([3]). *Let X and Y are two nonempty sets. Let* $f : X \to Y$ *be a mapping. If* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ *is an IFS in X, then the image of A under f, denoted by f(A), is the IFS in Y defined by* $f(A) = \{\langle y, f(\mu_A)(y), f_-(\nu_A)(y)\rangle \mid y \in Y\}$, *where* $f_-(\nu_A)(y) = 1 - f(1 - \nu_A)$.

Definition 2.20 ([4]). *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy closed mapping (IF closed map in short) if f(A) is an IFCS in (Y, σ) for every IFCS A of (X, τ).*

Definition 2.21 ([9]). *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an*

(i) *intuitionistic fuzzy regular closed mapping (IFR closed map in short) if f(A) is an IFRCS in (Y, σ) for every IFCS A of (X, τ),*

(ii) *intuitionistic fuzzy α-closed mapping (IFα closed map in short) if f(A) is an IFαCS in (Y, σ) for every IFCS A of (X, τ),*

(iii) *intuitionistic fuzzy pre closed mapping (IFP closed map in short) if f(A) is an IFPCS in (Y, σ) for every IFCS A of (X, τ),*

(iv) *intuitionistic fuzzy generalized closed mapping (IFG closed map in short) if f(A) is an IFGCS in (Y, σ) for every IFCS A of (X, τ),*

(v) *intuitionistic fuzzy generalized semi closed mapping (IFGS closed map in short) if f(A) is an IFGSCS in (Y, σ) for every IFCS A of (X, τ),*

Definition 2.22 ([12]). *An IFTS (X, τ) is said to be an*

(i) intuitionistic fuzzy ${}_{\pi\alpha a}T_{1/2}$ (IF${}_{\pi\alpha a}T_{1/2}$ in short) space if every IFπGαCS in X is an IFCS in X,

(ii) intuitionistic fuzzy ${}_{\pi\alpha b}T_{1/2}$ (IF${}_{\pi\alpha b}T_{1/2}$ in short) space if every IFπGαCS in X is an IFGCS in X,

3. Intuitionistic Fuzzy πgα Closed Mappings

In this section we introduce intuitionistic fuzzy πgα closed mappings and study some of their properties.

Definition 3.1. *A mapping f : (X, τ) $\to$ (Y, σ) is said to be an intuituionistic fuzzy $\pi g\alpha$ closed mapping (IFπGα closed map in short) if f(A) is an IFπGαCS in (Y, σ) for every IFCS A in (X, τ).*

Example 3.2. *Let X = {a, b} ans Y = {u, v}. Let T_1 = $\langle$x, (0.4, 0.5), (0.5, 0.4)$\rangle$ and T_2 = $\langle$y, (0.4, 0.3), (0.5, 0.6)$\rangle$. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IFπGα closed mapping.*

Definition 3.3. *A mapping f : (X, τ) $\to$ (Y, σ) is said to be an intuituionistic fuzzy $\pi g\alpha$ open mapping (IFπGα open map in short) if f(A) is an IFπGαOS in (Y, σ) for every IFOS A in (X, τ).*

Example 3.4. *Let X = {a, b} and Y = {u, v}. Let T_1 = $\langle$x, (0.6, 0.5), (0.3, 0.4)$\rangle$ and T_2 = $\langle$y, (0.2, 0.3), (0.7, 0.6)$\rangle$. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IFπGα open mapping.*

Theorem 3.5. *Every IF closed mapping is an IFπGα closed mapping, but not conversely.*

Proof. Let f : (X, τ) $\to$ (Y, σ) be an IF closed mapping. Let A be an IFCS in X. Since f is an IF closed mapping, f(A) is an IFCS in Y. Since every IFCS is an IFπGαCS, f(A) is an IFπGαCS in Y. Hence f is an IFπGα closed mapping. □

Example 3.6. *Let X = {a, b} and Y = {u, v}. Let T_1 = $\langle$x, (0.2, 0.1), (0.7, 0.8)$\rangle$ and T_2 = $\langle$y, (0.3, 0.2), (0.6, 0.7)$\rangle$. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Consider an IFS A = $\langle$x, (0.7, 0.8), (0.2, 0.1)$\rangle$ is an IFCS in X. Then f(A) is an IFπGαCS in Y. But f(A) is not an IFCS in Y. Therefore f is an IFπGα closed mapping but not an IF closed mapping.*

