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Compactness on Rough Intuitionisitic Fuzzy Structure Subgroup Space

Research Article

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Abstract: This paper aims to initiate the notion of rough intuitionistic fuzzy compact subgroup, rough intuitionistic fuzzy extremal compact spaces in the light of rough intuitionistic fuzzy sets. We characterize rough intuitionistic structure subspace space by applying various topological notions.Few of its properties are discussed.

Keywords: Rough intuitionistic fuzzy compact subgroup, Rough intuitionistic fuzzy extremal compact spaces, Intuitionistic fuzzy sets.

1. Introduction

Rough set theory, proposed by Pawlak [11] is a new mathematical tool that supports uncertainty reasoning. It may be seen as an extension of classical set theory and has been successfully applied to machine learning, intelligent systems, inductive reasoning, pattern recognition, image processing, signal analysis, knowledge discovery, decision analysis, expert systems and many other fields. In 1965, Zadeh [13] initiated the novel concept of fuzzy set theory. There have been attempts to fuzzify various mathematical structures like topological spaces, groups, rings, etc., also concepts like relations measure, probability and automata etc. Biswas and Nanda [2] in 1994 introduced the concept of rough ideal in semi group. Based on an equivalence relation in 1990, Dubois and Prade [5] introduced the lower and upper approximations of fuzzy sets in Pawlak approximation space to obtain an extended notion called rough fuzzy sets. In 2008, Kazanci and Davaaz [11] introduced rough prime ideals and rough fuzzy prime ideals in commutative rings. Recently Jayanta Ghosh and T.K. Samantha [7] introduced rough intuitionistic fuzzy sets in semigroups. Coker [3] introduced the concept of intuitionistic fuzzy compact sets. In this paper we define rough intuitionistic fuzzy compact subgroups and rough intuitionistic fuzzy subgroup extremal compact space. Some interesting properties are established.

2. Preliminaries

Definition 2.1 ([1]). *An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)/x \in X\rangle\}$ where the function $\mu : X \to [0,1]$ and $\nu : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely$\nu_A(x)$) of each element $x \in X$ to the set A, respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X) the set of all intuitionistic fuzzy set in X.*

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Definition 2.2 ([1]). *Let A and B be IFS's of the form $A = \{\langle x, \mu_A(x), \nu_A(x)/x \in X\rangle\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)/x \in X\rangle\}$. Then*

1. *$A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$.*
2. *A=B if and only if $A \subseteq B$ and $B \subseteq A$.*
3. *$\overline{A} = \{\langle x, \nu_A(x), \mu_A(x)/x \in X\rangle\}$.(Complement of A)*
4. *$A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)/x \in X\rangle\}$.*
5. *$A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)/x \in X\rangle\}$.*

For the sake of simplicity we use the notion $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)/x \in X\rangle\}$. The intuitionistic fuzzy set $0 \sim = \{\langle x, 0\sim, 1\sim\rangle /x \in X\}$ and $1 \sim = \{\langle x, 1\sim, 0\sim\rangle /x \in X\}$ are respectively the empty set and the whole set of X.

Note: For any IFS A in (X, τ), we have $cl(\overline{A}) = \overline{int(A)}$ and $int(\overline{A}) = \overline{cl(A)}$.

Definition 2.3 ([7]). *Let $A = (\mu_A, \nu_A)$ be an IFS in S and let $\alpha, \beta \in [0, 1]$ be such that $\alpha + \beta \leq 1$. Then the set*

$$A_{\alpha,\beta} = \{x \in S|\mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$$

is called a (α, β)-level subset of A. The set of all $(\alpha, \beta) \in Im(\mu_A) \times Im(\nu_A)$ such that $\alpha + \beta \leq 1$ is called the image of A denoted by $Im(A)$

Definition 2.4 ([7]). *Let θ be a congruence relation on G that is θ is an equivalence relation on G such that*

$$(a, b) \in \theta \Rightarrow (ax, bx) \in \theta \quad and \quad (xa, xb) \in \theta \quad forall \quad x \in S.$$

