International Journal of Mathematics And its Applications
Volume 3, Issue 4–C (2015), 9–16.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

On Decompositions of Generalized μ-α-sets

Research Article

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Abstract: The aim of this paper is to introduce the new notions called μ-α-locally closed sets, $\mu_{\alpha\text{-}t}$-sets and $\mu_{\alpha\text{-}B}$-sets and investigate their properties. Using these concepts we obtained some decompositions.

MSC: 54C05, 54C08, 54D10.

Keywords: μ-α-locally closed set, $\mu_\alpha g$ closed set, $\mu_{\alpha\text{-}t}$-set, $\mu_{\alpha\text{-}B}$-set.

1. Introduction

In the past few years, different forms of open sets have been studied. Recently a significant contribution to the theory of generalized open sets was extended by A. Csàszàr [1, 4]. Especially, Roy has defined some basic operators on generalized topological spaces [12]. On the other hand the notion of decomposition of continuity on topological spaces was introduced by Tong [14]. Recently Roy [13] studied on decomposition of generalized continuity via μ-open sets. The purpose of this paper is to introduce the new notions via μ-α-open sets called μ-α-locally closed sets, $\mu_{\alpha\text{-}t}$-sets and $\mu_{\alpha\text{-}B}$-sets and investigate their properties. Using these concepts we obtained some decompositions.

Recall some generalized topological concepts which are very useful in the sequel. Let X be a non-empty set and μ be a collection of subsets of X. Then μ is called a generalized topology [2] (briefly GT) on X if $\phi \in \mu$, and $G_i \in \mu$ for i $\in$ I $\neq \phi$ implies G $= \bigcup_{i\in I} \mathrm{G}_i \in \mu$. We say μ is strong [3] if X $\in \mu$ and we call the pair (X, μ) a generalized topological space (briefly GTS). The elements of μ are called μ-open sets and the complements of μ-open sets are called μ-closed sets. For A $\subseteq$ X we denote, by c_μ(A) the intersection of all μ-closed sets containing A [2] and by i_μ(A)the union of all μ-open sets contained in A.

A subset A of (X, μ) is called μ-α-open [4] iff A $\subseteq i_\mu(c_\mu(i_\mu(\mathrm{A})))$. The complement of a μ-α-open set is called μ-α-closed. We denote the family of μ-α-open sets of X by $\alpha(\mu)$. The intersection of all μ-α-closed sets containing A is called the μ-α-closure of a subset A of X and is denoted by $c_\alpha(A)$. The μ-α-interior of a subset A of X is the union of all μ-α-open sets contained in A and is denoted by $i_\alpha(A)$ [10].

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2. Preliminaries

Lemma 2.1 ([4]). *Let (X, μ) be a GTS and A, $B \subseteq X$, then the followings hold.*

(1). $i_\mu(A) \subseteq A \subseteq c_\mu(A)$;

(2). $A \subseteq B$ implies $i_\mu(A) \subseteq i_\mu(B)$ and $c_\mu(A) \subseteq c_\mu(B)$;

(3). $i_\mu(i_\mu(A)) = i_\mu(A)$ and $c_\mu(c_\mu(A)) = c_\mu(A)$;

(4). $i_\mu(X - A) = X - c_\mu(A)$ and $c_\mu(X - A) = X - i_\mu(A)$;

(5). $A \in \mu$ iff $A = i_\mu(A)$ and A is μ-closed iff $A = c_\mu(A)$.

Lemma 2.2 ([4]). *Let (X, μ) be a GTS and $A \subseteq X$, we have $c_\alpha(A) = A \cup c_\mu(i_\mu(c_\mu(A)))$ and $i_\alpha(A) = A \cap i_\mu(c_\mu(i_\mu(A)))$.*

Remark 2.3 ([7]). *In a GTS (X, μ) the followings hold:*

(1). $i_\mu(A \cap B) \subseteq i_\mu(A) \cap i_\mu(B)$;

(2). $i_\mu(A \cup B) \supseteq i_\mu(A) \cup i_\mu(B)$;

(3). $c_\mu(A \cap B) \subseteq c_\mu(A) \cap c_\mu(B)$;

(4). $c_\mu(A \cup B) \supseteq c_\mu(A) \cup c_\mu(B)$.