Theorem 3.7. *Every IFG closed mapping is an IFπGα closed mapping, but not conversely.*

Proof. Let f : (X, τ) $\to$ (Y, σ) be an IFG closed mapping. Let A be an IFCS in X. By hypothesis f(A) is an IFGCS in Y. Since every IFGCS is an IFπGαCS, f(A) is an IFπGαCS in Y. Hence f is an IFπGα closed mapping. □

Example 3.8. *Let X = {a, b} and Y = {u, v}. Let T_1 = $\langle$x, (0.5, 0.6), (0.4, 0.3)$\rangle$ and T_2 = $\langle$y, (0.6, 0.7), (0.3, 0.2)$\rangle$. Then τ = {$0_\sim$, T_1, $1_\sim$} and σ = {$0_\sim$, T_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Consider an IFS A = $\langle$x, (0.4, 0.3), (0.5, 0.6)$\rangle$ is an IFCS in X. Then f(A) is an IFπGαCS in Y. But f(A) is not an IFGCS in Y. Therefore f is an IFπGα closed mapping but not an IFG closed mapping.*

Theorem 3.9. *Every IFα closed mapping is an IFπGα closed mapping, but not conversely.*

Proof. Let f : (X, τ) $\to$ (Y, σ) be an IFα closed mapping. Let A be an IFCS in X. By hypothesis f(A) is an IFαCS in Y. Since every IFαCS is an IFπGαCS, f(A) is an IFπGαCS in Y. Hence f is an IFπGα closed mapping. □

Example 3.10. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.4, 0.5), (0.5, 0.4)\rangle$ and $T_2 = \langle y, (0.4, 0.3), (0.5, 0.6)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Consider an IFS $A = \langle x, (0.5, 0.4), (0.4, 0.5)\rangle$ is an IFCS in X. Then f(A) is an IFπGαCS in Y. But f(A) is not an IFαCS in Y. Therefore f is an IFπGα closed mapping but not an IFα closed mapping.*

Remark 3.11. *An IFπGα closed mapping is independent of an IFP closed mapping.*

Example 3.12. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.1, 0.2), (0.8, 0.7)\rangle$ and $T_2 = \langle y, (0.7, 0.6), (0.2, 0.3)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Consider an IFS $A = \langle x, (0.8, 0.7), (0.1, 0.2)\rangle$ is an IFCS in X. Then f(A) is an IFπGαCS in Y. But f(A) is not an IFPCS in Y. Therefore f is an IFπGα closed mapping but not an IFP closed mapping.*

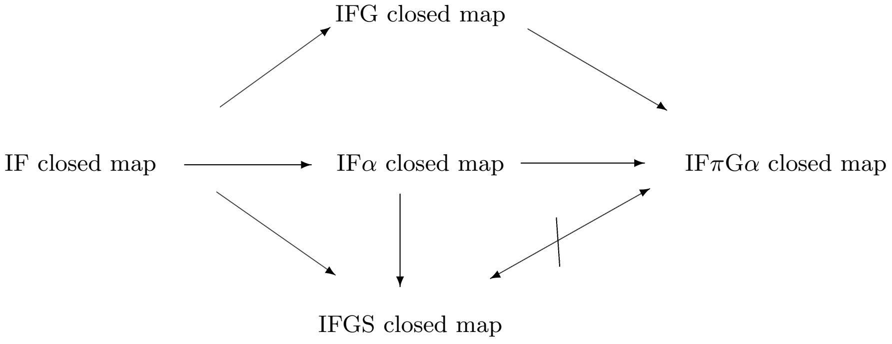
Example 3.13. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.7, 0.6), (0.3, 0.4)\rangle$ and $T_2 = \langle y, (0.4, 0.5), (0.6, 0.5)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Consider an IFS $A = \langle x, (0.3, 0.4), (0.7, 0.6)\rangle$ is an IFCS in X. Then f(A) is an IFPCS in Y. But f(A) is not an IFπGαCS in Y. Therefore f is an IFP closed mapping but not an IFπGα closed mapping.*

