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Alpha Cuts of Fuzzy Basis

Research Article

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Abstract: We introduce fuzzy basis, strong fuzzy basis and alpha cuts of fuzzy basis and strong fuzzy basis.

Keywords: Fuzzy basis, Strong fuzzy basis, Alpha cuts.

1. Introduction

In metric space and topological space the concept of base(basis) plays an important role. In the year 1965 Lotfi A.Zadeh [2] introduced the concept of fuzzy sets and fuzzy logic. In the year 1968 C.L.Chang [1] introduced Fuzzy topological spaces. In the year 2014 we [3] introduced fuzzy sequences in a metric space. In the same year we [6] introduced fuzzy subsequences and limit points. In the same year we [4], [5] defined fuzzy net and fuzzy filter.

In this paper we introduce fuzzy basis, strong fuzzy basis and alpha cuts.

2. Fuzzy Basis

Now we recall the definition base for a topology in a topological space. Let X be a non empty set. $\mathbf{B} \subset P(X)$ is called a crisp basis if (i) $\cup\{B/B \in \mathbf{B}\} = X$ (ii) $U, V \in \mathbf{B}$ and $x \in U \cap V$ implies there exists $W \in \mathbf{B}$ such that $x \in W \subset U \cap V$. Now we fuzzify this concept.

Definition 2.1 (Fuzzy Basis)**.** *Let X be a non empty set. A function $f : P(X) \to [0,1]$ is called a fuzzy basis if*

(1). $\cup\{B/f(B) = 1\} = X$.

(2). For each $\alpha \in (0,1]$. $f(U) \geq \alpha$, $f(V) \geq \alpha$ and $x \in U \cap V$ implies there exists W with $f(W) \geq \alpha$ and $x \in W \subset U \cap V$.

Definition 2.2 (Strong fuzzy basis)**.** *Let X be a non empty set. A function $f : P(X) \to [0,1]$ is called a strong fuzzy basis if*

(1). $\cup\{B/f(B) = 1\} = X$.

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(2). $f(U \cap V) \geq \min\{f(U), f(V)\}$ *for* $U, V \subset X$ *with* $U \cap V \neq \phi$.

Definition 2.3 (Strong crisp basis)**.** *Let* X *be a non empty set.* $\boldsymbol{B} \subset P(X)$ *is called a strong crisp basis if*

(1). $\cup\{B/B \in \boldsymbol{B}\} = X$.

(2). $U, V \in \boldsymbol{B}$, $U \cap V \neq \phi \Rightarrow U \cap V \in \boldsymbol{B}$.

Now we study the properties of α cuts of fuzzy basis and strong fuzzy basis.

Theorem 2.4. *Let* X *be a non empty set. Let* f *be a fuzzy basis. Let* $\alpha \in (0, l]$. *This* α *cut of* f *is a crisp basis.*

Proof. Let f be a fuzzy basis on a non empty set X. Take $\alpha \in (0, 1]$. $\alpha_f = \{A \subset X/f(A) \geq \alpha\}$. We claim that α_f is a crisp basis for X.

(1). Since f is a fuzzy basis $\cup\{B/f(B) = 1\} = X$. This implies $\cup\{B/f(B) \geq \alpha\} = X$. Hence $\cup\{B/B \in \alpha_f\} = X$.

(2). Take $U, V \in \alpha_f$. Then $f(U) \geq \alpha$ and $f(V) \geq \alpha$. Let $x \in U \cap V$. Since f is a fuzzy basis $\exists$ W such that $x \in W \subset U \cap V$ and $f(W) \geq \alpha$. This implies that $W \in \alpha_f$. Hence for $U, V \in \alpha_f$ and $x \in U \cap V$, $\exists$ W such that $x \in W \subset U \cap V$ and $W \in \alpha_f$. Hence α_f is a crisp basis.

□

Result 2.5. *Converse is not true.*

Let X *be a non empty set.* $f : P(X) \to [0, 1]$ *be a map.* α_f *is a crisp basis for some* $\alpha \in (0, 1]$. *Then* f *need not be a fuzzy basis.*

Example 2.6. *Let* $X = \{a, b, c\}$. *Define* $f : P(X) \to [0, 1]$ *as* $f(\phi) = 0$, $f\{a\} = 0.4$, $f\{b\} = 0$, $f\{c\} = 0.6$, $f\{a, b\} = 0.6$, $f\{a, c\} = 0.5$, $f\{b, c\} = 0.4$, $f(X) = 1$. *Take* $\alpha = 0.6$. $\alpha_f = \{\{c\}, \{a, b\}, X\}$. *Clearly* α_f *is a crisp basis. Take* $\alpha = 0.4$. *Take* $U = \{a, b\}$, $V = \{b, c\}$, $U \cap V = \{b\}$, $f(U) \geq \alpha$, $f(V) \geq \alpha$. *The only* W *with* $b \in W \subset U \cap V$ *is* $\{b\}$. *And* $f(W) = f\{b\} = 0$ *is not greater than or equal to* α. *Hence* f *is not a fuzzy basis. Therefore* α_f *is a crisp basis does not imply that* f *is a fuzzy basis.*

Now we try to make the converse true. We see that in the above example we have considered two values of α. For $\alpha = 0.6$, α_f is a crisp basis. But when $\alpha = 0.4$, α_f is not a crisp basis. This gives an idea.
Suppose α_f is a crisp basis for all values of α in $(0, 1]$ then we see that the converse is true.