Definition 2.4 ([7]). *A subset A of a GTS (X, μ) is said to be μ-locally closed if $A = U \cap F$ where U is μ-open and F is μ-closed in X.*

Definition 2.5 ([7]). *Let (X, μ) be a GTS and $A \subseteq X$. Then*

(1). A is said to be μ-dense if $c_\mu(A) = X$.

(2). (X, μ) is said to be μ-submaximal if each μ-dense subset of (X, μ) is a μ-open set.

Definition 2.6 ([12]). *Let (X, μ) be a GTS. Then a subset A of X is called μ-generalized closed set (in short μg-closed set) iff $c_\mu(A) \subseteq U$ whenever $A \subseteq U$ where U is μ-open in X. The complement of a μg-closed set is called a μg-open set.*

Definition 2.7. *Let (X, μ) be a GTS. Then a subset A of X is said to be a*

(1). μ_t-set [13] if $i_\mu(A) = i_\mu(c_\mu(A))$;

(2). μ_B-set [13] if $A = U \cap V$, $U \in \mu$, V is a μ_t-set;

(3). μ-semi-open [4] iff $A \subseteq c_\mu(i_\mu(A))$;

(4). μ-pre-open [4] iff $A \subseteq i_\mu(c_\mu(A))$.

Definition 2.8 ([10]). *A function $f : (X, \mu) \to (Y, \lambda)$ is said to be (α, λ)-continuous if $f^{-1}(U)$ is μ-α-open in X for every λ-open set U of Y.*

Definition 2.9 ([11]). *A function $f : (X, \mu) \to (Y, \lambda)$ is said to be contra-(α, λ)-continuous if $f^{-1}(V)$ is μ-α-closed in X for every λ-open set V of Y.*

3. μ-α-locally Closed Sets and $\mu_{\alpha\text{-}t}$-sets

Definition 3.1. *A subset A of a GTS (X, μ) is said to be μ-α-locally closed if $A = U \cap F$ where U is μ-α-open and F is μ-α-closed in X.*

Remark 3.2. *In a GTS (X, μ) the following properties hold.*

(1). Every μ-α-open set is μ-α-locally closed.

(2). If $X \in \mu$ then every μ-α-closed set is μ-α-locally closed.

Remark 3.3. *The condition $X \in \mu$ in Remark 3.2(2) cannot be dropped. This is shown by the following Example.*

Example 3.4. *Let $X = \{a, b, c, d\}$. If we take μ not containing X where $\mu = \{\phi, \{b\}, \{a, b, c\}\}$. Then $\alpha(\mu) = \{\phi, \{b\}, \{b, c\}, \{a, b, c\}\}$; μ-α-closed sets are $\{d\}$, $\{a, d\}$, $\{a, d, c\}$, X and μ-α-locally closed sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{b, c\}$, $\{a, c\}$, $\{a, b, c\}$. Clearly every μ-α-open set is μ-α-locally closed. Also we have $\{a, d\}$ is μ-α-closed but it is not μ-α-locally closed.*

Remark 3.5. *For a GTS (X, μ) we have the following:*

(1). the union of two μ-α-locally closed sets need not be a μ-α-locally closed;

(2). the intersection of two μ-α-locally closed sets need not be a μ-α-locally closed.

Example 3.6. *Consider $X = \{a, b, c\}$ with $\mu = \{\phi, \{a, b\}, \{b, c\}, X\}$. Then $\alpha(\mu) = \{\phi, \{a, b\}, \{b, c\}, X\}$; μ-α-closed sets are ϕ, $\{a\}$, $\{c\}$, X and μ-α-locally closed sets are ϕ, $\{a\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$, X.*

(1). Let $A = \{a\}$ and $B = \{c\}$. Clearly A and B are μ-α-locally closed sets but $A \cup B = \{a, c\}$ is not μ-α-locally closed.