Remark 3.14. *An IFπGα closed mapping is independent of an IFGS closed mapping.*

Example 3.15. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.45, 0.5), (0.45, 0.4)\rangle$ and $T_2 = \langle y, (0.5, 0.6), (0.4, 0.3)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Consider an IFS $A = \langle x, (0.45, 0.4), (0.45, 0.5)\rangle$ is an IFCS in X. Then f(A) is an IFπGαCS in Y. But f(A) is not an IFGSCS in Y. Therefore f is an IFπGα closed mapping but not an IFGS closed mapping.*

Example 3.16. *Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $T_1 = \langle x, (0.8, 0.7), (0.1, 0.2)\rangle$ and $T_2 = \langle y, (0.2, 0.3), (0.7, 0.6)\rangle$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Consider an IFS $A = \langle x, (0.1, 0.2), (0.8, 0.7)\rangle$ is an IFCS in X. Then f(A) is an IFGSCS in Y . But f(A) is not an IFπGαCS in Y. Therefore f is an IFGS closed mapping but not an IFπGα closed mapping.*

The above relations are given in the following diagram.



The reverse implications are not true in genereal.

Theorem 3.17. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. Let f(A) be an IFRCS in Y for every IFCS A in X. Then f is an IF$\pi g\alpha$ closed mapping.*

Proof. Let A be an IFCS in X. Then f(A) is an IFRCS in Y. Since every IFRCS is an IFπGαCS, f(A) is an IFπGαCS in Y. Hence f is an IFπGα closed mapping. ☐

Theorem 3.18. *Let f : (X, τ) $\rightarrow$ (Y, σ) be an IFπGα closed mapping. Then f is an IF closed mapping if Y is an IF$_{\pi\alpha a}$T$_{1/2}$ space.*

Proof. Let A be an IFCS in X. By hypothesis, f(A) is an IFπGαCS in Y. Since Y is an IF$_{\pi\alpha a}$T$_{1/2}$ space, f(A) is an IFCS in Y. Hence f is an IF closed mapping. □

Theorem 3.19. *Let f : (X, τ) $\rightarrow$ (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if Y is an IF$_{\pi\alpha a}$T$_{1/2}$ space.*

(i) f is an IFπGα open mapping,

(ii) If A is an IFOS in X then f(A) is an IFπGαOS in Y,

(iii) f(int(A)) $\subseteq$ int(cl(int(f(A)))) for every IFS A in X.

Proof. (i) $\Rightarrow$ (ii) : It is obviously true.
(ii) $\Rightarrow$ (iii) : Let A be an IFS in X. Then int(A) is an IFOS in X. Then f(int(A)) is an IFπGαOS in Y. Since Y is an IF$_{\pi\alpha a}$T$_{1/2}$ space, f(int(A)) is an IFOS in Y. Therefore f(int(A)) = int(f(int(A)) $\subseteq$ int(cl(int(f(A))).
(iii) $\Rightarrow$ (i) : Let A be an IFOS in X. By hypothesis, f(int(A)) $\subseteq$ int(cl(int(f(A)))). This implies f(A) $\subseteq$ int(cl(int(f(A)))). Hence f(A) is an IFαOS in Y. Since every IFαOS is an IFπGαOS, f(A) is an IFπGαOS in Y. Hence f is an IFπGα open mapping. □

Theorem 3.20. *Let f : (X, τ) $\rightarrow$ (Y, σ) be an IFπGα closed mapping. Then f is an IFG closed mapping if Y is an IF$_{\pi\alpha b}$T$_{1/2}$ space.*

Proof. Let A be an IFCS in X. Then f(A) is an IFπGαCS in Y, by hypothesis. Since Y is an IF$_{\pi\alpha b}$T$_{1/2}$ space, f(A) is an IFGCS in Y. Hence f is an IFG closed mapping. □

Theorem 3.21. *Let f : (X, τ) $\rightarrow$ (Y, σ) be an IF closed mapping and g : (Y, σ) $\rightarrow$ (Z, δ) be an IFπGα closed mapping. Then g $\circ$ f : (X, τ) $\rightarrow$ (Z, δ) is an IFπGα closed mapping.*

Proof. Let A be an IFCS in X. Then f(A) is an IFCS in Y, by hypothesis. Since g is an IFπGα closed mapping, g(f(A)) is an IFπGαCS in Z. Hence g $\circ$ f is an IFπGα closed mapping. □

Theorem 3.22. *Let f : (X, τ) $\rightarrow$ (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following are equivalent if Y is an IF$_{\pi\alpha a}$T$_{1/2}$ space.*

(i) f is an IFπGα closed mapping,

(ii) f(int(A)) $\subseteq$ αint(f(A)) for each IFCS A of X,

(iii) int(f^{-1}(B)) $\subseteq$ f^{-1}(αint(B)) for every IFS B of Y.

