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Some Results on Soft Sequences

Research Article

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Abstract: In this Paper, we discuss soft sequence, cluster point, limit point and convergence of soft sequence. We then prove some of the results related to these concepts.

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1. Introduction

Most of the problems arise in engineering, medical science, economics, sociology etc cannot be modeled using the classical mathematical methods. There are several theories such as probability theory, rough set theory, interval mathematics which are useful for solving the problem based on the uncertain data. But there are some difficulties with these theories as mentioned by Molodtsov [5] and he introduced Soft set theory in the year 1999 which will overcome all these difficulties. Main advantage of the soft set theory is that there is no need to introduce the membership function hence it is easy to handle. Several researchers worked on the soft set theory and introduced the new operations of the soft set theory such as Maji et al., Aktas and Cagman [4] have introduced the basic concepts of the soft set theory. P.K. Maji, R.Biswas and A. R. Roy [3] worked on the soft real sets and soft real numbers and their properties, Feng et al. [2] worked on soft sets combined with fuzzy sets and rough sets, the idea of soft metric space and soft points was first given by Sujoy Das and S.K. Samantha [7, 8]. Sadi Bayrramov, Cigdem Gunduz(Aras) and Murat I.Yazar [1] have worked on compact sets in Soft metric space. B. Surendranath Reddy and Sayyed Jalil worked on Soft totally bounded sets, soft equivalent, uniformly equivalent and Lipschitz equivalent metrics [9–11].

In this paper we will discuss about soft sequences and their properties such as convergent soft sequence, limit point of a soft sequence, cluster point of a soft sequence. Also we will prove some results based on the soft sequences.

1.1. Preliminary

We recall some basic definitions which are necessary in the next section.

Definition 1.1 ([5])**.** *Let U be an universal set, E be a set of parameters, then a pair (F, E) is called a soft set, where F is a mapping of E into the power set of U. In other words, a soft set is a parameterized family of subsets of U. For $x \in E, F(x)$ may be considered as the set of ϵ approximate elements of the soft set (F, E).*

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Definition 1.2 ([2]). *Let (F, A) and (G, B) be two soft sets over a common universal set U, then (F, A) is said to be a soft subset of (G, B), if the following condition holds,*

1. *$A \subset B$.*

2. *$\forall x \in A, F(x)$ and $G(x)$ are the identical approximations and denoted as $(F, A)\tilde{\subset}(G, B)$.*

In the same manner we say that (F, A) is superset of (G, B) over a common universal set U if (G, B) is a soft subset of (F, A). This relation will be denoted as $(F, A)\tilde{\supset}(G, B)$.

Definition 1.3 ([2]). *Let (F, A) and (G, B) be any two soft sets over a common universal set U then they are said to be equal if, $(F, A)\tilde{\subset}(G, B)$ and $(G, B)\tilde{\subset}(F, A)$.*

Definition 1.4 ([2]). *Let (F, A) be a soft set. Then the relative complement of (F, A) is denoted as $(F, A)^c$ and is defined as $(F, A)^c = (F^c, A)$, where $F^c : A \to P(U)$ is a mapping given by $F^c(x) = U \setminus F(x), \forall x \in A$.*

Definition 1.5 ([10]). *Let (F, A) be a soft set over a universal set U. Then (F, A) is said to be a null soft set, if $F(x) = \emptyset \quad \forall x \in A$.*

Definition 1.6. *[2] Let (F, A) be a soft set over the universal set U. Then (F, A) is said to be an absolute soft set if $\forall x \in A, F(x) = U$ and it is denoted as $\tilde{A}$.*

Definition 1.7 ([8]). *Let U be an universal set and A be a non empty set of parameters. A soft set (P, A) over U is called as a soft point if there is exactly one $\lambda \in A$ such that $P(\lambda) = \{x\}$ for some $x \in U$ and $P(\mu) = \emptyset$ for all $\mu \in A \setminus \{\lambda\}$. It will be denoted as P^x_λ.*

Definition 1.8 ([8]). *Two soft points P^x_λ, P^y_μ are called equal if $\lambda = \mu$ and $P(\lambda) = P(\mu)$ that is $x = y$. Thus $P^\lambda_x \neq P^y_\mu \Leftrightarrow x \neq y$ or $\lambda \neq \mu$.*

Definition 1.9 ([8]). *A mapping $\tilde{d} : SP(X') \times SP(X') \to \mathbb{R}(\mathbb{A})^*$ is said to be a soft metric on the soft set X' if $\tilde{d}$ satisfy the following conditions*

1. *$\tilde{d}(P^x_\lambda, P^y_\mu) \geq \bar{0}$ for all $P^x_\lambda, P^y_\mu \tilde{\in} X'$*

2. *$\tilde{d}(P^x_\lambda, P^y_\mu) = \bar{0}$ if and only if $P^x_\lambda = P^y_\mu$.*

3. *$\tilde{d}(P^x_\lambda, P^y_\mu) = d(P^y_\mu, P^x_\lambda)$ for all $P^x_\lambda, P^y_\mu \tilde{\in}(X', E)$.*

