Compactness on Rough Intuitionistic Fuzzy Structure Subgroup Space

H.Jude Immaculate\(^1\) and I.Arockiarani\(^1\)

1 Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu, India.

Abstract: This paper aims to initiate the notion of rough intuitionistic fuzzy compact subgroup, rough intuitionistic fuzzy extremal compact spaces in the light of rough intuitionistic fuzzy sets. We characterize rough intuitionistic structure subspace by applying various topological notions. Few of its properties are discussed.

Keywords: Rough intuitionistic fuzzy compact subgroup, Rough intuitionistic fuzzy extremal compact spaces, Intuitionistic fuzzy sets.

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1. Introduction


2. Preliminaries

Definition 2.1 ([1]). An intuitionistic fuzzy set (IFS in short) A in X is an object having the form A = \{⟨x, \mu_A(x), \nu_A(x)/x \in X⟩\} where the function \(\mu : X \to [0,1]\) and \(\nu : X \to [0,1]\) denote the degree of membership (namely \(\mu_A(x)\)) and the degree of non membership (namely \(\nu_A(x)\)) of each element \(x \in X\) to the set \(A\), respectively and \(0 \leq \mu_A(x) + \nu_A(x) \leq 1\) for each \(x \in X\). Denote by \(IFS(X)\) the set of all intuitionistic fuzzy set in \(X\).
Definition 2.2 ([1]). Let $A$ and $B$ be IFS’s of the form $A = \{(x, \mu_A(x), \nu_A(x)/x \in X)\}$ and $B = \{(x, \mu_B(x), \nu_B(x)/x \in X)\}$. Then

1. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$.

2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

3. $\overline{A} = \{(x, \mu_A(x), \nu_A(x)/x \in X)\}$ (Complement of $A$)

4. $A \cap B = \{(x, \mu_A(x) \land \mu_B(x), \nu_A(x) \lor \nu_B(x)/x \in X)\}$.

5. $A \cup B = \{(x, \mu_A(x) \lor \mu_B(x), \nu_A(x) \land \nu_B(x)/x \in X)\}$.

For the sake of simplicity we use the notion $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{(x, \mu_A(x), \nu_A(x)/x \in X)\}$. The intuitionistic fuzzy set $0 \sim = \{(x, 0 \sim, 1 \sim)/x \in X\}$ and $1 \sim = \{(x, 1 \sim, 0 \sim)/x \in X\}$ are respectively the empty set and the whole set of $X$.

Note: For any IFS $A$ in $(X, \tau)$, we have $\text{cl}(\overline{A}) = \overline{\text{int}(A)}$ and $\text{int}(\overline{A}) = \overline{\text{cl}(A)}$.

Definition 2.3 ([2]). Let $A = \langle \mu_A, \nu_A \rangle$ be an IFS in $S$ and let $\alpha, \beta \in [0, 1]$ be such that $\alpha + \beta \leq 1$. Then the set

$$A_{\alpha, \beta} = \{x \in S/\mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$$

is called a $(\alpha, \beta)$-level subset of $A$. The set of all $(\alpha, \beta) \in \text{Im}(\mu_A) \times \text{Im}(\nu_A)$ such that $\alpha + \beta \leq 1$ is called the image of $A$, denoted by $\text{Im}(A)$.

Definition 2.4 ([2]). Let $\theta$ be a congruence relation on $G$ that is $\theta$ is an equivalence relation on $G$ such that

$$(a, b) \in \theta \Rightarrow (ax, bx) \in \theta \quad \text{and} \quad (xa, xb) \in \theta \quad \forall x \in S.$$ 

For a congruence relation $\theta$ on $S$, we have $[a]_\theta [b]_\theta \subseteq [ab]_\theta$ for all $a, b \in S$, where $[a]_\theta$ denotes $\theta$-congruence class containing the element $a \in S$. A congruence relation $\theta$ on $S$ is called complete if $[a]_\theta [b]_\theta = [ab]_\theta$ for all $a, b \in S$. Let us consider $\theta$ to be a congruence relation of $S$. If $X$ is a nonempty subset of $S$ then the sets $\theta_* (X) = x \in S|x|_\theta \subseteq X$ and $\theta^*(X) = x \in S|x|_\theta \cap X \neq \phi$ are respectively called the $\theta$-lower and $\theta$-upper approximation set of the set $X$ and $\theta(X) = (\theta^*(X), \theta^*(X))$ is called rough set with respect to $\theta$ if $\theta_*(X) \neq \theta^*(X)$. If $A = \langle \mu_A, \nu_A \rangle$ be IFS of $S$. Then the IFS $\theta(A) = (\theta(\mu_A), \theta(\nu_A))$ and $\theta^*(A) = (\theta^*(\mu_A), \theta^*(\nu_A))$ are respectively called $\theta$-lower and $\theta$-upper approximation of the IFS $A = \langle \mu_A, \nu_A \rangle$ where for all $x \in S$