(2). Let $A = \{a, b\}$ and $B = \{b, c\}$. Clearly A and B are μ-α-locally closed sets but $A \cap B = \{b\}$ is not μ-α-locally closed.

Proposition 3.7. *Arbitrary union of μ-α-open sets is again μ-α-open.*

Proof. Let $G_i \in \alpha(\mu)$ for $i \in I \neq \phi$. Claim $G = \bigcup_{i\in I} G_i \in \alpha(\mu)$. Since $G_i \in \alpha(\mu)$ for $i \in I$, we have $G_i \subseteq i_\mu(c_\mu(i_\mu(G_i)))$ for each $i \in I$. Now consider $i_\mu(c_\mu(i_\mu(\bigcup_{i\in I} G_i))) \supseteq i_\mu(c_\mu(\bigcup_{i\in I} i_\mu(G_i))) \supseteq i_\mu(\bigcup_{i\in I} c_\mu(i_\mu(G_i))) \supseteq \bigcup_{i\in I} i_\mu(c_\mu(i_\mu(G_i))) \supseteq \bigcup_{i\in I} G_i$. Hence $G = \bigcup_{i\in I} G_i \in \alpha(\mu)$. □

Theorem 3.8. *If A is a μ-α-locally closed set in a GTS (X, μ), then there exists a μ-α-closed set K in X such that $A \cap K = \phi$.*

Proof. Let A be a μ-α-locally closed set in (X, μ) such that $A = U \cap F$, where U is μ-α-open and F is μ-α-closed. Claim $A \cap K = \phi$ where K is any μ-α-closed. Let $K = F \cap (X - U)$ and $X - K = (X - F) \cup U$. By Proposition 3.7, $X - K$ is μ-α-open in X and K is μ-α-closed in X. Also $A \cap K = (U \cap F) \cap (F \cap (X - U)) = F \cap (U \cap (X - U)) = \phi$. □

Lemma 3.9. *Let (X, μ) be a GTS and A, $B \subseteq X$, then the followings hold.*

(1). $i_\alpha(A) \subseteq A \subseteq c_\alpha(A)$;

(2). $A \subseteq B$ implies $i_\alpha(A) \subseteq i_\alpha(B)$ and $c_\alpha(A) \subseteq c_\alpha(B)$;

(3). $A \in \alpha(\mu)$ iff $A = i_\alpha(A)$ and A is μ-α-closed iff $A = c_\alpha(A)$;

(4). $i_\alpha(i_\alpha(A)) = i_\alpha(A)$ and $c_\alpha(c_\alpha(A)) = c_\alpha(A)$;

(5). $i_\alpha(X - A) = X - c_\alpha(A)$ *and* $c_\alpha(X - A) = X - i_\alpha(A)$.

Proof.

(1) By Lemma 2.2, we have $i_\alpha(A) \subseteq A$, and $A \subseteq c_\alpha(A)$. Hence $i_\alpha(A) \subseteq A \subseteq c_\alpha(A)$

(2) Let $A \subseteq B$. By Lemma 2.1 (2), we have $i_\mu(c_\mu(i_\mu(A))) \subseteq i_\mu(c_\mu(i_\mu(B)))$. Also $A \cap i_\mu(c_\mu(i_\mu(A))) \subseteq B \cap i_\mu(c_\mu(i_\mu(B)))$. This implies $i_\alpha(A) \subseteq i_\alpha(B)$. By the same way we can prove $c_\alpha(A) \subseteq c_\alpha(B)$.

(3) Let $A \in \alpha(\mu)$. Also $i_\alpha(A) = \{\cup\, G\colon\ G \subseteq A$ and $G \in \alpha(\mu)\}$. Always $A \subseteq A$, if $A \in \alpha(\mu)$ then we have $i_\alpha(A) = A$. Conversely if $A = i_\alpha(A) = \{\cup\, G\colon\ G \subseteq A$ and $G \in \alpha(\mu)\}$, then by Proposition 3.7, we have $A \in \alpha(\mu)$. Similaly we can prove A is μ-α-closed iff $A = c_\alpha(A)$.