4. *For all $P^x_\lambda, P^y_\mu, P^z_\gamma \tilde{\in}(X', E), \tilde{d}(P^x_\lambda, P^z_\gamma)\tilde{\leq}\tilde{d}(P^x_\lambda, P^y_\mu) + \tilde{d}(P^y_\mu, P^z_\gamma)$. The soft set (X',E) with a soft metric $\tilde{d}$ on X' is called soft metric space and denoted as $(X', \tilde{d}, E)$ or $(X', \tilde{d})$.*

Definition 1.10 ([8]). *Let $\{P^{x_n}_{\lambda_n}\}$ be a sequence of soft points in a soft metric space $(X^{'}, \tilde{d}, E)$. The soft sequence $\{P^{x_n}_{\lambda_n}\}$ is called as convergent soft sequence in $(X', \tilde{d}, E)$ if there is a soft point $P^x_e \tilde{\in}(X', E)$ such that $\tilde{d}(P^{x_n}_{\lambda_n}, P^x_e) \to 0$ as $n \to \infty$. i.e. for given $\tilde{\epsilon}\tilde{>}\tilde{0}$, there is a natural numbers N such that $0\tilde{\leq}\tilde{d}(P^{x_n}_{\lambda_n}, P^x_e)\tilde{<}\tilde{\epsilon}$, whenever $n \geq N$.*

Definition 1.11 ([8]). *Let $(X^{'}, \tilde{d}, E)$ be a soft metric space. A sequence of soft points $\{P^{x_n}_{\lambda_n}\}$ in (X', E) is said to be soft bounded if the set of points of the sequence $\{P^{x_n}_{\lambda_n} | n \in \mathbb{N}\}$ is a bounded set.*

Definition 1.12 ([8]). *Let $\{P^{x_n}_{\lambda_n}\}$ be a soft sequence in a soft metric space $(X', \tilde{d}, E)$. A soft point P^x_e in (X', E) is said to be a cluster point of the soft sequence $\{P^{x_n}_{\lambda_n}\}$ if for each soft real number $\tilde{\epsilon}\tilde{>}\tilde{0}$ and $m \in Z_+$, there is an $n \in Z_+$ such that $\tilde{d}(P^{x_n}_{\lambda_n}, P^x_e)\tilde{<}\tilde{\epsilon}\ \forall n > m$.*

Definition 1.13 ([8]). *Let $(X', \tilde{d}, E)$ be a soft metric space and $(Y, A)\tilde{\subset}(X', E)$. A soft point $P_e^a\tilde{\in}(X', E)$ is called as the soft limit point of (Y, A) if every soft open ball $B(P_e^a, \tilde{r})$ containing P_e^a contains at least one soft point of (Y, A) other than P_e^a.*

Definition 1.14 ([6]). *Let $(X', \tilde{d}, E)$ be a soft metric space and $(Y, A)\tilde{\subset}(X', E)$. Then the soft set generated by the collection of all soft points of (Y, A) and soft limit points of (Y, A) in $(X', \tilde{d}, E)$ is said to be the soft closure of (Y, A) in $(X', \tilde{d}, E)$. It is denoted as $\overline{(Y, A)}$.*

Definition 1.15 ([8]). *Let $(X', \tilde{d}, E)$ be a soft metric space and $P_e^a\tilde{\in}(X', E)$. A collection $N(P_e^a)$ of soft points containing the soft point P_e^a is said to a be neighbourhood of P_e^a, if there exists a positive soft real number $\tilde{r}$ such that $B(P_e^a, \tilde{r})\tilde{\subset}N(P_e^a)$.*

2. Properties of Soft Sequences

Example 2.1. *Let $X \subset \mathbb{R}$ be a non empty set and $A \subset \mathbb{R}$ be a non empty parameter set. Let $(\tilde{X}', A)$ be an absolute soft set where $(F, A) = \tilde{X}'$ with $\tilde{d}(P_\lambda^x, P_\mu^y) = |x - y| + |\lambda - \mu|$. Let $(Y, A)\tilde{\subset}(\tilde{X}', E)$ be such that $Y(\lambda) = (0, 1]$ in the real line, $Y(e) = \emptyset \quad \forall e \in A\backslash\{\lambda\}$. Let us choose the sequence $\{P_{\lambda_n}^{x_n}\}$ of soft points of (Y, A) where $P_{\lambda_n}^{x_n} = \frac{1}{n}$ then there is no soft point $P_\mu^y \in (Y, A)$ such that $P_{\lambda_n}^{x_n} \to P_\mu^y$ in (Y, d_Y, A). However the sequence $\{P_{\lambda_n}^{y_n}\}$ of soft points of (Y, A) where $P_{\lambda_n}^{y_n} = \frac{1}{2}$ for all $n \in N$ is convergent in (Y, d_Y, A).*

Theorem 2.2. *A Soft sequence $\{P_{\lambda_n}^{x_n}\}$ in $(X', \tilde{d}, E)$ converges to a soft point $P_e^x\tilde{\in}(X', E)$ if and only if for each soft real number $\tilde{\epsilon}\tilde{>}\tilde{0}$, there is a natural number N such that $P_{\lambda_n}^{x_n}\tilde{\in}B(P_e^x, \epsilon)$ for all $n \geq N$.*

