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$(1,2)^{\star}$-$r\omega$-Continuous and $(1,2)^{\star}$-$r\omega$-Irresolute Functions

Research Article

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Abstract: In this paper, we introduce two types of bitopological functions called $(1,2)^{\star}$-$r\omega$-continuous functions and $(1,2)^{\star}$-$r\omega$-irresolute functions and study their properties.

MSC: 54E55.

Keywords: $(1,2)^{\star}$-ω-continuity, $(1,2)^{\star}$-$r\omega$-continuity, $(1,2)^{\star}$-gpr-continuity.

1. Introduction

Recently Ravi, Lellis Thivagar, Ekici and Many others defined different weak forms of semi-open, preopen, regular open and regular semi-open in bitopological spaces.

In this paper, we introduce the notions of $(1,2)^{\star}$-$r\omega$-continuous and $(1,2)^{\star}$-$r\omega$-irresolute functions in bitopological spaces and study some of their basic properties. In most of the occasions our ideas are illustrated and substantiated by some suitable examples.

2. Preliminaries

Throughout this paper, X, Y and Z denote bitopological spaces (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) respectively.

Definition 2.1. *Let A be a subset of a bitopological space X. Then A is called $\tau_{1,2}$-open [4, 15] if $A = P \cup Q$, for some $P \in \tau_1$ and $Q \in \tau_2$. The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed. The family of all $\tau_{1,2}$-open (resp. $\tau_{1,2}$-closed) sets of X is denoted by $(1,2)^{\star}$-O(X) (resp. $(1,2)^{\star}$-C(X)).*

Definition 2.2 ([15, 18])**.** *Let A be a subset of a bitopological space X. Then*

(1). the $\tau_{1,2}$-interior of A, denoted by $\tau_{1,2}$-int(A), is defined by $\cup$ { U : $U \subseteq A$ and U is $\tau_{1,2}$-open}.

(2). the $\tau_{1,2}$-closure of A, denoted by $\tau_{1,2}$-cl(A), is defined by $\cap$ { U : $A \subseteq U$ and U is $\tau_{1,2}$-closed}.

Notice that $\tau_{1,2}$-open subsets of X need not necessarily form a topology.

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Definition 2.3. *A subset A of a bitopological space X is called*

(1). $(1,2)^{\star}$-regular open [14] if $A = \tau_{1,2}$-int($\tau_{1,2}$-cl(A)),

(2). $(1,2)^{\star}$-π-open [20] if the finite union of $(1,2)^{\star}$-regular open sets in X,

(3). $(1,2)^{\star}$-preopen [19] if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl(A)),

(4). $(1,2)^{\star}$-semi-open [13] if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(A)),

(5). regular $(1,2)^{\star}$-semi-open [21] if there is a $(1,2)^{\star}$-regular open set U such that $U \subseteq A \subseteq \tau_{1,2}$-cl($U$).

The complements of the above open sets are called their respective closed sets. The $(1,2)^{\star}$-preclosure of a subset A, $(1,2)^{\star}$-pcl(A) of X is the intersection of all $(1,2)^{\star}$-preclosed sets of X containing A.

Definition 2.4. *A subset A of a bitopological space X is called*

(1). $(1,2)^{\star}$-generalized closed (briefly $(1,2)^{\star}$-g-closed) [8] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X,

(2). $(1,2)^{\star}$-weakly closed (briefly $(1,2)^{\star}$-ω-closed) [16] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-semi-open in X,

(3). $(1,2)^{\star}$-regular generalized closed (briefly $(1,2)^{\star}$-rg-closed) [16] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-regular open in X,

(4). $(1,2)^{\star}$-weakly generalized closed (briefly $(1,2)^{\star}$-wg-closed) [20] if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq U$ and U is $\tau_{1,2}$-open in X,

(5). $(1,2)^{\star}$-generalized pre regular closed (briefly $(1,2)^{\star}$-gpr-closed) [21] if $(1,2)^{\star}$-pcl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-regular open in X,

(6). $(1,2)^{\star}$-π-generalized closed (briefly $(1,2)^{\star}$-πg-closed) [16] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^{\star}$-π-open in X,

(7). $(1,2)^{\star}$-regular ω-closed (briefly $(1,2)^{\star}$-$r\omega$-closed) [21] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is regular $(1,2)^{\star}$-semi-open in X.