Theorem 2.7. *Let* X *be a non empty set.* $f : P(X) \to (0, 1]$ *be a function. If* α_f *is a crisp basis for each* $\alpha \in (0, 1]$ *then* f *is a fuzzy basis.*

Proof. $X \neq \phi$. $f : P(X) \to [0, 1]$ is a function. α_f is a crisp basis for each $\alpha \in (0, 1]$.

(1). Take $\alpha = 1$. α_f is a crisp basis. Therefore $\cup\{B/f(B) = 1\} = X$.

(2). Take U and V subsets of X with $x \in U \cup V$ and let $f(U) \geq \alpha$, $f(V) \geq \alpha$. Now α_f is a crisp basis. $f(U) \geq \alpha$ and $f(V) \geq \alpha$ implies that $U, V \in \alpha_f$. Now α_f is a crisp basis, $U, V \in \alpha_f$, $x \in U \cap V$. Hence $\exists$ W such that $W \in \alpha_f$ and $x \in W \subset U \cap V$. $W \in \alpha_f \Rightarrow f(W) \geq \alpha$. Hence $\exists$ W such that $x \in W \subset U \cap V$ and $f(W) \geq \alpha$.

Therefore for each $\alpha(0, 1]$, $f(U) \geq \alpha$, $f(V) \geq \alpha$, $x \in U \cap V$ implies $\exists$ W such that $x \in W \subset U \cap V$ and $f(W) \geq \alpha$. This is true for each α. Hence f is a fuzzy basis. □

Now we try similar results in case of Strong fuzzy basis.

Theorem 2.8. *Let f be a strong fuzzy basis on X. Let $\alpha \in (0, 1]$. Then the α cut of f is a strong crisp basis.*

Proof. $X \neq \phi$. $f : P(X) \to [0, 1]$ be a strong fuzzy basis. Let $\alpha \in (0, 1]$. $\alpha_f = \{A/f(A) \geq \alpha\}$.

(1). Since f is a strong fuzzy basis $\cup\{B/f(B) = 1\} = X$. This implies $\cup\{B/f(B) \geq \alpha\} = X$. Therefore $\cup\{B/B \in \alpha_f\} = X$.

(2). Let $U, V \in \alpha_f$ and $U \cap V \neq \phi$. Now $f(U) \geq \alpha$ and $f(V) \geq \alpha$ and $U \cap V \neq \phi$. Since f is a strong basis $f(U \cap V) \geq \min\{f(U), f(V)\}$. Therefore $f(U \cap V) \geq \alpha$. Hence $U \cap V \in \alpha_f$. Hence $U, V \in \alpha_f$ and $U \cap V \neq \phi$ implies $U \cap V \in \alpha_f$. Hence α_f is a strong crisp basis. Hence α cut of a strong fuzzy basis is a strong crisp basis.

□

Result 2.9. *Converse is not true.*

Let X be a non empty set. $f : P(X) \to [0, 1]$ be a function. Let $\alpha \in (0, 1]$. α_f is a strong crisp basis does not imply that f is a strong fuzzy basis.

Example 2.10. *Let $X = \{a, b, c, d\}$. Define $f : P(X) \to (0, 1]$ as $f(\phi) = 0$, $f\{a, b, c\} = 1$, $f\{b, c, d\} = 1$, $f\{b, c\} = 0.6$, $f(X) = 1$ and $f(A) = 0$ for all other A. Take $\alpha = 0.6$. α cut of $f = \{\{a, b, c\}, \{b, c, d\}, \{b, c\}, X\}$. Clearly α cut of f is a strong crisp basis. We claim that f is not a strong fuzzy basis. Take $U = \{a, b, c\}$, $V = \{b, c, d\}$. $U \cap V = \{b, c\}$, $f(U \cap V) = f\{b, c\} = 0.6$. $\min\{f(U), f(V)\} = 1$. Now $f(U \cap V)$ is not greater than or equal to $\min\{f(U), f(V)\}$. Hence f is not a strong fuzzy basis.*

Theorem 2.11. *Let X be a non empty set. $f : P(X) \to [0, 1]$ be a function. If for each $\alpha \in (0, 1]$, α cut of f is a strong crisp basis then f is a strong fuzzy basis.*

Proof. $X \neq \phi$. $f : P(X) \to [0, 1]$ is function and for each $\alpha \in (0, 1]$. α cut of f is a strong crisp basis. Take $\alpha = 1$. α_f is a strong crisp basis. Hence $\cup\{B/B \in \alpha_f\} = X$. Hence $\cup\{B/f(B) = 1\} = X$. Take U, V where $U \cap V \neq \phi$. Let $f(U) = \alpha_1$, $f(V) = \alpha_2$. If $\alpha_1 = 0$ or $\alpha_2 = 0$ then whatever be the value of $f(U \cap V)$, we have $f(U \cap V) \geq \min\{f(U), f(V)\}$. Suppose $\alpha_1 \neq 0$ and $\alpha_2 \neq 0$ then let $\alpha = \min\{\alpha_1, \alpha_2\}$, $\alpha \in (0, 1]$. α cut of f is strong crisp basis $f(U) = \alpha_1 \geq \alpha$, $f(V) = \alpha_2 \geq \alpha$. Hence $f(U) \geq \alpha$ and $f(V) \geq \alpha$. Therefore $U, V \in \alpha_f$. Also $U \cap V \neq \phi$ and α_f is a strong crisp basis. Hence $U \cap V \in \alpha_f$. Therefore $f(U \cap V) \geq \alpha$. Hence $f(U \cap V) \geq \min\{f(U), f(V)\}$. Hence f is a strong fuzzy basis. □

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