Proof. Let a soft sequence $\{P_{\lambda_n}^{x_n}\}$ in (X', d, E) converges to $P_e^x\tilde{\in}(X', E)$. That is for every $\tilde{\epsilon}\tilde{>}\tilde{0}$ there is a natural number N such that $\tilde{d}(P_{\lambda_n}^{x_n}, P_e^x)\tilde{<}\tilde{\epsilon} \quad \forall n \geq N$. If and only if $P_{\lambda_n}^{x_n}\tilde{\in}B(P_e^x, \epsilon) \quad \forall n \geq N$. □

Lemma 2.3. *Let $(X', \tilde{d}, E)$ be a soft metric space. Then the following inequality holds*

$$|\tilde{d}(P_{e_1}^{x_1}, P_{e_1}^{y_1}) - \tilde{d}(P_{e_2}^{x_2}, P_{e_2}^{y_2})|\tilde{\leq}\tilde{d}(P_{e_1}^{x_1}, P_{e_2}^{x_2}) + \tilde{d}(P_{e_1}^{y_1}, P_{e_2}^{y_2})$$

for all $P_{e_1}^{x_1}, P_{e_2}^{x_2}, P_{e_1}^{y_1}, P_{e_2}^{y_2}\tilde{\in}(X', E)$.

Proof. By the triangle inequality

$$\tilde{d}(P_{e_1}^{x_1}, P_{e_1}^{y_1})\tilde{\leq}\tilde{d}(P_{e_1}^{x_1}, P_{e_2}^{x_2}) + \tilde{d}(P_{e_2}^{x_2}, P_{e_2}^{y_2}) + \tilde{d}(P_{e_2}^{y_2}, P_{e_1}^{y_1}).$$

therefore

$$\tilde{d}(P_{e_1}^{x_1}, P_{e_1}^{y_1}) - \tilde{d}(P_{e_2}^{x_2}, P_{e_2}^{x_2})\tilde{\leq}\tilde{d}(P_{e_1}^{x_1}, P_{e_2}^{x_2}) + \tilde{d}(P_e^{y_1}, P_{e_2}^{y_2})$$

Similarly we can write as

$$\tilde{d}(P_{e_2}^{x_2}, P_{e_2}^{y_2}) - \tilde{d}(P_{e_1}^{x_1}, P_{e_1}^{y_1})\tilde{\leq}\tilde{d}(P_{e_1}^{x_1}, P_{e_2}^{x_2}) + \tilde{d}(P_{e_1}^{y_1}, P_{e_2}^{y_2})$$

Thus

$$|\tilde{d}(P_{e_1}^{x_1}, P_{e_1}^{y_1}) - \tilde{d}(p_e^{x_2}, P_{e_2}^{y_2})|\tilde{\leq}\tilde{d}(p_{e_1}^{x_1}, P_{e_2}^{x_2}) + \tilde{d}(P_{e_1}^{y_1}, P_{e_2}^{y_2}).$$

□

Theorem 2.4. *Let $(X', \tilde{d}, E)$ be a soft metric space. A soft point $P_e^x\tilde{\in}(X', E)$ is a soft limit point of (F, A) if and only if there is a soft sequence in (F, A) converging to P_e^x.*

Proof. Let P_e^x be a limit point of (F, A). If P_e^x is in (F, A) then the constant sequence $\{P_e^x, P_e^x, P_e^x, ...P_e^x\}$ is in (F, A) that converges to P_e^x. If P_e^x is not in (F, A), then P_e^x is a limit point of (F, A). Therefore for each $n \in \mathbb{N}$, the open ball $B(P_e^x, \frac{1}{n})$ intersects with (F, A), and so there is a point P_λ^x in $B(P_e^x, \frac{1}{n})\tilde{\cap}(F, A)$. Then $\{P_{\lambda_n}^{x_n}\}$ is a soft sequence in (F, A) which converges to P_e^x.

Conversely, assume that $\{P_{\lambda_n}^{x_n}\}$ is a soft sequence in (F, A) that converges to P_e^x. Then for each soft real number $\epsilon > 0, \exists$ a natural number $N = N(\epsilon)$ such that $P_{\lambda_n}^{x_n}\tilde{\in}B(P_e^x, \epsilon) \quad \forall n \geq N$. Thus P_e^x is a limit point of (F, A). □

Theorem 2.5. *Let $(X', \tilde{d}, E)$ be a soft metric space. A sequence of soft points $\{P_{\lambda_n}^{x_n}\}$ in (X', E) converges to $P_e^x\tilde{\in}(X', E)$ if and only if every subsequence $\{P_{\lambda_{n_K}}^{x_{n_k}}\}$ of $\{P_{\lambda_n}^{x_n}\}$ converges to P_e^x.*

Proof. Let $\tilde{\epsilon}\tilde{>}\tilde{0}$ be given soft real number. If the soft sequence $\{P_{\lambda_n}^{x_n}\}$ converges to the soft points P_e^x, then there is a natural number N such that $\{P_{\lambda_n}^{x_n}\}\tilde{\in}B(P_e^x, \epsilon) \quad \forall n \geq N$. This implies that $\{P_{\lambda_{n_k}}^{x_{n_k}}\}\tilde{\in}B(P_e^x, \tilde{\epsilon}) \quad \forall k \geq N$,as $n_k \geq k \quad \forall k \in \mathbb{N}$. Converse, follows from the fact that $\{P_{\lambda_n}\}$ itself a soft subsequence. Since $n^k \geq k\forall k \in \mathbb{N}$. □