The complements of the above closed sets are called their respective open sets.

Definition 2.5. *A function $f : X \to Y$ is said to be*

(1). $(1,2)^{\star}$-g-open [8] if f(V) is $(1,2)^{\star}$-g-open in Y for each $\tau_{1,2}$-open set V in X,

(2). $(1,2)^{\star}$-ω-open [20] if f(V) is $(1,2)^{\star}$-ω-open in Y for each $\tau_{1,2}$-open set V in X.

Definition 2.6. *A function $f : X \to Y$ is said to be*

(1). $(1,2)^{\star}$-g-continuous [8] if $f^{-1}(V)$ is $(1,2)^{\star}$-g-closed in X for every $\sigma_{1,2}$-closed set V in Y,

(2). $(1,2)^{\star}$-ω-continuous [16] if $f^{-1}(V)$ is $(1,2)^{\star}$-ω-closed in X for every $\sigma_{1,2}$-closed set V in Y,

(3). $(1,2)^{\star}$-$r\omega$-continuous [21] if $f^{-1}(V)$ is $(1,2)^{\star}$-$r\omega$-closed in X for every $\sigma_{1,2}$-closed set V in Y,

(4). $(1,2)^{\star}$-rg-continuous [16] if $f^{-1}(V)$ is $(1,2)^{\star}$-rg-closed in X for every $\sigma_{1,2}$-closed set V in Y,

(5). $(1,2)^{\star}$-wg-continuous [20] if $f^{-1}(V)$ is $(1,2)^{\star}$-wg-closed in X for every $\sigma_{1,2}$-closed set V in Y,

(6). $(1,2)^\star$*-gpr-continuous* [21] *if* $f^{-1}(V)$ *is* $(1,2)^\star$*-gpr-closed in X for every* $\sigma_{1,2}$*-closed set V in Y,*

(7). $(1,2)^\star$-πg*-continuous* [16] *if* $f^{-1}(V)$ *is* $(1,2)^\star$-πg*-closed in X for every* $\sigma_{1,2}$*-closed set V in Y,*

(8). $(1,2)^\star$*-semi-continuous* [13] *if* $f^{-1}(V)$ *is* $(1,2)^\star$*-semi-open in X for every* $\sigma_{1,2}$*-open set V in Y.*

Definition 2.7. *A function* $f : X \to Y$ *is said to be*

(1). $(1,2)^\star$*-irresolute* [20] *if* $f^{-1}(V)$ *is* $(1,2)^\star$*-semi-open in X for every* $(1,2)^\star$*-semi-open set V in Y,*

(2). $(1,2)^\star$-ω*-irresolute* [16] *if* $f^{-1}(V)$ *is* $(1,2)^\star$-ω*-closed in X for every* $(1,2)^\star$-ω*-closed set V in Y.*

Definition 2.8 ([17])**.** *A bijective function* $f : X \to Y$ *is said to be*

(1). $(1,2)^\star$*-g-homeomorphism if f is both* $(1,2)^\star$*-g-continuous and* $(1,2)^\star$*-g-open,*

(2). $(1,2)^\star$-$\omega^\star$*-homeomorphism if both f and* f^{-1} *are* $(1,2)^\star$-ω*-irresolute,*

(3). $(1,2)^\star$-ω*-homeomorphism if f is both* $(1,2)^\star$-ω*-continuous and* $(1,2)^\star$-ω*-open.*

Proposition 2.9 ([17])**.** *Every* $(1,2)^\star$*-homeomorphism is* $(1,2)^\star$-ω*-homeomorphism but not conversely.*

Proposition 2.10 ([17])**.** *Every* $(1,2)^\star$-ω*-homeomorphism is* $(1,2)^\star$*-g-homeomorphism but not conversely.*

Remark 2.11 ([21])**.** *(1). Every* $\tau_{1,2}$*-closed set is* $(1,2)^\star$-$r\omega$*-closed but not conversely.*

(2). Every $\tau_{1,2}$*-closed set is* $(1,2)^\star$-ω*-closed but not conversely.*