$$\theta_*(\mu_A)(x) = \wedge_{a \in [x]_\theta} \mu_A(a), \theta_*(\nu_A)(x) = \vee_{a \in [x]_\theta} \nu_A(a)$$

$$\theta^*(\mu_A)(x) = \vee_{a \in [x]_\theta} \mu_A(a), \theta^*(\nu_A)(x) = \wedge_{a \in [x]_\theta} \nu_A(a)$$

For an IFS $A = \langle \mu_A, \nu_A \rangle$ of $S$, $\theta(A) = (\theta_*(A), \theta^*(A))$ is called rough intuitionistic fuzzy set with respect to $\theta$ if $\theta_*(A) \neq \theta^*(A)$.

3. Rough Intuitionistic Fuzzy Compact Structure Subgroup Space

Throughout this paper $G$ denotes a group.

Definition 3.1. Let $\theta$ be a congruence relation on $G$. An rough intuitionistic fuzzy set $A$ of $G$ is called upper rough intuitionistic fuzzy subgroup of $G$ if $\theta^*(A)$ is an rough intuitionistic fuzzy subgroup of $G$. (i.e.)
(i). $\theta^*(\mu_A)(xy) \leq \theta^*(\mu_A)(x) \land \theta^*(\mu_A)(y)$

(ii). $\theta^*(A)(x^{-1}) = \theta^*(A)(x)$

**Definition 3.2.** Let $\theta$ be a congruence relation on $G$. A rough intuitionistic fuzzy set $A$ of $G$ is called lower rough intuitionistic fuzzy subgroup of $G$ if $\theta_*(A)$ is a rough intuitionistic fuzzy subgroup of $G$ (i.e.)

(i). $\theta_*(\mu_A)(xy) \leq \theta_*(\mu_A)(x) \land \theta_*(\mu_A)(y)$

(ii). $\theta_*(A)(x^{-1}) = \theta_*(A)(x)$

**Definition 3.3.** If $\theta^*(A)$ and $\theta_*(A)$ are both intuitionistic fuzzy subgroup of $G$ then $A$ is called a rough intuitionistic fuzzy subgroup of $G$.

**Example 3.4.** Let $G = \{1, \omega, \omega^2\}$ where $\omega$ is the cubic root of unity with the binary operation which is defined as below

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Let $\theta$ be a congruence relation on $G$ such that the $\theta$-congruence classes are the subsets $\{1\}, \{\omega, \omega^2\}$. Let $A = \{x, \mu_A(x), \nu_A(x)\}/x \in G$ be an intuitionistic fuzzy subset of $G$ defined by $A = \{(1,0.5,0.4), (\omega,0.4,0.4), (\omega^2,0.5,0.4)\}$. Since for every $x \in G$, $\theta^*(\mu_A(x)) = \land_{a \in [\omega]} \mu_A(a)$ and $\theta^*(\nu_A(x)) = \land_{a \in [\omega]} \nu_A(a)$ so the upper approximation is $\theta^*(A) = \{x, \theta^*(\mu_A(x)), \theta^*(\nu_A(x))\}/x \in G$ is given by $\theta^*(A) = \{(1,0.5,0.4), (\omega,0.5,0.4), (\omega^2,0.5,0.4)\}$ and since for every $x \in G$, $\theta_*(\mu_A(x)) = \land_{a \in [\omega]} \mu_A(a)$ and $\theta_*(\nu_A(x)) = \land_{a \in [\omega]} \nu_A(a)$ so the lower approximation is $\theta_*(A) = \{x, \theta_*(\mu_A(x)), \theta_*(\nu_A(x))\}/x \in G$ is given by $\theta_*(A) = \{(1,0.5,0.4), (\omega,0.4,0.4), (\omega^2,0.4,0.4)\}$. Then it can be easily verified that