Theorem 2.6. *Let $(X'\tilde{d}, E)$ be a soft metric space. Then a soft sequence $\{P_{\lambda_n}^{x_n}\}$ in (X', E) converges to P_e^x in (X', E) if and only if every soft subsequence of $\{P_{\lambda_{n_k}}^{x_{n_k}}\}$ has a soft subsequence that converges to P_e^x.*

Proof. We know that soft subsequence of soft subsequence of a soft sequence $\{P_\lambda^x\}_n$ is a soft subsequence of $\{P_\lambda^x\}$, the necessity part follows by using the above theorem.

Conversely suppose that every soft subsequence of the soft sequence $\{P_{\lambda_n}^{x_n}\}$ has a soft subsequence that also converges to $P_e^x\tilde{\in}(X', E)$. Now suppose that $\{P_{\lambda_n}^{x_n}\}$ does not converges to P_e^x. So there is a soft real number $\tilde{\epsilon}\tilde{>}\tilde{0}$ such that for each $m \in Z_+$, there is a positive integer $n > m$ such that $\{P_{\lambda_n}^{x_n}\} \notin B(P_e^x, \tilde{\epsilon})$. But then we get a soft subsequence of a soft sequence $\{P_{\lambda_n}^{x_n}\}$ which has no soft subsequence that converges to the P_e^x. Which is a contradiction. Hence the soft sequence $\{P_{\lambda_n}^{x_n}\}$ converges to the P_e^x. □

Theorem 2.7. *A soft sequence $\{P_{\lambda_n}^{x_n}\}$ in a soft metric space $(X', \tilde{d}, E)$ converges to a soft point $P_e^x\tilde{\in}(X', E)$ if and only if for each neighbourhood N of P_e^x, there is a positive integer m such that $\{P_{\lambda_n}^{x_n}\}\tilde{\in}N$ for all $n \geq m$.*

Proof. If N is a neighbourhood of P_e^x then there is a soft real number $\tilde{\epsilon}\tilde{>}\tilde{0}$ such that $B(P_e^x, \tilde{\epsilon})\tilde{\subseteq}N$. If $\{P_{\lambda_n}^{x_n}\}$ converges to P_e^x, then there is a positive real number m such that $\{P_{\lambda_n}^{x_n}\}\tilde{\in}B(P_e^x, \epsilon)$ for all $n \geq m$.

Conversely, since for each soft real number $\tilde{\epsilon}\tilde{>}\tilde{0}, B(P_e^x, \tilde{\epsilon})$ is a neighbourhood of P_e^x, by our assumption, there is an $m \in \mathbb{N}$ such that $\{P_{\lambda_n}^{x_n}\}\tilde{\in}B(P_e^x, \tilde{\epsilon})$ for all $n \geq m$. Thus a soft sequence $\{P_{\lambda_n}^{x_n}\}$ converges to soft point P_e^x in $(X', \tilde{d}, E)$. □

Theorem 2.8. *The limit of a convergent soft sequence is a cluster point of that soft sequence.*

Proof. Let $(X', \tilde{d}, E)$ be a soft metric space and let $\{P_{\lambda_n}^{x_n}\}$ be a soft sequence in (X', E) converging to a soft point P_e^x in (X', E). Then for each soft real number $\tilde{\epsilon}\tilde{>}\tilde{0}$ there is an $m \in \mathbb{N}$ such that $\{P_{\lambda_n}^{x_n}\}\tilde{\in}B(P_e^x, \tilde{\epsilon})$ for all $n \geq m$. Now for each $p \in \mathbb{N}$, select $n > \max\{p, m\}$. Then $\{P_{\lambda_n}^{x_n}\}\tilde{\in}B(P_e^x, \tilde{\epsilon})$, and so P_e^x is a cluster point of the soft sequence $\{P_{\lambda_n}^{x_n}\}$. □

Theorem 2.9. *Every convergent soft sequence in a soft metric space has unique cluster point.*

Proof. Let $\{P_{\lambda_n}^{x_n}\}$ be a soft sequence in soft metric space $(X', \tilde{d}, E)$ converging to a soft point P_e^x then P_e^x is a cluster point of the soft sequence $P_{\lambda_n}^{x_n}$. Let P_e^y be another cluster point of the soft sequence $P_{\lambda_n\, n}^{x_n}$ different from P_e^x. Let $\tilde{\epsilon} = \frac{1}{2}\tilde{d}(P_e^x, P_e^y)$. Since P_e^x is a limit point of $P_{\lambda_n}^{x_n}$, there is an $m \in \mathbb{N}$such that $\tilde{d}(P_{\lambda\, n}^x, P_e^x) < \tilde{\epsilon}$ for all $n \geq m$. Since P_e^y is also a cluster point, there is an $n \in \mathbb{N}$ such that $n > m$ and $\tilde{d}(P_{\lambda_n}^{x_n}, P_e^x)\tilde{<}\tilde{\epsilon}$ then

$$\tilde{d}(P_e^x, P_e^y)\tilde{<}\tilde{d}(P_e^x, P_{\lambda_n}^{x_n}) + \tilde{d}(P_e^y, P_{\lambda_n}^{x_n})\tilde{<}2\tilde{\epsilon}.$$