For a congruence relation θ on S, we have $[a]_\theta[b]_\theta \subseteq [ab]_\theta$ for all $a, b \in S$, where $[a]_\theta$ denotes $\theta-$congruence class containing the element $a \in S$. A congruence relation θ on S is called complete if $[a]_\theta[b]_\theta = [ab]_\theta$ for all $a, b \in S$. Let us consider θ to be a congruence relation of S. If X is a nonempty subset of S then the sets $\theta_*(X) = x \in S|[x]_\theta \subseteq X$ and $\theta^*(X) = x \in S|[x]_\theta \cap X \neq \phi$ are respectively called the $\theta-$ lower and $\theta-$ upper approximation sof the set X and $\theta(X) = (\theta_*(X), \theta^*(X))$ is called rough set with respect to θ if $\theta_*(X) \neq \theta^*(X)$. If $A = (\mu_A, \nu_A)$ be IFS of S. Then the IFS $\theta_*(A) = (\theta_*(\mu_A), \theta_*(\nu_A))$ and $\theta^*(A) = (\theta^*(\mu_A), \theta^*(\nu_A))$ are respectively called $\theta-$lower and $\theta-$upper approximation of the IFS $A = (\mu_A, \nu_A)$ where for all $x \in S$

$$\theta_*(\mu_A)(x) = \wedge_{a\in[x]_\theta}\mu_A(a), \theta_*(\nu_A)(x) = \vee_{a\in[x]_\theta}\nu_A(a)$$
$$\theta^*(\mu_A)(x) = \vee_{a\in[x]_\theta}\mu_A(a), \theta_*(\nu_A)(x) = \wedge_{a\in[x]_\theta}\nu_A(a)$$

For an IFS $A = (\mu_A, \nu_A)$ of S, $\theta(A) = (\theta_*(A), \theta^*(A))$ is called rough intuitionistic fuzzy set with respect to θ if $\theta_*(A) \neq \theta^*(A)$.

3. Rough Intuitionistic Fuzzy Compact Structure Subgroup Space

Throughout this paper G denotes a group.

Definition 3.1. *Let θ be a congruence relation on G. An rough intuitionistic fuzzy set A of G is called upper rough intuitionistic fuzzy subgroup of G if $\theta^*(A)$ is an rough intuitionistic fuzzy subgroup of G.(i.e)* (i). $\theta^*(\mu_A)(xy) \leq \theta^*\mu_A(x) \wedge \theta^*\mu_A(y)$

(ii). $\theta^*(A)(x^{-1}) = \theta^*(A)(x)$

Definition 3.2. *Let θ be a congruence relation on G. A rough intuitionistic fuzzy set A of G is called lower rough intuitionistic fuzzy subgroup of G if $\theta_*(A)$ is an rough intuitionistic fuzzy subgroup of G.(i.e)*

(i). $\theta_*(\mu_A)(xy) \leq \theta_*\mu_A(x) \wedge \theta_*\mu_A(y)$

(ii). $\theta_*(A)(x^{-1}) = \theta_*(A)(x)$

Definition 3.3. *If $\theta^*(A)$ and $\theta_*(A)$ are both intuitionistic fuzzy subgroup of G then A is called a rough intuitionistic fuzzy subgroup of G.*

Example 3.4. *Let $G = \{1, \omega, \omega^2\}$ where ω is the cubic root of unity with the binary operation which is defined as below*

.	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

Let θ be a congruence relation on G such that the θ-congruence classes are the subsets $\{1\}, \{\omega, \omega^2\}$. Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in G\}$ be an intuitionistic fuzzy subset of G defined by $A = \{\langle 1, 0.5, 0.4\rangle, \langle \omega, 0.4, 0.4\rangle, \langle \omega^2, 0.5, 0.4\rangle\}$. Since for every $x \in G$, $\theta^*(\mu_A(x)) = \vee_{a\in[x]_\theta}\mu_A(a)$and $\theta^*(\nu_A(x)) = \wedge_{a\in[x]_\theta}\nu_A(a)$ so the upper approximation is $\theta^*(A) = \{\langle x, \theta^*(\mu_A(x)), \theta^*(\nu_A(x))\rangle / x \in G\}$ is given by $\theta^*(A) = \{\langle 1, 0.5, 0.4\rangle, \langle \omega, 0.5, 0.4\rangle, \langle \omega^2, 0.5, 0.4\rangle\}$ and since for every $x \in G, \theta_*(\mu_A)(x) = \wedge_{a\in[x]_\theta}\mu_A(a)$ and $\theta_*(\nu_A)(x) = \vee_{a\in[x]_\theta}\nu_A(a)$ so the lower approximation is $\theta_*(A) = \{\langle x, \theta_*(\mu_A(x)), \theta_*(\nu_A(x))\rangle / x \in G\}$ is given by $\theta_*(A) = \{\langle 1, 0.5, 0.4\rangle, \langle \omega, 0.4, 0.4\rangle, \langle \omega^2, 0.4, 0.4\rangle\}$. Then it can be easily verified that