(4) As $i_\alpha(A) \in \alpha(\mu)$, by (3), we have $i_\alpha(i_\alpha(A)) = i_\alpha(A)$. Similarly we can prove $c_\alpha(c_\alpha(A)) = c_\alpha(A)$.

(5) Now $i_\alpha(X - A) = (X - A) \cap i_\mu(c_\mu(i_\mu(X - A))) = (X - A) \cap (X - c_\mu(i_\mu(c_\mu(A)))) = X - (A \cup c_\mu(i_\mu(c_\mu(A)))) = X - c_\alpha(A)$. By similar argument we can show $c_\alpha(X - A) = X - i_\alpha(A)$. ☐

Theorem 3.10. *For a subset A of a GTS (X, μ), the following properties are equivalent:*

(1). *A is μ-α-locally closed;*

(2). *$A = U \cap c_\alpha(A)$ for some $U \in \alpha(\mu)$;*

(3). *$c_\alpha(A) - A$ is μ-α-closed;*

(4). *$A \cup (X - c_\alpha(A))$ is μ-α-open;*

(5). *$A \subseteq i_\alpha(A \cup (X - c_\alpha(A)))$.*

Proof. (1) ⇒ (2): Let A be a μ-α-locally closed subset of X. Claim $A = U \cap c_\alpha(A)$ for some $U \in \alpha(\mu)$. Since A is μ-α-locally closed, $A = U \cap F$ where U is μ-α-open and F is μ-α-closed. Then $A \subseteq F$ implies $c_\alpha(A) \subseteq F$. So $A = U \cap F \supseteq U \cap c_\alpha(A)$. Again $A \subseteq U$ and $A \subseteq c_\alpha(A)$ implies that $A = U \cap F \subseteq U \cap c_\alpha(A)$. Thus $A = U \cap c_\alpha(A)$.

(2) ⇒ (3): Let $A = U \cap c_\alpha(A)$ for some $U \in \alpha(\mu)$. Claim $c_\alpha(A) - A$ is μ-α-closed. Now $c_\alpha(A) - A = c_\alpha(A) - (U \cap c_\alpha(A))$ $= c_\alpha(A) \cap (X - (U \cap c_\alpha(A))) = c_\alpha(A) \cap ((X - U) \cup (X - c_\alpha(A)) = (c_\alpha(A) \cap (X - U)) \cup (c_\alpha(A) \cap (X - c_\alpha(A)) = c_\alpha(A) \cap (X - U)$ which is a μ-α-closed set.

(3) ⇒ (4): Let $c_\alpha(A) - A$ be μ-α-closed. Claim $A \cup (X - c_\alpha(A))$ is μ-α-open in X. Clearly $X - (c_\alpha(A) - A)$ is μ-α-open and $X - (c_\alpha(A) - A) = X - (c_\alpha(A) \cap (X - A)) = A \cup (X - c_\alpha(A))$. Hence $A \cup (X - c_\alpha(A)) \in \alpha(\mu)$.

(4) ⇒ (5): Let $A \cup (X - c_\alpha(A))$ be μ-α-open. This implies $A \subseteq (A \cup (X - c_\alpha(A))) = i_\alpha(A \cup (X - c_\alpha(A)))$.