Which is a contradiction. Hence a convergent soft sequence in a soft metric space has unique cluster point. □

Theorem 2.10. *A soft point P^x_e in a soft metric space $(X', \tilde{d}, E)$ is a cluster point of the soft sequence $\{P^{x_n}_{\lambda_n}\}$ in (X', E) if and only if there is a soft subsequence $\{P^{x_{nk}}_{\lambda_{nk}}\}$ of $\{P^{x_n}_{\lambda_n}\}$ that converges to the soft point P^x_e.*

Proof. First suppose that P^x_e be the cluster point of the soft sequence $\{P^{x_n}_{\lambda_n}\}$. Then there is an $n_1 \in \mathbb{N}$ such that $\tilde{d}(P^{x_{n1}}_{\lambda_{n1}}, P^x_e)\tilde{<}\tilde{1}$. Again there is n_2 such that $n_2 > n_1$, and $\tilde{d}(P^{x_{n2}}_{\lambda_{n2}}, P^x_e)\tilde{<}\frac{\tilde{1}}{2}$. By the induction method there exist $n_k \in Z_+$ such that $n_k > n_k - 1$ and $\tilde{d}(P^{x_{nk}}_{\lambda_{nk}}, P^x_e)\tilde{<}\frac{\tilde{1}}{k}$. Thus $\{P^{x_{nk}}_{\lambda_{nk}}\}$ is a soft subsequence of of the soft sequence $\{P^{x_n}_{\lambda_n}\}$ which converges to P^x_e.

Conversely assume that the soft subsequence $\{P^{x_{nk}}_{\lambda_{nk}}\}$ of the soft sequence $\{P^{x_n}_{\lambda_n}\}$ converges to the soft point P^x_e in (X', E). Let $\tilde{\epsilon}\tilde{>}\tilde{0}$ be a soft real number and $m \in Z_+$. Since the soft sub sequence $\{P^{x_{nk}}_{\lambda_{nk}}\}$ converges to the soft point P^x_e, there is a $p \in \mathbb{N}$ such that $\tilde{d}(P^{x_{nk}}_{\lambda_{nk}}, P^x_e)\tilde{<}\tilde{\epsilon}$, for all $k \geq p$. Then for each $r > \max\{p, m\}, n_r > m$, and $\tilde{d}(P^{x_{nr}}_{\lambda_{nr}}, P^x_e)\tilde{<}\tilde{\epsilon}$. Thus P^x_e is a cluster point of the soft sequence $\{P^{x_n}_{\lambda_n}\}$. □

Theorem 2.11. *A soft point $P^x_e\tilde{\in}(X', E)$ is an adherent point of a soft subset (B, A) of (X', E) in a soft metric space (X', d, E) if and only if there is a soft sequence in B that converges to the soft point P^x_e.*

Proof. Suppose that P^x_e be an adherent point of (B, A). If P^x_e belongs to (B, A) then the constant soft sequence $\{P^x_e, P^x_e, P^x_e, P^x_e, P^x_e, P^x_e, ...\}$ belongs to (B, A) that converges to the soft point P^x_e. If suppose that P^x_e not belongs to the set (B, A), then P^x_e be an limit point of the set (B, A). Hence for each $m \in \mathbb{N}$ the open ball $B(P^x_e, \frac{1}{m})$ intersects to (B, A), and so there is a point $P^{x_n}_{\lambda_n}$ in $B(P^x_e, \frac{1}{m})\tilde{\cap}(B, A)$. Then $\{P^{x_n}_{\lambda_n}\}$ is a soft sequence in (B, A) that converges to the soft point P^x_e.

Conversely suppose that $\{P^{x_n}_{\lambda_n}\}$ be a soft sequence in the set B such that it converges to the soft point P^x_e. Then for any soft real number $\tilde{\epsilon}\tilde{>}\tilde{0}$, there exist $p \in \mathbb{N}$ such that $\{P^{x_n}_{\lambda_n}\}\tilde{\in}B(P^x_e, \tilde{\epsilon})$ for all $n \geq p$. Thus P^x_e be an adherent point of the set B. □

Theorem 2.12. *Let $(X', \tilde{d}, E)$ be a soft metric space. Let $\{P^{y_n\lambda_n}\}_n$ be the soft sequence in (X', E) which converges to the soft point P^y_e if and only if the soft sequence $\{P^{x_{nj}}_{\lambda_{nj}}\}$ in the j^th co-ordinates converges in X'_j to the soft point P^x_e for all $j = 1, 2, 3, ...k$.*

Proof. If the soft sequence $\{P^{y_n}_{\lambda_n}\}$ converges to the soft point P^y_e then for all soft real number $\tilde{\epsilon}\tilde{>}\tilde{0}$ there exist an $m \in \mathbb{N}$ such that $\tilde{d}(P^{y_n}_{\lambda_n}, P^y_e)\tilde{<}\tilde{\epsilon}$ for all $n \geq m$. This implies that $\max_j\{d(P^{x_{nj}}_{\lambda_{nj}}, P^{x_j}_{e_j})\}\tilde{<}\tilde{\epsilon}$ for all $n \geq m$.Therefore $\tilde{d}(P^{x_{nj}}_{\lambda_{nj}}, P^{x_j}_{e_j})\tilde{<}\tilde{\epsilon} \quad \forall n \geq m$. That is $\{P^{x_{nj}}_{\lambda_{nj}}\}$ is a convergent soft sequence which converges to the soft point $P^{x_j}_{e_j}$ for each j.