(i). $\theta^*(\mu_A)(xy) \geq \theta^*(\mu_A)(y)$
$\theta^*(\nu_A)(xy) \leq \theta^*(\nu_A)(y)$

(ii). $\theta_*(\mu_A)(xy) \geq \theta_*(\mu_A)(y)$
$\theta_*(\nu_A)(xy) \leq \theta_*(\nu_A)(y)$

Also it can be verified that

(i). $\theta^*(\mu_A)(x^{-1}) = \theta^*(\mu_A)(x)$
$\theta^*(\nu_A)(x^{-1}) = \theta^*(\nu_A)(x)$

(ii). $\theta_*(\mu_A)(x^{-1}) = \theta_*(\mu_A)(x)$
$\theta_*(\nu_A)(x^{-1}) = \theta_*(\nu_A)(x)$

Definition 3.5. *Let $(G, \Im)$ be any intuitionistic fuzzy rough structure group space and A be a rough intuitionistic fuzzy subgroup in G. Then A is said to be a rough intuitionistic fuzzy rough compact subgroup in $(G, \Im)$ if for every family of $\{A_i | i \in J\}$ of rough intuitionistic fuzzy open subgroups in $(G, \Im)$ satisfies the condition $A \subseteq \cup_{i\in J}A_i$, there exists a finite subfamily $J_0 = \{1, ..., n\} \subseteq J$ such that $A \subseteq \cup_{i=1}^n A_i$. The complement of a rough intuitionistic fuzzy compact group in $(G, \Im)$ is a rough intuitionistic fuzzy closed compact subgroup in $(G, \Im)$.*

Definition 3.6. *Let G be a group. A family of a rough intuitionistic fuzzy subgroup in G is said to be a rough intuitionistic fuzzy structure subgroup on G if it satisfies the following axioms*

(i). $0\sim, 1\sim \in \Im$.

(ii). finite intersection of elements of $\Im$ *is in* $\Im$.

(iii). arbitrary union of elements of $\Im$ *is in* $\Im$.

Then the ordered pair $(G, \Im)$ *is called an rough intuitionistic fuzzy structure subgroup space.*
Every member of $\Im$ *is called an rough intuitionistic fuzzy open subgroup in* $(G, \Im)$ *. The complement of a rough intuitionistic fuzzy open subgroup in* $(G, \Im)$ *is a rough intuitionistic fuzzy closed subgroup in* $(G, \Im)$.

Notation 3.7. *Let* $(G, \Im)$ *be any rough intuitionistic fuzzy structure subgroup space. Then*

(i). O(SG) denotes the family of all rough intuitionistic fuzzy open subgroup in $(G, \Im)$.

(ii). C(SG)denotes the family of all rough intuitionistic fuzzy closed subgroup in $(G, \Im)$

Definition 3.8. *Let* $(G, \Im)$ *be a rough intuitionistic fuzzy structure subgroup space. Let* $A = \langle x, \mu_A, \nu_A \rangle$ *be an intuitionistic fuzzy subgroup in G. Then*

(i). the rough intuitionistic fuzzy subgroup interior of A is defined and denoted as

$$RIF_{SG}int(A) = \bigcup\{B = \langle x, \mu_B, \nu_B \rangle / B \in O(SG) and B \subseteq A\}$$

(ii). the rough intuitionistic fuzzy subgroup closure of A is defined and denoted as

$$RIF_{SG}cl(A) = \bigcap\{B = \langle x, \mu_B, \nu_B \rangle / B \in C(SG) and B \supseteq A\}$$