(3). Every $(1,2)^\star$-ω*-closed set is* $(1,2)^\star$-$r\omega$*-closed but not conversely.*

(4). Every $(1,2)^\star$-$r\omega$*-closed set is* $(1,2)^\star$*-rg-closed but not conversely.*

(5). Every $(1,2)^\star$-$r\omega$*-closed set is* $(1,2)^\star$*-gpr-closed but not conversely.*

3. $(1,2)^\star$-$r\omega$-continuous Functions

Definition 3.1. *A function* $f : X \to Y$ *is said to be* $(1,2)^\star$-$r\omega$*-continuous if* $f^{-1}(V)$ *is* $(1,2)^\star$-$r\omega$*-closed in X, for every* $\sigma_{1,2}$*-closed set V in Y.*

Theorem 3.2. *Every* $(1,2)^\star$*-continuous function is* $(1,2)^\star$-$r\omega$*-continuous.*

Proof. Let f : X → Y be $(1,2)^\star$-continuous and V be any $\sigma_{1,2}$-closed set in Y. Then $f^{-1}(V)$ is $\tau_{1,2}$-closed set in X. Then $f^{-1}(V)$ is $(1,2)^\star$-$r\omega$-closed in X. Therefore, f is $(1,2)^\star$-$r\omega$-continuous. □

Remark 3.3. *The converse of Theorem 3.2 need not be true as shown in the following example.*

Example 3.4. *Let* $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{a, b\}\}$, $\sigma_1 = \{\phi, Y, \{c\}\}$ *and* $\sigma_2 = \{\phi, Y\}$*. Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^\star$-$r\omega$*-continuous but not* $(1,2)^\star$*-continuous.*

Theorem 3.5. *If* $f : X \to Y$ *is* $(1,2)^\star$-ω*-continuous function then it is* $(1,2)^\star$-$r\omega$*-continuous.*

Proof. Let V be any $\sigma_{1,2}$-closed set of Y. Then by hypothesis $f^{-1}(V)$ is $(1,2)^\star$-ω-closed set in X. But every $(1,2)^\star$-ω-closed set is $(1,2)^\star$-$r\omega$-closed. Therefore, f is $(1,2)^\star$-$r\omega$-continuous. □

Remark 3.6. *The converse of Theorem 3.5 need not be true as shown in the following Example.*

Example 3.7. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{c, d\}\}$ *and* $\sigma_2 = \{\phi, Y\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^{\star}$-$r\omega$*-continuous but not* $(1,2)^{\star}$-ω*-continuous.*

Theorem 3.8. *If* $f : X \to Y$ *is* $(1,2)^{\star}$-$r\omega$*-continuous function then it is* $(1,2)^{\star}$-rg*-continuous.*

Proof. Let V be any $\sigma_{1,2}$-closed set of Y. Then by hypothesis $f^{-1}(V)$ is $(1,2)^{\star}$-$r\omega$-closed set in X. But every $(1,2)^{\star}$-$r\omega$-closed set is $(1,2)^{\star}$-rg-closed. Therefore, f is $(1,2)^{\star}$-rg-continuous. □

Remark 3.9. *The converse of Theorem 3.8 need not be true as shown in the following Example.*

Example 3.10. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{b, d\}\}$ *and* $\sigma_2 = \{\phi, Y\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^{\star}$-rg*-continuous but not* $(1,2)^{\star}$-$r\omega$*-continuous.*

Theorem 3.11. *If* $f : X \to Y$ *is* $(1,2)^{\star}$-$r\omega$*-continuous function then it is* $(1,2)^{\star}$-*gpr-continuous.*

Proof. Let V be any $\sigma_{1,2}$-closed set of Y. Then by hypothesis $f^{-1}(V)$ is $(1,2)^{\star}$-$r\omega$-closed set in X. But every $(1,2)^{\star}$-$r\omega$-closed set is $(1,2)^{\star}$-*gpr*-closed. Therefore, f is $(1,2)^{\star}$-*gpr*-continuous. □

Remark 3.12. *The converse of Theorem 3.11 need not be true as shown in the following Example.*

Example 3.13. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a, b, d\}\}$ *and* $\sigma_2 = \{\phi, Y\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^{\star}$-*gpr-continuous but not* $(1,2)^{\star}$-$r\omega$*-continuous.*