Conversely suppose that $\tilde{\epsilon}\tilde{>}\tilde{0}$ be any soft real number. For every j, there is an $m_j \in \mathbb{N}$ using our assumption such that $\tilde{d}(P^{x_{nj}}_{\lambda_{nj}}, P^{x_j}_{e_j})\tilde{<}\tilde{\epsilon}$ for each $n \geq m_j$. Let m_0 be the maximum among $m_1, m_2, m_3, ...m_k$. It follows that $(P^{y_n}_{\lambda_n}, P^y_e)\tilde{<}\tilde{\epsilon}$ for each $n \geq m_0$ and therefore the soft sequence $\{P^{y_n}_{\lambda_n}\}$ converges to the soft point P^y_e. □

Theorem 2.13. *Suppose that $(X', \tilde{d}_1, E), (X', \tilde{d}_2, E)$ be two soft metric spaces. Then the soft metrics $\tilde{d}_1, \tilde{d}_2$ are said to be equivalent if and only if every subset of (X', E) has the same soft closure in $(X', \tilde{d}_1, E), (X', \tilde{d}_2, E)$.*

Proof. First suppose that $\tilde{d}_1, \tilde{d}_2$ are the equivalent soft metrics on the soft metric space (X', E), and let (M, A) be a subset of (X', E). Next suppose that $P^x_e\tilde{\in}X' - M$ be the limit point of the subset (M, A) in the metric space $(X', \tilde{d}_2, E)$. Then there is a soft sequence $\{P^{x_n}_{\lambda_n}\}$ in the set (M, A) which converges to the soft point P^x_e in the soft metric space $(X', \tilde{d}_2, E)$. Then the soft sequence $\{P^{x_n}_{\lambda_n}\}$ converges to the soft point P^x_e in the soft metric space $(X', \tilde{d}_1, E)$. Hence the soft point P^x_e is the soft limit point of the set (M, A) in the soft metric space $(X', \tilde{d}_1, E)$. Hence the soft closure of a set (M, A) in the soft metric space $(X', \tilde{d}_1, E), (X', \tilde{d}_2, E)$ are same.

Conversely, assume that $\tilde{d}_1, \tilde{d}_2$ are not equivalent. Also suppose that the soft sequence $\{P^{x_n}_{\lambda_n}\}$ in (X', E) converges to the soft point P^x_e in the soft metric space $(X', \tilde{d}_1, E)$ and suppose that the soft sequence $\{P^{x_n}_{\lambda_n}\}$ does not converges to the soft point P^x_e in the soft metric space $(X', \tilde{d}_2)$. Then there is a soft real number $\tilde{\epsilon}\tilde{>}\tilde{0}$ such that for each $m \in Z_+$ there is an $n > m$ such that $\tilde{d}_2(P^x_{\lambda\,n}, P^x_e)\tilde{\geq}\tilde{\epsilon}$. Consequently, there is a soft subsequence $\{P^{x_{n\,k}}_{\lambda_{n\,k}}\}$ of the soft sequence $\{P^{x_n}_{\lambda_n}\}$ such that $P^{x_{n\,k}}_{\lambda_{n\,k}}\tilde{\notin}S(P^x_e, \tilde{\epsilon}, \tilde{d}_2)$ for all $k \in Z_+$. This means that the soft subsequence $\{P^{x_{n\,k}}_{\lambda_{n\,k}}\}\tilde{\subset}S - S(P^x_e, \tilde{\epsilon}, \tilde{d}_2)$. Now the set $X - S(P^x_e, \tilde{\epsilon}, \tilde{d}_2)$ is a closed set in the soft metric space $(X', \tilde{d}_2, E)$. Let M be the set of points of of the soft subsequence $\{P^{x_{n\,k}}_{\lambda_{n\,k}}\}$. Then we have $(M, A)\tilde{\subset}X - S(P^x_e, \epsilon, \tilde{d}_2, E)$ so that the soft closure of of (M, A) in the soft metric space $(X', \tilde{d}_2, E)$ is contained in the set $X - S(P^x_e, \tilde{\epsilon}, \tilde{d}_2)$. The soft closure of the set (M, A) in $(X', \tilde{d}_2, E)$ does not contain the soft point P^x_e. But the soft point P^x_e is the is in the soft closure of (M, A) in the soft metric space $(X', \tilde{d}_1, E)$, since $\{P^{x_{n\,k}}_{\lambda_{n\,k}}\}$ is the soft subsequence of the soft sequence $\{P^{x_n}_{\lambda_n}\}$ which converges to the soft point P^x_e and hence $\{P^{x_{n\,k}}_{\lambda_{n\,k}}\}$ is itself a soft soft convergent to the soft point P^x_e in the soft metric space $(X', \tilde{d}_1, E)$. This shows that the soft closure of (M, A) in $(X', \tilde{d}_1, E), (X', \tilde{d}_2, E)$ are not identical. This completes the proof of the theorem □