(i). $\theta^*(\mu_A)(xy) \geq \theta^*(\mu_A)(y)$

(ii). $\theta^*(\mu_A)(xy) \leq \theta^*(\mu_A)(y)$

Also it can be verified that

(i). $\theta^*(\mu_A)(x^{-1}) = \theta^*(\mu_A)(x)$

(ii). $\theta^*(\mu_A)(x^{-1}) = \theta^*(\mu_A)(x)$

**Definition 3.5.** Let $(G, \Sigma)$ be any intuitionistic fuzzy rough structure group space and $A$ be a rough intuitionistic fuzzy subgroup in $G$. Then $A$ is said to be a rough intuitionistic fuzzy rough compact subgroup in $(G, \Sigma)$ if for every family of $\{A_i\}_{i \in J}$ of rough intuitionistic fuzzy open subgroups in $(G, \Sigma)$ satisfies the condition $A \subseteq \cup_{i \in J} A_i$, there exists a finite subfamily $J_0 = \{1, \ldots, n\} \subseteq J$ such that $A \subseteq \cup_{i = 1}^n A_i$. The complement of a rough intuitionistic fuzzy compact group in $(G, \Sigma)$ is a rough intuitionistic fuzzy closed compact subgroup in $(G, \Sigma)$.
Definition 3.6. Let G be a group. A family of a rough intuitionistic fuzzy subgroup in G is said to be a rough intuitionistic fuzzy structure subgroup on G if it satisfies the following axioms

(i). \(0 \sim, 1 \sim \in \mathcal{I}\).

(ii). Finite intersection of elements of \(\mathcal{I}\) is in \(\mathcal{I}\).

(iii). Arbitrary union of elements of \(\mathcal{I}\) is in \(\mathcal{I}\).

Then the ordered pair \((G, \mathcal{I})\) is called a rough intuitionistic fuzzy structure subgroup space.

Every member of \(\mathcal{I}\) is called a rough intuitionistic fuzzy open subgroup in \((G, \mathcal{I})\). The complement of a rough intuitionistic fuzzy open subgroup in \((G, \mathcal{I})\) is a rough intuitionistic fuzzy closed subgroup in \((G, \mathcal{I})\).

Notation 3.7. Let \((G, \mathcal{I})\) be any rough intuitionistic fuzzy structure subgroup space. Then

(i). \(O(SG)\) denotes the family of all rough intuitionistic fuzzy open subgroup in \((G, \mathcal{I})\).

(ii). \(C(SG)\) denotes the family of all rough intuitionistic fuzzy closed subgroup in \((G, \mathcal{I})\).

Definition 3.8. Let \((G, \mathcal{I})\) be a rough intuitionistic fuzzy structure subgroup space. Let \(A = \langle x, \mu_A, \nu_A \rangle\) be an intuitionistic fuzzy subgroup in G. Then

(i). The rough intuitionistic fuzzy subgroup interior of \(A\) is defined and denoted as

\[
RIF_{SG} int(A) = \bigcup \{B = \langle x, \mu_B, \nu_B \rangle / B \in O(SG) \text{ and } B \subseteq A\}
\]

(ii). The rough intuitionistic fuzzy subgroup closure of \(A\) is defined and denoted as

\[
RIF_{SG} cl(A) = \bigcap \{B = \langle x, \mu_B, \nu_B \rangle / B \in C(SG) \text{ and } B \supseteq A\}
\]

Remark 3.9. Let \((G, \mathcal{I})\) be any rough intuitionistic fuzzy structure subgroup space. Let \(A = \langle x, \mu_A, \nu_A \rangle\) be any rough intuitionistic fuzzy subgroup in G. Then the following statements hold:

(i). \(RIF_{SG} cl(A) = A\) if and only if \(A\) is an rough intuitionistic fuzzy closed subgroup.

(ii). \(RIF_{SG} int(A) = A\) if and only if \(A\) is an rough intuitionistic fuzzy open subgroup.

(iii). \(RIF_{SG} int(A) \subseteq A \subseteq RIF_{SG} cl(A)\).

(iv). \(RIF_{SG} int(1 \sim) = 1 \sim\) and \(RIF_{SG} int(0 \sim) = 0 \sim\).