(5) ⇒ (1): Let $A \subseteq i_\alpha(A \cup (X - c_\alpha(A)))$. Since $A \subseteq c_\alpha(A)$, $A \subseteq i_\alpha(A \cup (X - c_\alpha(A))) \cap c_\alpha(A)$. As $i_\alpha(A \cup (X - c_\alpha(A)))$ $\subseteq A \cup (X - c_\alpha(A)))$ we have $i_\alpha(A \cup (X - c_\alpha(A))) \cap c_\alpha(A) \subseteq (A \cup (X - c_\alpha(A))) \cap c_\alpha(A) = A$. Hence $A = i_\alpha(A \cup (X - c_\alpha(A))) \cap c_\alpha(A)$ where $i_\alpha(A \cap (X - c_\alpha(A)))$ is μ-α-open and $c_\alpha(A)$ is μ-α-closed. Hence A is μ-α-locally closed. ☐

Theorem 3.11. *Let (X, μ) be a GTS. If $A \subseteq B \subseteq X$ and B is μ-α-locally closed, then there exists μ-α-locally closed set C such that $A \subseteq C \subseteq B$.*

Proof. Let $A \subseteq B \subseteq X$ and B be μ-α-locally closed. Claim there exists a μ-α-closed set C such that $A \subseteq C \subseteq B$. Since B is μ-α-locally closed by Theorem 3.10, $B = U \cap c_\alpha(B)$ for some $U \in \alpha(\mu)$. Now $B \subseteq U$ implies that $A \subseteq B \subseteq U$. Then by Lemma 3.9, $A \subseteq U \cap c_\alpha(A) \subseteq U \cap c_\alpha(B) = B$. Hence $A \subseteq C \subseteq B$ where $C = U \cap c_\alpha(A)$ which is μ-α-locally closed. ☐

Definition 3.12. *Let (X, μ) be a GTS. Then a subset A of X is called μ-α-dense if $c_\alpha(A) = X$. The space (X, μ) is called μ-α-submaximal if every μ-α-dense subset is μ-α-open in X.*

Theorem 3.13. *A GTS (X, μ) is μ-α-submaximal if and only if every subset of X is μ-α-locally closed.*

Proof. Suppose that (X, μ) is a μ-α-submaximal GTS . Claim every subset of X is μ-α-locally closed. Let $A \subseteq X$. Consider $X - (c_\alpha(A) - A)$. To prove that $X - (c_\alpha(A) - A)$ is μ-α-dense. Now $c_\alpha(X - (c_\alpha(A) - A)) = c_\alpha(X - (c_\alpha(A) \cap (X - A)))$ $= c_\alpha(A \cup (X\text{-}c_\alpha(A))) \supseteq c_\alpha(A) \cup c_\alpha(X - c_\alpha(A)) \supseteq c_\alpha(A) \cup (X - i_\alpha(c_\alpha(A))) \supseteq c_\alpha(A) \cup (X - c_\alpha(A)) = X$. Therefore $X - (c_\alpha(A) - A)$ is μ-α-dense subset of X. Since X is μ-α-submaximal, by definition, $X - (c_\alpha(A) - A)$ is μ-α-open set. It follows that $(c_\alpha(A) - A)$ is μ-α-closed. Consequently, $X - (c_\alpha(A) - A) = (c_\alpha(A) - A)^c = (c_\alpha(A) \cap A^c)^c = A \cup (X - c_\alpha(A))$ is μ-α-open. Thus, $A = (A \cup (X - c_\alpha(A))) \cap c_\alpha(A)$ is μ-α-locally closed.
Conversely, let every subset of X be μ-α-locally closed. Let A be μ-α-dense subset of X. Claim A is μ-α-open in X. Since $A \subseteq X$, A is μ-α-locally closed. Hence $A = U \cap c_\alpha(A)$ where U is μ-α-open. By assumption A is μ-α-dense. It becomes that $A = U \cap X = U$. This implies that A is μ-α-open. Hence (X, μ) is μ-α-submaximal. ☐

Definition 3.14. *Let (X, μ) be a GTS. If a subset A of X is called μ_α-generalized closed set (in short $\mu_\alpha g$-closed set) iff $c_\alpha(A) \subseteq U$ whenever $A \subseteq U$ and U is μ-α-open in X. The complement of a $\mu_\alpha g$-closed set is called $\mu_\alpha g$-open set.*

Theorem 3.15. *Let (X, μ) be a GTS. If A is both $\mu_\alpha g$-closed and μ-α-locally closed, then it is μ-α-closed. The converse is also true if $X \in \mu$.*