Remark 3.14. *The concepts of*

(1). $(1,2)^{\star}$-$r\omega$*-continuous and* $(1,2)^{\star}$-g*-continuous are independent.*

(2). $(1,2)^{\star}$-$r\omega$*-continuous and* $(1,2)^{\star}$*-semi-continuous are independent.*

(3). $(1,2)^{\star}$-$r\omega$*-continuous and* $(1,2)^{\star}$-wg*-continuous are independent.*

(4). $(1,2)^{\star}$-$r\omega$*-continuous and* $(1,2)^{\star}$-πg*-continuous are independent.*

Example 3.15. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{c, d\}\}$ *and* $\sigma_2 = \{\phi, Y\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^{\star}$-$r\omega$*-continuous but not* $(1,2)^{\star}$-g*-continuous.*

Example 3.16. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a, c\}\}$ *and* $\sigma_2 = \{\phi, Y\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^{\star}$-g*-continuous but not* $(1,2)^{\star}$-$r\omega$*-continuous.*

Example 3.17. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{c, d\}\}$ *and* $\sigma_2 = \{\phi, Y\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^{\star}$-$r\omega$*-continuous but not* $(1,2)^{\star}$*-semi-continuous.*

Example 3.18. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a, c, d\}\}$ *and* $\sigma_2 = \{\phi, Y\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^{\star}$*-semi-continuous but not* $(1,2)^{\star}$-$r\omega$*-continuous.*

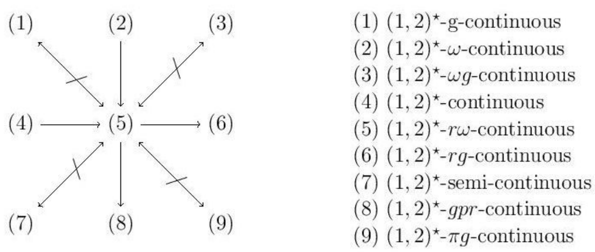
Example 3.19. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{c, d\}\}$ *and* $\sigma_2 = \{\phi, Y\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^\star$*-rω-continuous but not* $(1,2)^\star$*-wg-continuous.*

Example 3.20. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a, b, d\}\}$ *and* $\sigma_2 = \{\phi, Y\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^\star$*-wg-continuous but not* $(1,2)^\star$*-rω-continuous.*

Example 3.21. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{c, d\}\}$ *and* $\sigma_2 = \{\phi, Y\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^\star$*-rω-continuous but not* $(1,2)^\star$*-πg-continuous.*

Example 3.22. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{b, d\}\}$ *and* $\sigma_2 = \{\phi, Y\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^\star$*-πg-continuous but not* $(1,2)^\star$*-rω-continuous.*

Remark 3.23. *The following diagram summarizes the above discussions.*



Remark 3.24. *The following Example shows that the composition of two* $(1,2)^\star$*-rω-continuous functions need not be a* $(1,2)^\star$*-rω-continuous.*

Example 3.25. *Let* $X = Y = Z = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a, b\}\}$, $\sigma_2 = \{\phi, Y, \{c, d\}\}$, $\eta_1 = \{\phi, Z, \{a, b, d\}\}$ *and* $\eta_2 = \{\phi, Z\}$. *Let the functions* $f : X \to Y$ *and* $g : Y \to Z$ *be the identity functions. Then f and g are* $(1,2)^\star$*-rω-continuous but* $g \circ f$ *is not* $(1,2)^\star$*-rω-continuous, since* $(g \circ f)^{-1}(\{c\}) = \{c\}$ *is not* $(1,2)^\star$*-rω-closed set in X.*

4. $(1,2)^\star$-$r\omega$-irresolute Functions

Definition 4.1. *A function* $f : X \to Y$ *is called* $(1,2)^\star$*-rω-irresolute if the inverse image of every* $(1,2)^\star$*-rω-closed set in Y is* $(1,2)^\star$*-rω-closed in X.*

Theorem 4.2. *Every* $(1,2)^\star$*-rω-irresolute function is* $(1,2)^\star$*-rω-continuous but not conversely.*