(v). \(RIF_{SG} cl(1 \sim) = 1 \sim\) and \(RIF_{SG} cl(0 \sim) = 0 \sim\).

(vi). \(RIF_{SG} cl(\bar{A}) = \overline{RIF_{SG} int(A)}\) and \(RIF_{SG} int(\bar{A}) = \overline{RIF_{SG} cl(A)}\).

Definition 3.10. Let \((G, \mathcal{I})\) be a rough intuitionistic fuzzy rough structure subgroup space

(i). If a family \(\{(x, \mu_{G_i}, \nu_{G_i}) : i \in J\}\) of rough intuitionistic fuzzy open subgroups in G satisfies the condition

\[
\bigcup \{(x, \mu_{G_i}, \nu_{G_i}) : i \in J\} = 1 \sim,
\]

then it is called a rough intuitionistic fuzzy open cover of G. A finite subfamily of a rough intuitionistic fuzzy open cover of G is called a finite subcover of \(\{(x, \mu_{G_i}, \nu_{G_i}) : i \in J\}\). A finite subfamily of a rough fuzzy open subgroup cover \(\{(x, \mu_{G_i}, \nu_{G_i}) : i \in J\}\) which is also a fuzzy open subgroup cover of G is called a finite subgroup subcover of \(\{(x, \mu_{G_i}, \nu_{G_i}) : i \in J\}\).
(ii). A family \( \{ \langle x_i, \mu_{K_i}, \nu_{K_i} \rangle : i \in J \} \) of rough intuitionistic fuzzy rough closed subgroup in \( G \) satisfies the finite intersection property if and only if every finite subfamily \( \{ \langle x_i, \mu_{K_i}, \nu_{K_i} \rangle : i = 1, 2, \ldots, n \} \) of the family satisfies the condition \( \bigcup_{i=1}^{n} \{ \langle x_i, \mu_{K_i}, \nu_{K_i} \rangle \} \neq 0 \sim \)

**Definition 3.11.** Let \((G, \Im)\) be any rough intuitionistic fuzzy structure group space and \( A \) be a rough intuitionistic fuzzy subgroup in \( G \). Then \( A \) is said to be a intuitionistic fuzzy rough open compact subgroup in \((G, \Im)\) if it is both rough intuitionistic fuzzy open and rough intuitionistic fuzzy compact.

The complement of rough intuitionistic fuzzy compact subgroup in \((G, \Im)\) is a rough intuitionistic fuzzy closed compact subgroup in \((G, \Im)\).

**Notation 3.12.** Let \((G, \Im)\) be any rough intuitionistic fuzzy structure subgroup space. Then

(i). \( \text{SG}(OC) \) denotes the collection of all rough intuitionistic fuzzy open compact subgroups in \((G, \Im)\).

(ii). \( \text{SG}(CCmpt) \) denotes the collection of all rough intuitionistic fuzzy closed compact subgroups in \((G, \Im)\).

**Definition 3.13.** Let \((G, \Im)\) be any rough intuitionistic fuzzy structure subgroup space. Let \( A = \langle x, \mu_A, \nu_A \rangle \) be a rough intuitionistic fuzzy subgroup in \( G \). Then

(i). the rough intuitionistic fuzzy compact SG-interior of \( A \) is defined and denoted by

\[
\text{RIFC}_{SG} \text{int}(A) = \bigcup \{ B = \langle x_i, \mu_{B_i}, \nu_{A_i} \rangle / B \in \text{SG}(OC) \text{ and } B \subseteq A \}.
\]

(ii). the rough intuitionistic fuzzy compact SG-closure of \( A \) is defined and denoted by

\[
\text{RIFC}_{SG} \text{cl}(A) = \bigcap \{ B = \langle x_i, \mu_{B_i}, \nu_{A_i} \rangle / B \in \text{SG}(CCmpt) \text{ and } B \supseteq A \}.
\]

**Theorem 3.14.** Let \((G, \Im)\) be any rough intuitionistic fuzzy structure group space. Let \( A = \langle x, \mu_A, \nu_A \rangle \) be a rough intuitionistic fuzzy subgroup in \( G \). Then the following statement holds:

(i). \( \text{RIFC}_{SG} \text{cl}(A) = A \) if and only if \( A \) is a rough intuitionistic fuzzy closed compact subgroup.

(ii). \( \text{RIFC}_{SG} \text{int}(A) = A \) if and only if \( A \) is a rough intuitionistic fuzzy open compact subgroup.