Proof. (i) $\Rightarrow$ (ii) : Let f be an IFπGα closed mapping. Let A be any IFS in X. Then int(A) is an IFOS in X. By hypothesis, f(int(A)) is an IFπGαOS in Y. Since Y is an IF$_{\pi\alpha a}$T$_{1/2}$ space, f(int(A)) is an IFαOS in Y. Therefore αint(f(int(A))) = f(int(A)). Now f(int(A)) = αint(f(int(A))) $\subseteq$ αint(f(A)).
(ii) $\Rightarrow$ (iii) : Let B be an IFS in Y. Then f^{-1}(B) is an IFS in X. By hypothesis, f(int(f^{-1}(B))) $\subseteq$ αint(f(f^{-1}(B))) $\subseteq$ αint(B). Therefore int(f^{-1}(B)) $\subseteq$ f^{-1}(αint(B)). (iii) $\Rightarrow$ (i) : Let A be an IFOS in X. Then int(A) = A and f(A) is an IFS in Y. Then int(f^{-1}(f(A))) $\subseteq$ f^{-1}(αint(f(A))), by hypothesis. Now A = int(A) $\subseteq$ int(f^{-1}(f(A))) $\subseteq$ f^{-1}(αint(f(A))). Therefore f(A) $\subseteq$ f(f^{-1}(αint(f(A)))) = αint(f(A)) $\subseteq$ f(A). Therefore αint(f(A)) = f(A) is an IFαOS in Y. Hence f(A) is an IFπGαOS in Y. This implies f is an IFπGα closed mapping. □

Theorem 3.23. *Let f : (X, τ) → (Y, σ) be an IFπGα closed mapping and Y is an* $IF_{\pi\alpha c}T_{1/2}$ *space, then f is an IFGS closed mapping.*

Proof. Let A be an IFCS in X. By hypothesis, f(A) is an IFπGαCS in Y. Since Y is an $IF_{\pi\alpha c}T_{1/2}$ space, f(A) is an IFGSCS in Y. This implies f is an IFGS closed mapping. □

Theorem 3.24. *A mapping f : X → Y is an IFπGα open mapping if f(αint(A)) ⊆ αint(f(A))for every A ⊆ X.*

Proof. Let A be an IFOS in X. Then int(A) = A. Now f(A) = f(int(A)) ⊆ f(αint(A)) ⊆ αint(f(A)), by hypothesis. But αintf(A) ⊆ f(A). Hence αint(f(A)) = f(A). That is f(A) is an IFαOS in X. This implies f(A) is an IFπGαOS in X. Hence f is an IFπGα open mapping. □

Theorem 3.25. *Let f : (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if Y is an* $IF_{\pi\alpha a}T_{1/2}$ *space.*

(i) f is an IFπGα closed mapping,

(ii) cl(int(cl(f(A)))) ⊆ f(cl(A)) for every IFS A in X.

Proof. (i) ⇒ (ii) Let A be an IFS in X. Then cl(A) is an IFCS in X. By hypothesis, f(cl(A)) is an IFπGαCS in Y. Since Y is an $IF_{\pi\alpha a}T_{1/2}$ space, f(cl(A)) is an IFCS in Y. Therefore cl(f(cl(A))) = f(cl(A)). Now clearly cl(int(cl(f(A)))) ⊆ cl(f(cl(A))) = f(cl(A)). Hence cl(int(cl(f(A)))) ⊆ f(cl(A)).