Proof. Assume that f : X → Y is $(1,2)^\star$-$r\omega$-irresolute and V is $\sigma_{1,2}$-closed set in Y. So it is $(1,2)^\star$-$r\omega$-closed set in Y. By our assumption $f^{-1}(V)$ is a $(1,2)^\star$-$r\omega$-closed set in X. Therefore, f is $(1,2)^\star$-$r\omega$-continuous. □

Example 4.3. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a, b\}\}$ *and* $\sigma_2 = \{\phi, Y, \{c, d\}\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is a* $(1,2)^\star$*-rω-continuous but not* $(1,2)^\star$*-rω-irresolute, because* $f^{-1}(\{a, c\}) = \{a, c\}$ *is not an* $(1,2)^\star$*-rω-closed set in X.*

Theorem 4.4. *Let $f : X \to Y$ and $g : Y \to Z$ be any two functions. Then $g \circ f$ is $(1,2)^\star$-$r\omega$-continuous if g is $(1,2)^\star$-continuous and f is $(1,2)^\star$-$r\omega$-continuous.*

Proof. Let V be any $\eta_{1,2}$-closed set in Z. Then g^{-1}(V) is $\sigma_{1,2}$-closed in Y, since g is $(1,2)^\star$-continuous. Then f^{-1}(g^{-1}(V)) is $(1,2)^\star$-$r\omega$-closed in X, as f is $(1,2)^\star$-$r\omega$-continuous. That is, $(g \circ \text{f})^{-1}$(V) is $(1,2)^\star$-$r\omega$-closed in X. Hence $g \circ$ f is $(1,2)^\star$-$r\omega$-continuous. □

Theorem 4.5. *Let $f : X \to Y$ and $g : Y \to Z$ be any two functions. Then $g \circ f$ is $(1,2)^\star$-$r\omega$-irresolute if g is $(1,2)^\star$-$r\omega$-irresolute and f is $(1,2)^\star$-$r\omega$-irresolute.*

Proof. Let V be any $(1,2)^\star$-$r\omega$-closed set in Z. Since g is $(1,2)^\star$-$r\omega$-irresolute, g^{-1}(V) is $(1,2)^\star$-$r\omega$-closed in Y. Then f^{-1}(g^{-1}(V)) = $(g \circ \text{f})^{-1}$(V) is $(1,2)^\star$-$r\omega$-closed in X, as f is $(1,2)^\star$-$r\omega$-irresolute. Therefore, $g \circ$ f is $(1,2)^\star$-$r\omega$-irresolute. □

Theorem 4.6. *Let $f : X \to Y$ and $g : Y \to Z$ be any two functions. Then g o f is $(1,2)^\star$-$r\omega$-continuous if g is $(1,2)^\star$-$r\omega$-continuous and f is $(1,2)^\star$-$r\omega$-irresolute.*

Proof. Let V be any $\eta_{1,2}$-closed set in Z. Since g is $(1,2)^\star$-$r\omega$-continuous, g^{-1}(V) is $(1,2)^\star$-$r\omega$-closed in Y. Then f^{-1}(g^{-1}(V)) = $(g \circ \text{f})^{-1}$(V) is $(1,2)^\star$-$r\omega$-closed in X, as f is $(1,2)^\star$-$r\omega$-irresolute. Therefore, $g \circ$ f is $(1,2)^\star$-$r\omega$-continuous. □

5. $(1,2)^\star$-$r\omega$-homeomorphisms

We introduce the following definition.

Definition 5.1. *A function $f : X \to Y$ is called $(1,2)^\star$-$r\omega$-open (resp. $(1,2)^\star$-$r\omega$-closed) if f(V) is $(1,2)^\star$-$r\omega$-open (resp. $(1,2)^\star$-$r\omega$-closed) in Y for each $\tau_{1,2}$-open set V in X.*

Definition 5.2. *A bijection $f : X \to Y$ is called $(1,2)^\star$-$r\omega$-homeomorphism if f is both $(1,2)^\star$-$r\omega$-continuous and $(1,2)^\star$-$r\omega$-open. We denote the family of all $(1,2)^\star$-$r\omega$-homeomorphisms of a bitopological space X onto itself by $(1,2)^\star$-$r\omega$-h(X).*