(iii). \( \text{RIFC}_{SG} \text{int}(A) \subseteq A \subseteq \text{RIFC}_{SG} \text{cl}(A) \).

(iv). \( \text{RIFC}_{SG} \text{cl}(A) = \text{RIFC}_{SG} \text{int}(A) \) and \( \text{RIFC}_{SG} \text{int}(A) = \text{RIFC}_{SG} \text{cl}(A) \).

(v). \( \text{RIFC}_{SG} \text{cl}(0 \sim) = 0 \sim \) and \( \text{RIFC}_{SG} \text{int}(1 \sim) = 1 \sim \).

(vi). \( \text{RIFC}_{SG} \text{cl}(1 \sim) = 1 \sim \) and \( \text{RIFC}_{SG} \text{int}(0 \sim) = 0 \sim \).

**Definition 3.15.** Let \((G, \Im)\) be any rough intuitionistic fuzzy subgroup space. Then \((G, \Im)\) is called a rough intuitionistic fuzzy subgroup extremal compact space if the rough intuitionistic fuzzy SG-closure of every rough intuitionistic fuzzy open compact subgroup is an rough intuitionistic fuzzy open compact subgroup.

**Proposition 3.16.** Let \((G, \Im)\) be any rough intuitionistic fuzzy structure subgroup space. Then the following are equivalent:

(i). \((G, \Im)\) is a rough intuitionistic fuzzy subgroup extremal compact space.
(ii). For each rough intuitionistic fuzzy closed compact subgroup $A$, $RIF_{SG\text{int}}(A)$ is a rough intuitionistic fuzzy closed compact subgroup.

(iii). For each rough intuitionistic fuzzy open compact subgroup $A$, we have $RIFC_{SG\text{cl}}(RIFC_{SG\text{cl}}(A)) = RIFC_{SG\text{cl}}(A)$.

(iv). For every pair of rough intuitionistic fuzzy compact subgroup $A$ and $B$ with $RIFC_{SG\text{cl}}(A) = \overline{B}$, we have $RIFC_{SG\text{cl}}(B) = RIFC_{SG\text{cl}}(A)$.

**Proof.**  
(i) $\Rightarrow$ (ii) Let $A$ be a rough intuitionistic fuzzy closed compact subgroup in $(G, \mathcal{I})$. Then $\overline{A}$ is a rough intuitionistic fuzzy open compact subgroup in $(G, \mathcal{I})$. By assumption, $RIFC_{SG\text{cl}}(\overline{A})$ is a rough intuitionistic fuzzy open compact subgroup in $(G, \mathcal{I})$. Now $RIFC_{SG\text{cl}}(\overline{A}) = RIFC_{SG\text{int}}(A)$. Therefore $RIFC_{SG\text{int}}(A)$ is a rough intuitionistic fuzzy closed compact subgroup in $(G, \mathcal{I})$.

(ii) $\Rightarrow$ (iii) Let $A$ be a rough intuitionistic fuzzy open compact subgroup in $(G, \mathcal{I})$. Then $\overline{A}$ is a rough intuitionistic fuzzy closed compact subgroup in $(G, \mathcal{I})$. By assumption $RIFC_{SG\text{int}}(\overline{A}) = RIFC_{SG\text{cl}}(A)$ is a rough intuitionistic fuzzy closed compact subgroup in $(G, \mathcal{I})$.

Now $RIFC_{SG\text{cl}}(RIFC_{SG\text{cl}}(A)) = RIFC_{SG\text{cl}}(A).

(iii) $\Rightarrow$ (iv) Let $A$ and $B$ be any rough intuitionistic fuzzy open compact subgroup in $(G, \mathcal{I})$ such that $RIFC_{SG\text{cl}}(A) = \overline{B}$. By (iii)

\[ RIFC_{SG\text{cl}}(RIFC_{SG\text{cl}}(A)) = RIFC_{SG\text{cl}}(A) \]

$\Rightarrow$ $RIFC_{SG\text{cl}}(B) = RIFC_{SG\text{cl}}(A)$

(iv) $\Rightarrow$ (i) Let $A$ and $B$ be any two rough intuitionistic fuzzy compact subgroup in $(G, \mathcal{I})$ such that $RIFC_{SG\text{cl}}(A) = B$. By (iv) it follows that $RIFC_{SG\text{cl}}(B) = RIFC_{SG\text{cl}}(A)$. That is $RIFC_{SG\text{cl}}(A)$ is a rough intuitionistic fuzzy closed compact subgroup in $(G, \mathcal{I})$. This implies that $RIFC_{SG\text{cl}}(A)$ is a rough intuitionistic fuzzy open compact subgroup in $(G, \mathcal{I})$. Hence $(G, \mathcal{I})$ is a rough intuitionistic fuzzy subgroup extremal compact space.