Remark 3.9. *Let* $(G, \Im)$ *be any rough intuitionstic fuzzy structure subgroup space. Let* $A = \langle x, \mu_A, \nu_A \rangle$ *be any rough intuitionistic fuzzy subgroup in G. Then the following statements hold:*

(i). $RIF_{SG}cl(A) = A$ *if and only if A is an rough intuitionistic fuzzy closed subgroup.*

(ii). $RIF_{SG}int(A) = A$ *if and only if A is an rough intuitionistic fuzzy open subgroup.*

(iii). $RIF_{SG}int(A) \subseteq A \subseteq RIF_{SG}cl(A)$.

(iv). $RIF_{SG}int(1\sim) = 1\sim$ *and* $RIF_{SG}int(0\sim) = 0\sim$.

(v). $RIF_{SG}cl(1\sim) = 1\sim$ *and* $RIF_{SG}cl(0\sim) = 0\sim$.

(vi). $RIF_{SG}cl(\overline{A}) = \overline{RIF_{SG}int(A)}$ *and* $RIF_{SG}int(\overline{A}) = \overline{RIF_{SG}cl(A)}$.

Definition 3.10. *Let* $(G, \Im)$ *be a rough intuitionistic fuzzy rough structure subgroup space*

(i). If a family $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$ *of rough intuitionistic fuzzy open subgroups in G satisfies the condition* $\bigcup\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\} = 1\sim$*, then it is called a rough intuitionistic fuzzy open cover of G. A finite subfamily of a rough intuitionistic fuzzy open cover of G is called a finite subcover of* $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$*. A finite subfamily of a rough fuzzy open subgroup cover* $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$ *which is also a fuzzy open subgroup cover of G is called a finite subgroup subcover of* $\{\langle x, \mu_{G_i}, \nu_{G_i} \rangle : i \in J\}$

(ii). A family $\{\langle x, \mu_{K_i}, \nu_{K_i}\rangle : i \in J\}$ of rough intuitionistic fuzzy rough closed subgroup in G satisfies the finite intersection property if and only if every finite subfamily $\{\langle x, \mu_{K_i}, \nu_{K_i}\rangle : i = 1, 2, ...n\}$ of the family satisfies the condition $\bigcup_{1=1}^{n} \{\langle x, \mu_{K_i}, \nu_{K_i}\rangle\} \neq 0\sim$

Definition 3.11. *Let $(G, \Im)$ be any rough intuitionistic fuzzy structure group space and A be an rough intuitionistic fuzzy subgroup in G. Then A is said to be a intuitionistic fuzzy rough open compact subgroup in $(G, \Im)$ if it is both rough intuitionistic fuzzy open and rough intuitionistic fuzzy compact.*
The complement of rough intuitionistic fuzzy compact subgroup in $(G, \Im)$ is a rough intuitionistic fuzzy closed compact subgroup in $(G, \Im)$.

Notation 3.12. *Let $(G, \Im)$ be any rough intuitionistic fuzzy structure subgroup space. Then*

(i). SG(OC) denotes the collection of all rough intuitionistic fuzzy open compact subgroups in $(G, \Im)$.

(ii). SG(CCmpt)denotes the collection of all rough intuitionistic fuzzy closed compact subgroups in $(G, \Im)$.

Definition 3.13. *Let $(G, \Im)$ be any rough intuitionistic fuzzy structure subgroup space. Let $A = \langle x, \mu_A, \nu_A\rangle$ be a rough intuitionistic fuzzy subgroup in G. Then*

(i). the rough intuitionistic fuzzy compact SG-interior of A is defined and denoted by

$$RIFC_{SG}int(A) = \bigcup\{B = \langle x, \mu_B, \nu_A\rangle / B \in SG(OC) and B \subseteq A\}.$$

(ii). the rough intuitionistic fuzzy compact SG-closure of A is defined and denoted by

$$RIFC_{SG}cl(A) = \bigcap\{B = \langle x, \mu_B, \nu_A\rangle / B \in SG(CCmpt) and B \supseteq A\}.$$

Theorem 3.14. *Let $(G, \Im)$ be any rough intuitionistic fuzzy structure group space. Let $A = \langle x, \mu_A, \nu_A\rangle$ be a rough intuitionistic fuzzy subgroup in G. Then the following statement holds:*

(i). $RIFC_{SG}cl(A) = A$ if and only if A is a rough intuitionistic fuzzy closed compact subgroup.