Example 5.3. *Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a, b\}\}$ and $\sigma_2 = \{\phi, Y, \{c, d\}\}$. Let the function $f : X \to Y$ be the identity function. Then f is bijective, $(1,2)^\star$-$r\omega$-continuous and f is $(1,2)^\star$-$r\omega$-open. Therefore f is $(1,2)^\star$-$r\omega$-homeomorphism.*

Theorem 5.4. *Every $(1,2)^\star$-homeomorphism is an $(1,2)^\star$-$r\omega$-homeomorphism.*

Proof. Let f : X → Y be a $(1,2)^\star$-homeomorphism. Then f is both $(1,2)^\star$-continuous and $(1,2)^\star$-open and f is bijection. As every $(1,2)^\star$-continuous function is $(1,2)^\star$-$r\omega$-continuous and every $(1,2)^\star$-open function is $(1,2)^\star$-$r\omega$-open, we have f is both $(1,2)^\star$-$r\omega$-continuous and $(1,2)^\star$-$r\omega$-open. Therefore f is $(1,2)^\star$-$r\omega$-homeomorphism. □

Remark 5.5. *The converse of Theorem 5.4 need not be true as shown in the following example.*

Example 5.6. *Let $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a, b\}\}$ and $\sigma_2 = \{\phi, Y, \{c, d\}\}$. Let the function $f : X \to Y$ be the identity function. Then f is $(1,2)^\star$-$r\omega$-homeomorphism but it is not $(1,2)^\star$-homeomorphism.*

Theorem 5.7. *Every $(1,2)^\star$-ω-homeomorphism is an $(1,2)^\star$-$r\omega$-homeomorphism.*

Proof. Let f : X → Y be a $(1,2)^\star$-ω-homeomorphism. Then f is $(1,2)^\star$-ω-continuous and $(1,2)^\star$-ω-open and f is bijection. As every $(1,2)^\star$-ω-continuous function is $(1,2)^\star$-$r\omega$-continuous and every $(1,2)^\star$-ω-open function is $(1,2)^\star$-$r\omega$-open, we have f is both $(1,2)^\star$-$r\omega$-continuous and $(1,2)^\star$-$r\omega$-open. Therefore f is $(1,2)^\star$-$r\omega$-homeomorphism. □

Remark 5.8. *The converse of Theorem 5.7 need not be true as shown in the following Example.*

Example 5.9. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a, b\}\}$ *and* $\sigma_2 = \{\phi, Y, \{c, d\}\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is* $(1, 2)^\star$*-rω-homeomorphism but it is not* $(1, 2)^\star$*-ω-homeomorphism.*

Theorem 5.10. *For any bijection function* $f : X \to Y$ *the following statements are equivalent :*

(1). $f^{-1} : Y \to X$ *is* $(1, 2)^\star$*-rω-continuous.*

(2). f is $(1, 2)^\star$*-rω-open function.*

(3). f is $(1, 2)^\star$*-rω-closed function.*

Theorem 5.11. *Let* $f : X \to Y$ *be a bijection* $(1, 2)^\star$*-rω-continuous function. Then the following statements are equivalent*

(1). f is an $(1, 2)^\star$*-rω-open function.*

(2). f is an $(1, 2)^\star$*-rω-homeomorphism.*

(3). f is an $(1, 2)^\star$*-rω-closed function.*

Proof. Follows from Theorem 5.10. □

Remark 5.12. *The composition of two* $(1, 2)^\star$*-rω-homeomorphism functions need not be a* $(1, 2)^\star$*-rω-homeomorphism function as shown in the following Example.*

Example 5.13. *Let* $X = Y = Z = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{a, c, d\}\}$, $\sigma_1 = \{\phi, Y, \{a, b\}\}$, $\sigma_2 = \{\phi, Y, \{c, d\}\}$, $\eta_1 = \{\phi, Z, \{a\}, \{b\}, \{a, b\}\}$ *and* $\eta_2 = \{\phi, Z\}$. *Let* $f : X \to Y$ *and* $g : Y \to Z$ *be the identity functions. Then f and g are* $(1, 2)^\star$*-rω-homeomorphism but their* $g \circ f : X \to Z$ *is not* $(1, 2)^\star$*-rω-homeomorphism, since for the* $\tau_{1,2}$*-open set* $V = \{a, c, d\}$ *in X,* $(g \circ f)(V) = f(g(V)) = f(g(\{a, c, d\})) = f(\{a, c, d\}) = \{a, c, d\}$ *is not* $(1, 2)^\star$*-rω-open in Z.*