Proof. Suppose that A is $\mu_\alpha g$-closed and μ-α-locally closed. Thus $A = U \cap F$, where $U \in \alpha(\mu)$ and F is μ-α-closed. So $A \subseteq U$ and $A \subseteq F$. By hypothesis $c_\alpha(A) \subseteq U$ and $c_\alpha(A) \subseteq c_\alpha(F) = F$. Thus $c_\alpha(A) \subseteq U \cap F = A$. Thus A is μ-α-closed.
Conversely, suppose that A is μ-α-closed in X. Let $A \subseteq U$ where $U \in \alpha(\mu)$. Then $c_\alpha(A) = A \subseteq U$. Thus A is $\mu_\alpha g$-closed. Since A is μ-α-closed, by Remark 3.2(2), it is μ-α-locally closed. ☐

Remark 3.16. *The following Examples show that μ-α-locally closed sets and $\mu_\alpha g$-closed sets are independent.*

Example 3.17. *Consider $X = \{a, b, c\}$ with $\mu = \{\phi, \{a, b\}, \{a, c\}, X\}$. Then $\alpha(\mu) = \{\phi, \{a, b\}, \{a, c\}, X\}$; μ-α-locally closed sets are ϕ, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, X and $\mu_\alpha g$-closed sets are ϕ, $\{b\}$, $\{c\}$, $\{b, c\}$, X. We obtain*

(1). $\{a, b\}$ is μ-α-locally closed but not $\mu_\alpha g$-closed.

(2). $\{b, c\}$ is $\mu_\alpha g$-closed but not μ-α-locally closed.

Remark 3.18. *Every μ-α-closed set in a GTS (X, μ) is $\mu_\alpha g$-closed. If A is a μ-α-closed set in X such that $A \subseteq U$ and $U \in \alpha(\mu)$. Then $c_\alpha(A) = A \subseteq U$. Hence A is $\mu_\alpha g$-closed.*

The converse of the remark is not true as seen from the following Example.

Example 3.19. *Consider the GTS in Example 3.17. It is clear that $\{b, c\}$ is $\mu_\alpha g$-closed but not μ-α closed.*

Definition 3.20. *Let (X, μ) be a GTS. Then a subset A of X is said to be a*

(1). $\mu_{\alpha\text{-}t}$-set if $i_\alpha(A) = i_\alpha(c_\alpha(A))$;

(2). $\mu_{\alpha\text{-}B}$-set if $A = U \cap V$, $U \in \alpha(\mu)$ and V is a $\mu_{\alpha\text{-}t}$-set.

Remark 3.21. *For any generalized topological space (X, μ), X is always $\mu_{\alpha\text{-}t}$-set. By the definition of μ, ϕ is always μ-open and hence μ-α-open. Then $\phi^c = X$ which is always μ-α-closed. Therefore $c_\alpha(X) = X$. This implies $i_\alpha(c_\alpha(X)) = i_\alpha(X)$. Hence X is always $\mu_{\alpha\text{-}t}$-set.*

Proposition 3.22. *Let (X, μ) be a GTS. Then*

(1). If A is a μ-α-closed set then it is a $\mu_{\alpha\text{-}t}$-set;

(2). If $X \in \mu$, every $\mu_{\alpha\text{-}t}$-set is a $\mu_{\alpha\text{-}B}$-set;

(3). Every μ-α-locally closed set is a $\mu_{\alpha\text{-}B}$-set.

Proof.

(1) Let A be a μ-α-closed set. Then $A = c_\alpha(A)$. Thus $i_\alpha(A)= i_\alpha(c_\alpha(A))$. Therefore A is a $\mu_{\alpha\text{-}t}$-set.

(2) Let A be a $\mu_{\alpha\text{-}t}$-set. Then $A = X \cap A$, X is μ-α-open. Hence A is $\mu_{\alpha\text{-}B}$-set.