**Proposition 3.17.** Let $(G, \mathcal{I})$ be any rough intuitionistic fuzzy rough intuitionistic fuzzy subgroup space. Then $(G, \mathcal{I})$ is an intuitionistic fuzzy subgroup extremal compact space if and only if for each rough intuitionistic fuzzy open compact subgroup $A$ and rough intuitionistic fuzzy closed compact subgroup $B$ such that $A \subseteq B$, $RIFC_{SG\text{cl}}(A) \subseteq RIFC_{SG\text{int}}(A)$

**Proof.** Let $A$ be a rough intuitionistic fuzzy compact subgroup and $B$ be a rough intuitionistic fuzzy closed compact subgroup in $(G, \mathcal{I})$ such that $A \subseteq B$. Then by (ii) of Proposition 3.14 $RIFC_{SG\text{int}}(B)$ is a rough intuitionistic fuzzy closed compact subgroup in $(G, \mathcal{I})$. Therefore $RIFC_{SG\text{cl}}(RIFC_{SG\text{cl}}(B))) = RIFC_{SG\text{int}}(B)$. Since $A$ is a rough intuitionistic fuzzy open compact group and $A \subseteq B$, $A \subseteq RIFC_{SG\text{int}}(B)$. Now $RIFC_{SG\text{cl}}(A) \subseteq RIFC_{SG\text{cl}}(RIFC_{SG\text{int}}(B)) = RIFC_{SG\text{int}}(B)$.

Conversely, let $B$ be a rough intuitionistic fuzzy closed compact subgroup in $(G, \mathcal{I})$. Then $RIFC_{SG\text{int}}(B)$ is a rough intuitionistic fuzzy open compact subgroup in $(G, \mathcal{I})$ and $RIFC_{SG\text{cl}}(B) \subseteq B$. By assumption $RIFC_{SG\text{cl}}(A(RIFC_{SG\text{cl}}(B))) \subseteq RIFC_{SG\text{int}}(B)$. Also $RIFC_{SG\text{cl}}(B) \subseteq RIFC_{SG\text{cl}}(RIFC_{SG\text{cl}}(B))$. This implies $RIFC_{SG\text{cl}}(RIFC_{SG\text{cl}}(B))) = RIFC_{SG\text{cl}}(B)$. Thus $RIFC_{SG\text{cl}}(B)$ is a rough intuitionistic fuzzy closed compact-compact subgroup in $(G, \mathcal{I})$. By (ii) of Proposition 3.13 $(G, \mathcal{I})$ is a rough intuitionistic fuzzy subgroup extremal compact space.

**Definition 3.18.** Let $(G, \mathcal{I})$ be any rough intuitionistic fuzzy structure subgroup space. A rough intuitionistic fuzzy subgroup $A$ in $(G, \mathcal{I})$ is said to be an RIFC co subgroup in $(G, \mathcal{I})$ if it is both rough intuitionistic fuzzy open compact and rough intuitionistic fuzzy closed compact.
Remark 3.19. Let \((G, \mathfrak{S})\) be any rough intuitionistic fuzzy extremal compact space. Let \(\{A_i, B_i/i \in N\}\) be a collection such that \(A_i\)'s are rough intuitionistic fuzzy open compact subgroup and \(B_i\)'s are rough intuitionistic fuzzy closed compact subgroup and let \(A\) and \(B\) be any two rough intuitionistic fuzzy co subgroup. If \(A_i \subseteq A \subseteq B_j\) and \(A_i \subseteq B_i \subseteq B_j\) for all \(i, j \in N\) then there exists a RIFC co subgroup \(C\) such that \(RIFC_{SGcl}(A_i) \subseteq C \subseteq RIFC_{SGint}(B_j)\) for all \(i, j \in N\). By Proposition 3.13 \(RIFC_{SGcl}(A_i) \subseteq RIFC_{SGcl}(A) \cap RIFC_{SGint}(B) \subseteq RIFC_{SGint}(B_j)\) for all \(i, j \in N\). Therefore, \(C = RIFC_{SGcl}(A) \cap RIFC_{SGint}(B)\) is a RIFC co subgroup in \((G, \mathfrak{S})\) satisfying the required condition.