Theorem 2.14. *Let $(X'_i, \tilde{d}_i, E)$ be a soft metric spaces where $i = 1, \ldots, k$, and $X = \prod X_i$. Then for any real number $n \geq 1$, the function defined by by*

$$\rho_n(P^x_e, P^y_e) = [\sum_{i=1}^{i=n}\{\tilde{d}_i(P^{x_i}_{e_i}, P^{y_i}_{e_i})\}^n]^{\frac{1}{n}}$$

For all $P^x_e = (P^{x_1}_{e_1}, P^{x_2}_{e_2}, P^{x_3}_{e_3} \ldots P^{x_k}_{e_k})$, $P^y_e = (P^{y_1}_{e_1}, P^{y_2}_{e_2}, P^{y_3}_{e_3}, \ldots, P^{y_k}_{e_k})$ in X' is a product metric on X'.

Proof. We will prove that ρ_n is a soft metric on X'. The triangle inequality for ρ_n is

$$[\sum_{i=1}^{i=k}\{d_i(P^{x_i}_{e_i}, P^{y_i}_{e_i})\}^n]^{\frac{1}{n}}\tilde{\leq}[\sum_{i=1}^{k}\{\tilde{d}_i(P^{x_i}_{e_i}, P^{z_i}_{e_i}) + \tilde{d}_i(P^{z_i}_{e_i}], P^{y_i}_{e_i})\}^n]^{\frac{1}{n}}$$

$$\tilde{\leq}[\sum_{i=1} k\{\tilde{d}_i(P^{x_i}_{e_i}, P^{z_i}_{e_i})\}^n]^{\frac{1}{n}} + [\sum_{i=1} k\{\tilde{d}_i(P^{z_i}_{e_i}, P^{y_i}_{e_i})\}^n]^{\frac{1}{n}}$$

using Minkowkis inequality. We will prove that $\tilde{d}$ is a soft metric space. Let $\{P^{z_n}_{\lambda_n}\}$ be a soft sequence in (X', E) converges to the soft point P^z_e where $P^{x_n}_{\lambda_n} = (P^{x_{n1}}_{\lambda_{n1}}, P^{x_{n2}}_{\lambda_{n2}}, P^{x_{n3}}_{\lambda_{n3}}, \ldots\ldots P^{x_{nk}}_{\lambda_{nk}}$ and $P^x_e = (P^{x_1}_{e_1}, P^{x_2}_{e_2}, P^{x_3}_{e_3}, \ldots, P^{x_k}_{e_k})$ in X'. Since $d_i(P^{x_{n\,i}}_{\lambda_{n\,i}}, P^{x_i}_{e_i}) \leq \rho_n(P^{z_n}_{\lambda_n}, P^z_e)$, hence the convergence of the soft sequence $\{P^{x_{n\,i}}_{\lambda_{n\,i}}\}$ in i-th co-ordinates to the soft point $P^{x_i}_{e_i}$ for each $i = 1, \ldots, k$. Conversely, suppose that each co-ordinate soft sequence $\{P^{x_{n\,i}}_{\lambda_{n\,i}}\}$, converges to the soft point $P^{x_i}_{e_i}$ where $i = 1, \ldots, k$. Let $\tilde{\epsilon}\tilde{>}\tilde{0}$ be any soft real number. Then there is an $m_i \in \mathbb{N}$ such that

$$\tilde{d}_i(P^{x_{n\,i}}_{\lambda_{n\,i}}, P^{x_i}_{e_i})\tilde{\leq}(\frac{\tilde{\epsilon}^p}{k})^{\frac{1}{p}}$$

for each $n \geq m_i$.

Let $m_0 = max\{m_1, m_2, m_3, \ldots.m_k\}$, then we have $\left[\sum_{i=1}^{k}\{\tilde{d}_i(\{P^{x_{n\,i}}_{\lambda_{n\,i}}\}, P^{x_i}_{e_i})\}^p\right]^{\frac{1}{p}}\tilde{<}\tilde{\epsilon}\ \forall n \geq m_0$. □

Theorem 2.15. *The soft metric ρ in the general Frechet soft metric space $(X', \tilde{\rho}, E)$ is a soft product metric.*