Definition 5.14. *A bijection* $f : X \to Y$ *is said to be* $(1, 2)^\star$*-rωc-homeomorphism if both f and* f^{-1} *are* $(1, 2)^\star$*-rω-irresolute. We say that bitopological spaces X and Y are* $(1, 2)^\star$*-rωc-homeomorphic if there exists a* $(1, 2)^\star$*-rωc-homeomorphism from X onto Y.*
We denote the family of all $(1, 2)^\star$*-rωc-homeomorphisms of a bitopological space X onto itself by* $(1, 2)^\star$*-rωc-h(X).*

Theorem 5.15. *Every* $(1, 2)^\star$*-rωc-homeomorphism is an* $(1, 2)^\star$*-rω-homeomorphism.*

Proof. Let $f : X \to Y$ be an $(1, 2)^\star$-$r\omega c$-homeomorphism. Then f and f^{-1} are $(1, 2)^\star$-$r\omega$-irresolute and f is bijection. By Theorem 4.2, f and f^{-1} are $(1, 2)^\star$-$r\omega$-continuous. Therefore f is $(1, 2)^\star$-$r\omega$-homeomorphism. □

Remark 5.16. *The converse of Theorem 5.15 need not be true as shown in the following Example.*

Example 5.17. *Let* $X = Y = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a, b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a, b\}\}$ *and* $\sigma_2 = \{\phi, Y, \{c, d\}\}$. *Let the function* $f : X \to Y$ *be the identity function. Then f is* $(1, 2)^\star$*-rω-homeomorphism but it is not* $(1, 2)^\star$*-rωc-homeomorphism, since f is not* $(1, 2)^\star$*-rω-irresolute.*

Remark 5.18. *The following diagram summarizes the above discussions.*

$$(1,2)^{\star}\text{-}\omega\text{-}homeomorphism \rightarrow (1,2)^{\star}\text{-}g\text{-}homeomorphism$$

$$\uparrow \qquad \searrow$$

$$(1,2)^{\star}\text{-}homeomorphism \rightarrow (1,2)^{\star}\text{-}r\omega\text{-}homeomorphism$$

$$\uparrow$$

$$(1,2)^{\star}\text{-}r\omega c\text{-}homeomorphism$$

Theorem 5.19. *Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be $(1,2)^{\star}$-$r\omega c$-homeomorphism, then their composition $g \circ f : X \rightarrow Z$ is also $(1,2)^{\star}$-$r\omega c$-homeomorphism.*

Proof. Let U be a $(1,2)^{\star}$-$r\omega$-open set in Z. Since g is $(1,2)^{\star}$-$r\omega$-irresolute, g^{-1}(U) is $(1,2)^{\star}$-$r\omega$-open in Y. Since f is $(1,2)^{\star}$-$r\omega$-irresolute, $\mathrm{f}^{-1}(g^{-1}(\mathrm{U})) = (g \circ \mathrm{f})^{-1}(\mathrm{U})$ is $(1,2)^{\star}$-$r\omega$-open set in X. Therefore $g \circ$ f is $(1,2)^{\star}$-$r\omega$-irresolute. Also for a $(1,2)^{\star}$-$r\omega$-open set G in X, we have $(g \circ \mathrm{f})(\mathrm{G}) = g(\mathrm{f}(\mathrm{G})) = g(\mathrm{W})$, where W = f(G). By hypothesis, f(G) is $(1,2)^{\star}$-$r\omega$-open in Y and so again by hypothesis, g(f(G)) is a $(1,2)^{\star}$-$r\omega$-open set in Z. That is $(g \circ \mathrm{f})(\mathrm{G})$ is a $(1,2)^{\star}$-$r\omega$-open set in Z and therefore $(g \circ \mathrm{f})^{-1}$ is $(1,2)^{\star}$-$r\omega$-irresolute. Also $g \circ$ f is a bijection. Hence $g \circ$ f is $(1,2)^{\star}$-$r\omega c$-homeomorphism. □

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