Note RIFC(SG) denotes the collection of all rough intuitionistic fuzzy subgroup in \(G\).

Proposition 3.20. Let \((G, \mathfrak{S})\) be any rough intuitionistic fuzzy subgroup extremal compact space. Let \(\{A_i\}_{i \in \mathbb{Q}}\) and \(\{B_i\}_{i \in \mathbb{Q}}\) (\(\mathbb{Q}\) set of all rational numbers) be monotonic increasing collection of rough intuitionistic fuzzy open compact subgroup and rough intuitionistic fuzzy closed compact subgroup of \((G, \mathfrak{S})\) respectively and suppose that \(A_{q_1} \subseteq B_{q_2}\) whenever \(q_1 \leq q_2\). Then there exists a monotonic increasing collection \(\{C_i\}_{i \in \mathbb{Q}}\) of rough intuitionistic fuzzy co subgroup of \((G, \mathfrak{S})\) such that \(RIFC_{SGcl}(A_{q_1}) \subseteq C_{q_2}\) and \(C_{q_1} \subseteq RIFC_{SGint}(B_{q_2})\) whenever \(q_1 < q_2\).

Proof. Let us arrange all rational numbers into a sequence \(\{q_n\}\) without repetitions. For every \(n \geq 2\), we shall define inductively a collection \(\{C_{q_i}/1 \leq i < n\}\) \(\subseteq RIFC(G)\) of rough intuitionistic fuzzy co subgroup such that

\[
RIFC_{SGcl}(A_{q_i}) \subseteq C_{q_i}/q_i < q_i, C_{q_i} \subseteq RIFC_{SGint}(B_{q_i})
\]

if \(q_i < q\) for all \(i < n\).

The countable collection \(\{RIFC_{SGcl}(A_{q_i})\}\) and \(\{RIFC_{SGint}(B_{q_i})\}\) satisfy \(RIFC_{SGcl}(A_{q_1}) \subseteq RIFC_{SGint}(B_{q_2})\) if \(q_1 < q_2\). By remark 3.17 there exists a rough intuitionistic fuzzy co subgroup \(D_1\) such that \(RIFC_{SGcl}(A_{q_1}) \subseteq D_1 \subseteq RIFC_{SGint}(B_{q_2})\). Letting \(C_{q_1} = D_1\) we get \(S_2\). Assume that the rough intuitionistic fuzzy subgroup \(C_{q_i}\) are already defined for \(i < n\) and satisfy \((S_n)\). Define \(E = \cup\{C_i/i < n, q_i < q_n\} \cup A_{q_n}\). \(F = \cap\{C_i/j < n, q_i > q_n\} \cap B_{q_n}\). Then we have \(RIFC_{SGcl}(C_{q_i}) \subseteq RIFC_{SGcl}(E) \subseteq RIFC_{SGint}(C_{q_i})\) and \(RIFC_{SGcl}(E) \subseteq RIFC_{SGcl}(F) \subseteq RIFC_{SGint}(C_{q_i})\) whenever \(q_i < q_n < q(i, j < n)\) as well as \(A_i \subseteq RIFC_{SGcl}(E) \subseteq B_q\) and \(A_i \subseteq RIFC_{SGcl}(F) \subseteq B_q\), whenever \(q < q_n < q_i\). This shows that the countable collections \(\{C_i/i < n, q < q_n\}\cup\{A_i/q < q_n\}\) and \(\{C_i/j < n, q_i > q_n\}\cup B_i/q > q_n\) together with \(E\) and \(F\) full fill the condition of remark. Hence there exists a rough intuitionistic fuzzy co subgroup \(D_n\) such that \(RIFC_{SGcl}(D_n) \subseteq B_q\) if \(q_n < q, A_q \subseteq RIFC_{SGint}(D_n)\) if \(q < q_n\). \(RIFC_{SGcl}(C_{q_i}) \subseteq RIFC_{SGint}(D_n)\) if \(q_i < q_n\) and \(RIFC_{SGcl}(D_n) \subseteq RIFC_{SGint}(C_{q_i})\) if \(q_n < q_j\) where \(1 \leq i, j \leq n - 1\). Setting \(C_{q_n} = D_n\) we obtain a rough intuitionistic fuzzy subgroup \(C_{q_1}, C_{q_2}, \ldots, C_{q_n}\) that satisfy \((S_{n+1})\). Therefore the collection \(\{C_{q_i}/1 = 1, 2, \ldots, n\}\) the required property. \(\square\)