(3) Let A be a μ-α-locally closed subset of X. Then $A = U \cap F$, where U is μ-α-open set and F is μ-α-closed set. Then by (1), F is $\mu_{\alpha\text{-}t}$-set and hence A is $\mu_{\alpha\text{-}B}$-set. □

Remark 3.23. *The condition $X \in \mu$ cannot dropped in Proposition 3.22(2) as shown by the following Example.*

Example 3.24. *Let $X = \{a, b, c, d\}$, if we take μ not containing X where $\mu = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}\}$, then $\alpha(\mu) = \{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$; $\mu_{\alpha\text{-}t}$-sets are ϕ, $\{b\}$, $\{c\}$, $\{d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$, $\{b, c, d\}$, $\{a, b, c\}$, X and $\mu_{\alpha\text{-}B}$-sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, b, c\}$. Clearly $\{b, d\}$ is a $\mu_{\alpha\text{-}t}$-set but not $\mu_{\alpha\text{-}B}$-set.*

Remark 3.25. *The union of two $\mu_{\alpha\text{-}t}$-sets need not be a $\mu_{\alpha\text{-}t}$-set. This can be shown by the following Example.*

Example 3.26. *Consider the GTS as in Example 3.6. Then $\mu_{\alpha\text{-}t}$-sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$. Then $\{a\}$ and $\{b\}$ are two $\mu_{\alpha\text{-}t}$-sets but their union $\{a, b\}$ is not a $\mu_{\alpha\text{-}t}$-set.*

Remark 3.27. *For a GTS (X, μ) the following properties hold.*

(1). μ-α-locally closed sets and $\mu_{\alpha\text{-}t}$-sets are independent;

(2). $\mu_{\alpha\text{-}B}$-set need not be a $\mu_{\alpha\text{-}t}$-set.

Example 3.28. *Let $X = \{a, b, c, d\}$ with $\mu = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}\}$. Then $\alpha(\mu) = \{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$; $\mu_{\alpha\text{-}t}$-sets are ϕ, $\{b\}$, $\{c\}$, $\{d\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$, $\{a, b, c\}$, $\{b, c, d\}\}$, X; μ-α-locally closed sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{a, b, c\}$ and $\mu_{\alpha\text{-}B}$-sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{a, b, c\}$.*

(1). $\{c, d\}$ is a $\mu_{\alpha\text{-}t}$-set which is not μ-α-locally closed set.

(2). $\{a\}$ is a μ-α-locally closed set which is not a $\mu_{\alpha\text{-}t}$-set.

(3). Let $A = \{a, b\}$. Then $i_\alpha(A) \neq i_\alpha(c_\alpha(A))$. Hence it is not $\mu_{\alpha\text{-}t}$-set but $A = A \cap X$ where X is $\mu_{\alpha\text{-}t}$-set and $A \in \alpha(\mu)$. So it is $\mu_{\alpha\text{-}B}$-set.

4. Decomposition of μ-α-continuity

Definition 4.1. *A function f: $(X, \mu) \to (Y, \lambda)$ is said to be $\mu_\alpha g$-continuous (resp. μ_α-lc-continuous) if $f^{-1}(F)$ is $\mu_\alpha g$-closed (resp. μ-α-locally closed) in (X, μ) for every λ-closed set F of (Y, λ).*

Theorem 4.2. *A function f: $(X, \mu) \to (Y, \lambda)$ is both $\mu_\alpha g$-continuous and μ_α-lc-continuous, then it is (α, λ)-continuous. The converse is true if $X \in \mu$.*

Proof. Let f be both $\mu_\alpha g$-continuous and μ_α-lc-continiuius and U be λ-open subset of (Y, λ), then $Y - U$ is λ-closed subset of Y. Since f is both $\mu_\alpha g$-continuous and μ_α-lc-continuous, then $f^{-1}(Y - U) = X - f^{-1}(U)$ is both $\mu_\alpha g$-closed and

μ-α-loccally closed in X, then by Theorem 3.15, $X - f^{-1}(U)$ is μ-α-closed in X. Hence $f^{-1}(U)$ is μ-α-open in X. Therfore f is (α, λ)-continuous.