(ii) ⇒ (i) Let A be an IFCS in X. By hypothesis cl(int(cl(f(A)))) ⊆ f(cl(A)) = f(A). This implies f(A) is an IFαCS in Y and hence f(A) is an IFπGαCS in Y. That is f is an IFπGα closed mapping. □

Definition 3.26. *A mapping f : X → Y is said to be an intuitionistic fuzzy i-πgα closed mapping (IFi-πGα closed mapping in short) if f(A) is an IFπGαCS in Y for every IFπGαCS A in X.*

Example 3.27. *Let X = {a, b} and Y = {u, v}. Let* $T_1 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ *and* $T_2 = \langle y, (0.2, 0.3), (0.7, 0.6)\rangle$*. Then* $\tau = \{0_\sim, T_1, 1_\sim\}$ *and* $\sigma = \{0_\sim, T_2, 1_\sim\}$ *are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFi-πGα closed mapping.*

Theorem 3.28. *Every IFi-πGα closed mapping is an IFπGα closed mapping but not conversely.*

Proof. Let f : X → Y be an IFi-πGα closed mapping. Let A be an IFCS in X. Then A is an IFπGαCS in X. By hypothesis, f(A) is an IFπGαCS in Y. Hence f is an IFπGα closed mapping. □

Example 3.29. *Let X = {a, b} and Y = {u, v}. Let* $T_1 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ *and* $T_2 = \langle y, (0.4, 0.5), (0.5, 0.5)\rangle$*. Then* $\tau = \{0_\sim, T_1, 1_\sim\}$ *and* $\sigma = \{0_\sim, T_2, 1_\sim\}$ *are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFπGα closed mapping. But f is not an IFi πGα closed mapping. Since the IFS A = ⟨x, (0.35, 0.45), (0.55, 0.5)⟩ is an IFπGαCS in X, but f(A) is not an IFπGαCS in Y. Hence f is an IFπGα closed mapping, but not an IFi-πGα closed mapping.*

Theorem 3.30. *If f : X → Y be a bijective mapping then the following are equivalent.*

(i) f is an IFi-πGα closed mapping,

(ii) f(A) is an IFπGαCS in Y for every IFπGαCS A in X,

(iii) f(A) is an IFi-πGαOS in Y for every IFi-πGαOS A in X.

Proof. (i) $\Leftrightarrow$ (ii): It is obviously true.

(ii) $\Rightarrow$ (iii): Let A be an IFπGαOS in X. By hypothesis, f(A^c) is an IFπGαCS in Y. That is $f(A)^c$ is an IFπGαCS in Y. Hence f(A) is an IFπGαOS in Y. (iii) $\Rightarrow$ (i): Let A be an IFπGαCS in X. Then A^c is an IFπGαOS in X. By hypothesis, f(A^c) is an IFπGαOS in Y. That is $f(A)^c$ is an IFπGαOS in Y. Hence f(A) is an IFπGαCS in Y. Thus f is an IFi-πGα closed mapping. □

Theorem 3.31. *Let $f : X \to Y$ be a mapping where X and Y are $IF_\alpha T_{1/2}$ spaces, then the following are equivalent.*

(i) f is an IFi-$\pi G\alpha$ closed mapping,

(ii) f(A) is an IF$\pi G\alpha OS$ in Y for every IF$\pi G\alpha OS$ A in X,

(iii) f(αint(B)) $\subseteq$ αint(f(B)) for every IFS B in X,

(iv) αcl(f(B)) $\subseteq$ f(αcl(B)) for every IFS B in X.

Proof. (i) $\Rightarrow$ (ii): It is obviously true.

(ii) $\Rightarrow$ (iii) Let B be any IFS in X. Since αint(B) is an IFαOS, it is an IFπGαOS in X. Then by hypothesis, f(αint(B)) is an IFπGαOS in Y. Since Y is an $IF_\alpha T_{1/2}$ space, f(αint(B)) is an IFαOS in Y. Therefore f(αint(B)) = αint(f(αint(B))) $\subseteq$ αint(f(B)). (iii) $\Rightarrow$ (iv): It can be proved by taking complement in (iii).

(iv) $\Rightarrow$ (i): Let A be an IFπGαCS in X. By hypothesis, αcl(f(A)) $\subseteq$ f(αcl(A)). Since X is an $IF_\alpha T_{1/2}$ space, A is an IFαCS in X. Therefore αcl(f(A)) $\subseteq$ f(αcl(A)) = f(A) $\subseteq$ αcl(f(A)). Hence f(A) is an IFαCS in Y. This implies f(A) is an IFπGαCS in Y. Thus f is an IFi-πGα closed mapping. □

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