(ii). $RIFC_{SG}int(A) = A$ if and only if A is a rough intuitionistic fuzzy open compact subgroup.

(iii). $RIFC_{SG}int(A) \subseteq A \subseteq RIFC_{SG}cl(A)$.

(iv). $RIFC_{SG}cl(\overline{A}) = \overline{RIFC_{SG}int(A)}$ and $RIFC_{SG}int(\overline{A}) = \overline{RIFC_{SG}cl(A)}$.

(v). $RIFC_{SG}cl(0\sim) = 0\sim$ and $RIFC_{SG}int(1\sim) = 1\sim$.

(vi). $RIFC_{SG}cl(1\sim) = 1\sim$ and $RIFC_{SG}int(0\sim) = 0\sim$.

Definition 3.15. *Let $(G, \Im)$ be any rough intuitionistic fuzzy subgroup space. Then $(G, \Im)$ is called a rough intuitionistic fuzzy subgroup extremal compact space if the rough intuitionistic fuzzy SG-closure of every rough intuitionistic fuzzy open compact subgroup is an rough intuitionistic fuzzy open compact subgroup.*

Proposition 3.16. *Let $(G, \Im)$ be any rough intuitinistic fuzzy structure subgroup space. Then the following are equivalent:*

(i). $(G, \Im)$ is a rough intuitionistic fuzzy subgroup extremal compact space.

(ii). For each rough intuitionistic fuzzy closed compact subgroup A, $RIF_{SG}int(A)$ is a rough intuitionistic fuzzy closed compact subgroup.

(iii). For each rough intuitionistic fuzzy open compact subgroup A, we have $RIFC_{SG}cl(\overline{RIFC_{SG}cl(A)}) = \overline{RIFC_{SG}cl(A)}$.

(iv). For every pair of rough intuitionistic fuzzy compact subgroup A and B with $RIF_{SG}cl(A) = \overline{B}$, we have $RIFC_{SG}cl(B) = \overline{RIFC_{SG}cl(A)}$.

Proof. $(i) \Rightarrow (ii)$ Let A be a rough intuitionistic fuzzy closed compact subgoup in $(G, \Im)$. Then $\overline{A}$ is a rough intuitionistic fuzzy open compact subgroup in $(G, \Im)$. Then by assumption, $RIFC_{SG}cl(\overline{A})$ is a rough intuitionistic fuzzy open compact subgroup in $(G, \Im)$. Now $RIFC_{SG}cl(\overline{A}) = \overline{RIFC_{SG}int(A)}$. Therefore $RIFC_{SG}int(A)$ is a rough intuitionistic fuzzy closed compact subgroup in $(G, \Im)$.
$(ii) \Rightarrow (iii)$ Let A be a rough intuitionistic fuzzy open compact subgroup in $(G, \Im)$. Then $\overline{A}$ is a rough intuitionistic fuzzy closed compact group in $(G, \Im)$. By assumption $RIFC_{SG}int(\overline{A}) = \overline{RIF_{SG}cl(A)}$ is a rough intuitionistic fuzzy closed compact subgroup in $(G, \Im)$.
Now $RIFC_{SG}cl(\overline{RIFC_{SG}cl(A)}) = \overline{RIFC_{SG}cl(A)}$.
$(iii) \Rightarrow (iv)$ Let A and B be any rough intuitionistic fuzzy open compact subgroup in $(G, \Im)$ such that $RIFC_{SG}cl(A) = \overline{B}$. By (iii)

$$RIFC_{SG}cl(\overline{RIFC_{SG}cl(A)}) = \overline{RIFC_{SG}cl(A)}$$
$$\Rightarrow RIFC_{SG}cl(B) = \overline{RIFC_{SG}cl(A)}$$