Definition 3.21. Let \((G, \mathfrak{S})\) be any rough intuitionistic fuzzy structure subgroup space. Let \(A = (x, \mu_A, \nu_A)\) and \(B = (x, \mu_B, \nu_B)\) be any two rough intuitionistic fuzzy rough subgroup in \(G\). Then \(A\) is a rough intuitionistic fuzzy subgroup quasi-coincident with \(B(AqB)\) if there is a \(x \in G\) such that \(\mu_A(x) + \mu_B(x) > 1\) and \(\nu_A(x) + \nu_B(x) < 1\). Otherwise \(A\) is not a rough intuitionistic fuzzy subgroup quasi-coincident with \(B(AqB)\).

Proposition 3.22. Let \((G, \mathfrak{S})\) be any rough intuitionistic fuzzy structure subgroup space. Then \((G, \mathfrak{S})\) is a rough intuitionistic fuzzy subgroup extremal compact space if and only if every rough intuitionistic fuzzy open compact subgroup \(A = (x, \mu_A, \nu_A)\) and \(B = (x, \mu_B, \nu_B)\) with \(AqB\),

\[
RIFC_{SGcl}(A) \cap RIFC_{SGcl}(B).
\]
Proof. Let $A = \langle x, \mu_A, \nu_A \rangle$ and $B = \langle x, \mu_B, \nu_B \rangle$ be any two rough intuitionistic fuzzy open compact subgroups with $A \sqcup B$. Since $(G, \mathfrak{I})$ is a rough intuitionistic fuzzy subgroup extremal compact space $RIFC_{SGcl}(A)$ and $RIFC_{SGcl}(B)$ are rough intuitionistic fuzzy open compact subgroups. Hence $RIFC_{SGcl}(A) \sqcup RIFC_{SGcl}(B)$. Conversely let $A$ be a rough intuitionistic fuzzy open compact group and $RIFC_{SGcl}(A)$ be an intuitionistic fuzzy open compact subgroup in $(G, \mathfrak{I})$ such that $A \sqcup RIFC_{SGcl}(A)$. Then by hypothesis $RIFC_{SGcl}(A) \sqcup RIFC_{SGcl}(RIFC_{SGcl}(A))$. That is for all $x \in G \mu_{RIFC_{SGcl}(A)}(x) + \mu_{RIFC_{SGcl}(RIFC_{SGcl}(A))}(x) \leq 1$ and $\nu_{RIFC_{SGcl}(A)}(x) + \nu_{RIFC_{SGcl}(RIFC_{SGcl}(A))}(x) \geq 1$ \[ \Rightarrow \mu_{RIFC_{SGcl}(A)}(x) \leq 1 - \nu_{RIFC_{SGcl}(RIFC_{SGcl}(A))}(x) \]
\[= 1 - \mu_{RIFC_{SGcl}(RIFC_{SGcl}(A))}(x) = \mu_{RIFC_{SGcl}(RIFC_{SGcl}(A))}(x) \text{ and } \nu_{RIFC_{SGcl}(A)}(x) \geq 1 - \nu_{RIFC_{SGcl}(RIFC_{SGcl}(A))}(x) = \nu_{RIFC_{SGcl}(RIFC_{SGcl}(A))}(x). \]
Hence $RIFC_{SGcl}(A) \subseteq RIFC_{SGcl}(RIFC_{SGcl}(A))$ for all $x \in G$ but $RIFC_{SGcl}(A) \supseteq RIFC_{SGcl}(RIFC_{SGcl}(A)) \Rightarrow RIFC_{SGcl}(A) = RIFC_{SGcl}(RIFC_{SGcl}(A))$ is an open compact sub rough in $(G, \mathfrak{I})$. Therefore $(G, \mathfrak{I})$ is a rough intuitionistic fuzzy subgroup extremal compact space.

References