Proof. Let $\{P^{y_n}_{\lambda_n}\}$ be a soft sequence in (X', E), where $P^{y_n}_{\lambda_n} = \{P^{x_{n\,1}}_{\lambda_{n\,1}}, P^{x_{n\,2}}_{\lambda_{n\,2}}, P^{x_{n\,3}}_{\lambda_{n\,3}} \ldots\}$ converging to the soft point $P^y_e = \{P^{x_1}_{e_1}, P^{x_2}_{e_2}, P^{x_3}_{e_3} \ldots\}$. We have

$$0\tilde{\leq}\frac{\tilde{d}_j(\{P^{x_{n\,j}}_{\lambda_{n\,j}}\}, P^{x_j}_{e_j})}{2^j(1 + \tilde{d}_j(\{P^{x_{n\,j}}_{\lambda_{n\,j}}\}, P^{x_j}_{e_j}))}\tilde{\leq}\tilde{\rho}(P^{y_n}_{\lambda_n}, P^y_e), \tag{1}$$

which implies that $\lim \frac{\tilde{d}_j(P^{x_{n\,j}}_{\lambda_{n\,j}}, P^{x_j}_{e_j})}{2^j(1+\tilde{d}_j(P^{x_{n\,j}}_{\lambda_{n\,j}}, P^{x_j}_{e_j})} = 0$ as $n \to \infty$.The above equation can be written as $\lim \tilde{d}_j(P^{x_{n\,j}}_{\lambda_{n\,j}}, P^{x_j}_{e_j}) = 0$, as $n \to \infty$. Hence the soft sequence $\{P^{x_{n\,j}}_{\lambda_{n\,j}}\}$ of the oft sequence $\{P^{y_n}_{\lambda_n}\}$ in the j-th co-ordinate of converges to the soft point

$P_{e_j}^{x_j}$ for all j. Conversely assume that the soft sequence $\{P_{\lambda_{nj}}^{x_{nj}}\}$ converges to the soft point $P_{e_j}^{x_j}$ for all j. Let $\tilde{\epsilon}\tilde{>}\tilde{0}$ be a soft real number. then there is an $n_0 \in \mathbb{N}$ which depends on $\tilde{\epsilon}$ such that $\sum_{i=n_0+1}^{i=\infty} \frac{1}{2^i} < \frac{\tilde{\epsilon}}{2}$. Again the the soft sequence $\{P_{\lambda_{ni}}^{x_{ni}}\}$ converges to the soft point $P_{e_i}^{x_i}$, For all $j = 1, \ldots, n_0$, then there is $p_i \in \mathbb{N}$ such that

$$\tilde{d}_i(P_{\lambda_{ni}}^{x_{ni}}, P_{e_i}^{x_i})\tilde{<}\frac{\tilde{\epsilon}}{2n_0},$$

for all $n \geq p_i$. Let $p_0 = max\{p_1, p_2, p_3, \ldots, p_{n0}\}$. Then we have

$$\tilde{\rho}(P_{\lambda_n}^{y_n}, P_e^y)\tilde{\leq}\sum_{i=1} n_0\tilde{d}_i(P_{\lambda_{ni}}^{x_{ni}}, P_{e_i}^{x_i}) + \sum_{i=n_0+1}^{i=\infty} \frac{1}{2^i}$$

$$\tilde{<}\frac{n_0\tilde{\epsilon}}{2n_0} + \frac{\tilde{\epsilon}}{2}$$

for $n \geq p_0$. Hence $\tilde{\rho}(P_{\lambda_n}^{y_n}, P_e^y)\tilde{<}\tilde{\epsilon}$ for each $n \geq p_0$. □

Theorem 2.16. *Let $\{P_{\lambda_n}^{y_n}\}$ be a soft sequence in l_p spaces where $\{P_{\lambda_n}^{y_n}\} = \{P_{\lambda_{n1}}^{y_{n1}}, P_{\lambda_{n2}}^{y_{n2}}, P_{\lambda_{n3}}^{y_{n3}} \ldots\}$ converges to the soft point $P_e^y = \{P_{e_1}^{x_1}, P_{e_2}^{x_2}, P_{e_3}^{x_3} \ldots\}$ in the l_p spaces. Then the soft sequence $P_{\lambda_{nj}}^{x_{nj}}$ in the j^th coordinate of the soft sequence $\{P_{\lambda_n}^{y_n}\}$ converges to the the soft point $P_{e_j}^{x_j}$. For all $j \in \mathbb{N}$.*

Proof. since we have

$$0\tilde{\leq}|P_{\lambda_{nj}}^{y_{nj}} - P_{e_j}^{x_j}|\tilde{\leq}[\sum_{i=1}^{\infty}|P_{\lambda_{nj}}^{y_{nj}} - P_{e_j}^{x_j}|^p]^{\frac{1}{p}} = \tilde{d}_p(P_{\lambda_n}^{y_n}, P_e^y),$$

Now conversely suppose that the above theorem is not true; let define the soft sequence $\{P_{\lambda_n}^{y_n}\}$ in the l_p space given bellow. Assume that $P_{\lambda_n}^{y_n} = \{P_{\lambda_{n1}}^{y_{n1}}, P_{\lambda_{n2}}^{y_{n2}}, \ldots\}$ where

$$P_{\lambda_{nj}}^{y_{nj}} = \begin{cases} 1, & if n = j; \\ 0, & if n \neq j. \end{cases}$$

Then $\lim P_{\lambda_{nj}}^{y_{nj}} = 0$ as $n \to \infty$ for all $j \in \mathbb{N}$, but the soft sequence $\{P_{\lambda_n}^{y_n}\}$ not converges to $0 = \{0, 0, 0, 0, 0, \ldots\}$ in the l_2 space since $\tilde{d}_p(\{P_{\lambda_n}^{y_n}\}, 0) = 1$ for each n. □

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