$(iv) \Rightarrow (i)$ Let A and B be any two rough intuitionistic fuzzy compact subgroup in $(G, \Im)$ such that $\overline{RIFC_{SG}cl(A)} = B$. By (iv)it follows that $RIFC_{SG}cl(B) = \overline{RIFC_{SG}cl(A)}$. That is $\overline{RIFC_{SG}cl(A)}$ is a rough intuitionistic fuzzy closed compact subgroup in $(G, \Im)$. This implies that $RIFC_{SG}cl(A)$ is a rough intuiotnistic fuzzy open compact subgroup in $(G, \Im)$. Hence $(G, \Im)$ is a rough intuitionistic fuzzy subgroup extremal compact space. □

Proposition 3.17. *Let $(G, \Im)$ be any rough intutionistic fuzzy rough intuitinistic fuzzy subgroup space. Then $(G, \Im)$ is an intuitionistic fuzzy subgroup extremal compact space if and only if for each rough intuitionistic fuzzy open compact subgroup A and rough intuitionistic fuzzy closed compact subgroup B such that $A \subseteq B$, $RIFC_{SG}cl(A) \subseteq RIFC_{SG}int(A)$*

Proof. Let A be a rough intuitionistic fuzzy compact subgroup and B be a rough intutuionistic fuzzy closed compact subgroup in $(G, \Im)$ such that $A \subseteq B$. Then by (ii) of Proposition 3.14 $RIFC_{SG}int(B)$ is a rough intuitionistic fuzzy closed compact subgroup in $(G, \Im)$. Therefore $RIFC_{SG}cl(RIFC_{SG}(int(B))) = RIFC_{SG}int(B)$. Since A is a rough intuitionistic fuzzy open compact group and $A \subseteq B$, $A \subseteq RIFC_{SG}int(B)$. Now $RIFC_{SG}cl(A) \subseteq RIFC_{SG}cl(RIFC_{SG}int(B)) = RIFC_{SG}int(B)$.
Conversely, let B be a rough intuitionistic fuzzy closed compact subgroup in $(G, \Im)$. Then $RIFC_{SG}int(B)$ is a rough intuitionistic fuzzy open compact subgroup in $(G, \Im)$ and $RIFC_{SG}int(B) \subseteq B$. By assumption $RIFC_{SG}cl(A(RIFC_{SG}int(B))) \subseteq RIFC_{SG}int(B)$. Also $RIFC_{SG}int(B) \subseteq RIFC_{SG}cl(RIFC_{SG}int(B))$. This implies $RIFC_{SG}cl(RIFC_{SG}(int(B))) = RIFC_{SG}int(B)$. Thus $RIFC_{SG}int(B)$ is a rough intuitionistic fuzzy closed cmmpt-compact subgroup in $(G, \Im)$. By (ii) of Proposition 3.13 $(G, \Im)$ is a rough intuitionistic fuzzy subgroup extremal compact space. □

Definition 3.18. *Let $(G, \Im)$ be any rough intuitionistic fuzzy structure subgroup space. A rough intuitionistic fuzzy subgroup A in $(G, \Im)$ is said to be an RIFC co subgroup in $(G, \Im)$ if it is both rough intuitionistic fuzzy open compact and rough intuitionistic fuzzy closed compact*

Remark 3.19. *Let $(G, \Im)$ be any rough intuitionisitic fuzzy extremal compact space. Let $\{A_i, B_i/i \in N\}$ be a collection such that $A_i's$ are rough intuitionistic fuzzy open compact subgroup and $B_i's$ are rough intuitionistic fuzzy closed compact subgroup and let A and B be any two rough intutitionistic fuzzy co subgroup. If $A_i \subseteq A \subseteq B_j$ and $A_i \subseteq B \subseteq B_j$ for all $i, j \in N$ then there exists a RIFC co subgroup C such that $RIFC_{SG}cl(A_i) \subseteq C \subseteq RIFC_{SG}int(B_j)$ for all $i, j \in N$. By Proposition 3.13 $RIFC_{SG}cl(A_i) \subseteq RIFC_{SG}cl(A) \cap RIFC_{SG}int(B) \subseteq RIFC_{SG}int(B_j)$ for all $i, j \in N$. Therefore, $C = RIFC_{SG}cl(A) \cap RIFC_{SG}int(B)$ is a RIFC co subgroup in $(G, \Im)$ satisfying the required condition.*

Note $RIFC(SG)$ denotes the collection of all rough intuitionstc fuzzy subgroup in G.