Convesely let F be λ-closed subset of Y then $Y - F$ is λ-open in Y, since f is (α, λ)-continuous $f^{-1}(Y - F) = X - f^{-1}(F)$ is μ-α-open in X. Therefore $f^{-1}(F)$ is μ-α-closed in X, by Theorem 3.15 $f^{-1}(F)$ is both $\mu_\alpha g$-closed and μ-α-locally closed, if $X \in \mu$. Hence f is both $\mu_\alpha g$-continuous and μ_α-lc-continuous. □

Example 4.3. *Let $X = \{a, b, c\}$, $\mu = \{\phi, \{a\}, \{a, b\}, \{b, c\}, X\}$ and $\lambda = \{\phi, \{b,\}, \{b, c\}, Y\}$. Then the identity function f: (X, μ) $\to$ (Y, λ) is $\mu_\alpha g$-continuous but not μ_α-lc-continuous as $f^{-1}(\{a, c\}) = \{a, c\}$ is not μ-α-locally closed.*

Example 4.4. *Let $X = \{a, b, c\}$, $\mu = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, X\}$ and $\lambda = \{\phi, \{b, c\}, \{a, c\}, Y\}$. Then the identity function f: (X, μ) $\to$ (Y, λ) is μ_α-lc-continuous but not $\mu_\alpha g$-continuous as $f^{-1}(\{a\} = \{a\}$ is not $\mu_\alpha g$-closed in X*

Theorem 4.5. *A contra-(α, λ)-continuous function f: (X, μ) $\to$ (Y, λ) is (α, λ)-continuous if and only if it is $\mu_\alpha g$-continuous.*

Proof. Let f be both contra-(α, λ)-continuous and $\mu_\alpha g$-continuous. Let F be a λ-closed set in Y. Then by contra-(α, λ)-continuity of f, $f^{-1}(F)$ is μ-α-open in X, by Remark 3.2(1) it is μ-α-locally closed. Also since f is μ_αg-continuous, $f^{-1}(F)$ is $\mu_\alpha g$-closed. Thus by Theorem 3.15, $f^{-1}(F)$ is μ-α-closed. Hence f to be (α, λ)-continuous.

Converse part is obvious as every μ-α-closed set is $\mu_\alpha g$-closed set. □

Definition 4.6. *A mapping f: (X, μ) $\to$ (Y, λ) is said to be contra-$\mu_\alpha g$-continuous (resp. μ_α-contra-lc-continuous) if $f^{-1}(V)$ is $\mu_\alpha g$-closed (resp. μ-α-locally closed) in X for each λ-open set V of Y.*

Theorem 4.7. *If a mapping (X, μ) $\to$ (Y, λ) is μ_α-contra-lc-continuous and contra $\mu_\alpha g$-continuous, then it is contra-(α, λ)-continuous. The converse is true if $X \in \mu$.*

Proof. Follows from Theorem 3.15. □

Example 4.8. *Let $X = \{a, b, c\}$, $\mu = \{\phi, \{a, b\}, \{a, c\}, X\}$ and $\lambda = \{\phi, \{b\}, \{b, c\}, Y\}$. Then the identity function f: (X, μ) $\to$ (Y, λ) is contra-$\mu_\alpha g$-continuous but not contra-μ_α-lc-continuous as the inverse image of the λ-open set $\{b, c\}$ is not μ-α-locally closed.*

Example 4.9. *Let $X = \{a, b, c, d\}$, $\mu = \{\phi, \{d\}, \{a, b\}, \{a, b, d\}, X\}$ and $\lambda = \{\phi, \{a, b\}, \{a, c\}, \{a, b, c\}, Y\}$. Then the identity function f: (X, μ) $\to$ (Y, λ) is contra-μ_α-lc-continuous but not contra $\mu_\alpha g$-continuous as inverse image of the λ-open set $\{a, b\}$ is not $\mu_\alpha g$-closed.*

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