Proposition 3.20. *Let $(G, \Im)$ be any rough intuitionistic fuzzy subgroup extremal compact space. Let $\{A_q\}_{q\in Q}$ and $\{B_q\}_{q\in Q}$ (Q set of all rational numbers) be monotonic increasing collection of rough intuitionistic fuzzy open compact subgroup and rough intuitionistic fuzzy closed compact subgroup of $(G, \Im)$ respectively and suppose that $A_{q1} \subseteq Bq_2$ whenever $q_1 \leq q_2$ Then there exists a monotone increasing collection $\{C_q\}_{q\in Q}$ of rough intuitionistic fuzzy co subgroup of $(G, \Im)$ such that $RIFC_{Sg}cl(A_{q1}) \subseteq C_{q2}$ and $C_{q1} \subseteq RIFC_{SG}int(B_{q2})$ whenever $q_1 < q_2$.*

Proof. Let us arrange all rational numbers into a sequence $\{q_n\}$ without repetitions. for every $n \geq 2$, we shall define inductively a collection $\{C_{q_i}/1 \leq i < n\} \subseteq RIFC(G)$ of rough intuitioistic fuzzy co subgroup such that

$$RIFC_{SG}cl(A_q) \subseteq C_{q_i} \text{if} q < q_i, C_{qi} \subseteq RIFC_{SG}int(B_q)$$

if $q_i < q$ for all $i < n$.

The countable collection $\{RIFC_{SG}cl(A_q)\}$ and $\{RIFC_{SG}int(B_q)\}$ satisfy $RIFC_{SG}cl(A_{q_1}) \subseteq RIFC_{SG}int(B_{q_2})$ if $q_1 < q_2$. By remark 3.17 there exists a rough intuitionistic fuzzy co subgroup D_1 such that $RIFC_{SG}cl(A_{q_1}) \subseteq D_1 \subseteq RIFC_{SG}int(B_{q_2})$. Letting $C_{q_1} = D$ we get S_2. Assume that the rough intuitionistic fuzzy subgroup C_{q_1} are already defined for $i < n$ and satisfy (S_n). Define $E = \cup\{C_{q_i}/i < n, q_i < q_n\} \cup A_{q_n}$. $F = \cap\{C_{q_j}/j < n, q_j > q_n\} \cap B_{q_n}$. Then we have $RIFC_{SG}cl(C_{q_i}) \subseteq RIFC_{SG}cl(E) \subseteq RIFC_{SG}int(C_{q_j})$ and $RIFC_{SG}cl(C_{q_i}) \subseteq RIFC_{SG}int(F) \subseteq RIFC_{SG}int(C_{q_j})$ whenever $q_i < q_n < q_j (i, j < n)$ as well as $A_q \subseteq RIFC_{SG}cl(E) \subseteq B_{q_i}$ and $A_q \subseteq RIFC_{SG}int(F) \subseteq B_{q_i}$ whenever $q < q_n < q_1$. This shows that the contable collections $\{C_{q_i}/i < n, q_i < q_n\} \cup \{A_q/q < q_n\}$ and $\{C_{q_j}/j < n, q_j > q_n\} \cup B_q/q > q_n\}$ together with E and F full fill the condition of remark. Hence there exists a rough intuitionistic fuzzy co subgroup D_n such that $RIFC_{SG}cl(D_n) \subseteq B_q$ if $q_n < q, A_q \subseteq RIFC_{SG}int(D_n)$ if $q < q_n$. $RIFC_{SG}cl(C_{q_i}) \subseteq RIFC_{SG}int(D_n)$ if $q_i < q_n$ and $RIFC_{SG}cl(D_n) \subseteq RIFC_{SG}int(C_{q_i})$ if $q_n < q_j$ where $1 \leq i, j \leq n-1$. Setting $C_{q_n} = D_n$ we obtain a rough intuitionistic fuzzy subgroup $C_{q_1}, C_{q_2}, \ldots, C_{q_n}$ that satisfy (S_{n+1}). Therefore the collection $\{C_{q_i}1 = 1, 2, \}$ the required property. ☐

Definition 3.21. *Let $(G, \Im)$ be any rough intuitonistic fuzzy structure subgroup space. Let $A = \langle x, \mu_A, \nu_A\rangle$ and $B = \langle x, \mu_B, \nu_B\rangle$ be any two rough intuitionistic fuzzy rough subgroup in G. Then A is a rough intuitionistic fuzzy subgroup quasi-coincident with $B(AqB)$ if there is a $x \in G$ such that $\mu_A(x) + \mu_B(x) > 1$ and $\nu_A(x) + \nu_B(x) < 1$. Otherwise A is not a rough intuitionistic fuzzy subgroup quasi-coincident with $B(A\tilde{q}B)$*

Proposition 3.22. *Let $(G, \Im)$ be any rough intuitionistic fuzzy structure subgroup space. Then $(G, \Im)$ is a rough intuitionistic fuzzy subgroup extremal compact space if and only if every rough intuitionistic fuzzy open compact subgroup $A = \langle x, \mu_A, \nu_A\rangle$ and $B = \langle x, \mu_B, \nu_B\rangle$ with $A\tilde{q}B$,*

$$RIFC_{SG}cl(A)\tilde{q}RIFC_{SG}cl(B).$$

Proof. Let $A = \langle x, \mu_A, \nu_A \rangle$ and $B = \langle x, \mu_B, \nu_B \rangle$ be any two rough intuitionistic fuzzy open compact subgroup with $A\tilde{q}B$. Since $(G, \Im)$ is a rough intuitionistic fuzzy subgroup extremal compact space $RIFC_{SG}cl(A)$and $RIFC_{SG}cl(B)$ are rough intuitionistic fuzzy open compact subgroups. Hence $RIFC_{SG}cl(A)\tilde{q}RIFC_{SG}cl(B)$.

Conversely let A be a rough intuitionistic fuzzy open compact group and $\overline{RIFC_{SG}cl(A)}$ be an intuitionistic fuzzy open compact subgroup in $(G, \Im)$ such that $A\tilde{q}\overline{RIFC_{SG}cl(A)}$. Then by hypothesis $RIFC_{SG}cl(A)\tilde{q}RIFC_{SG}cl(\overline{RIFC_{SG}cl(A)})$. That is for all $x \in G\mu_{RIFC_{SG}cl(A)}(x)+\mu_{RIFC_{SG}cl(\overline{RIFC_{SG}cl(A)})}(x) \leq 1$ and $\nu_{RIFC_{SG}cl(A)}(x)+\nu_{RIFC_{SG}cl(\overline{RIFC_{SG}cl(A)})}(x) \geq 1$

$\Rightarrow \mu_{RIFC_{SG}cl(A)}(x) \leq 1 - \mu_{RIFC_{SG}cl(\overline{RIFC_{SG}cl(A)})}(x)$

$= 1 - \mu_{RIFC_{SG}cl(RIFC_{SG}int(\overline{A}))}(x)$

$= \mu_{RIFC_{SG}int(\overline{RIFC_{SG}int(\overline{A})})}(x) = \mu_{RIFC_{SG}int(RIFC_{SG}cl(A))}(x)$ and $\nu_{RIFC_{SG}cl(A)}(x) \geq 1 - \nu_{RIFC_{SG}cl(\overline{RIFC_{SG}cl(A)})}(x) = \nu_{RIFC_{SG}int(RIFC_{SG}cl(A))}(x)$. Hence $RIFC_{SG}cl(A) \subseteq RIFC_{SG}int(RIFC_{Sg}cl(A))$ for all $x \in G$ but $RIFC_{SG}cl(A) \supseteq RIFC_{SG}int(RIFC_{SG}cl(A)) \Rightarrow RIFC_{SG}cl(A) = RIFC_{SG}int(RIFC_{SG}cl(A))$ is an open compact sub rough in $(G, \Im)$. Therefore $(G, \Im)$ is a rough intuitionistic fuzzy subgroup extremal compact space. □

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